

# DC-DC Converters For Renewable Energy Sources Using Fourier Series Technique

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**In this system we use fourier series Techniques for modeling of power electronic DC-DC converters. The Renewable source output can be used and converted into efficient DC Source to Load. The conventional sub-circuit formed by the voltage source, switching transistor and diode is replaced by an independent square-pulse current source, while not making any changes to the rest of the circuit. Now the independent square pulse current wave is then approximated by Fourier series in terms of sinusoidal currents, thus making the system behavior linear. Next we derive the state-space representation in both the time domain and s-domain using state-space averaging techniques and numerically the validations of each of the DC-DC converter models have also been presented.**

*Keywords: Renewable energy sources, Power Electronics, DC-DC convertors, Fourier series, circuit modeling, linear systems*

## INTRODUCTION & BACKGROUND

In the modern power systems industry the extensive use of renewable into the system have demanded the need for conversion of electrical power from one domain to the other and this has brought the notion of power electronics. The power electronics equipments form the link between the power producer and the power consumer; and thus converters have to be viewed with their power sources interface and loads. Now as the power electronics industry progressed different challenges have come up so as to deliver an efficient, clean and high quality power to the consumer and thus there is need to understand the behavior of each of the power electronic devices in depth so as to improve the power quality that is being delivered to the consumer. The power electronic devices falls into various categories depending on their application such as DC-AC Inverters, DC-DC converters, Phase changer, Voltage booster etc.,. In this work we shall consider the DC- DC power converters and shall develop the models for such Convertors

DC converters must be viewed along with their power sources and loads; the power source for a DC power system

can be solar-cell, fuel-cell or any type of battery which can provide electrical power at a nominal voltage and current for a specified amount of time. A typical DC power system is shown in fig 1. It has a dc power source, DC power converter and a dc motor load[1]. In such a system sub-harmonics and electronic chaos occur during the system start-up, shut-down and load changes which can have damaging effects on the power source and DC power controller. Studies has revealed that there are specific switching frequency ranges where in we can have adhering effects on the power electronic equipments and also the sub-harmonics will be of very high amplitudes leading to undesirable transient responses.

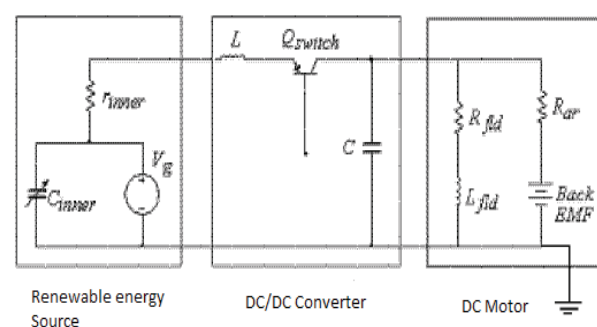


Fig.1: Equivalent circuit of a DC power system

## RENEWABLE ENERGY SOURCES

The world population in the year 2100 will be in excess of 12 billion. If the current trends in technological progress and innovation continue, the demand for energy then will be five times greater than what it is now. If we continue the policy of using coal, oil and gas at the present rate, then by the year 2020 the global temperature will have increased by two degrees Celsius. We do not need reminding of the adverse effects of this: the increased risk of flooding in lowland areas, the processes of desertification, and changing climate all over the world. It is a known fact that at the present moment renewable energy contributes only 11%-15% to our primary energy. If we intend to do something about our planet, to safeguard our future and to create a healthy environment for the generations to come, then we must all actively utilize renewable energy in our daily life. It is expected that 60% of all our energy will come from renewable energy in upcoming Years. The renewable sources like Solar cell, Piezo-electric and Chemical Fuel cell Energy are good source of DC source if we can improve the voltage magnitude either increasing or decreasing the level of voltage magnitude by reducing the DC transient and Harmonics. For this purpose a DC-DC converters with Fourier series techniques by which we can eliminate un-wanted harmonics.

## DC-DC CONVERTERS

These are power electronic devices which are used to change the DC electrical power efficiently from one voltage level to another. Generally speaking the use of a switch or switches for the purpose of power conversion can be regarded as an SMPS. A few applications of interest of DC-DC converters are where 5V DC on a personal computer motherboard must be stepped down to 3V, 2V or less for one of the latest CPU chips; where 1.5V from a single cell must be stepped up to 5V or more, to operate electronic circuitry. In these devices our main focus is to alter the dc energy from a particular level to other with minimum loss. The need for such a converter has risen due to the fact that transformers are unable to function on dc supply. It is worth mentioning to mention that a power converter does not manufacture power instead it transforms dc power from one voltage level to another. Thus the efficiency of a dc power converter cannot be equal to 100 and hence the input power is always greater than the output power.

There are three basic types of DC-DC converters which are studied in detail in this work. They are:

- DC Buck Converter
- DC Boost Converter
- DC Buck-Boost Converter

### a. DC Buck Converter

A Buck converter is normally a step-down DC-DC converter consisting mainly of inductor and two switches (usually a transistor switch and a diode) for controlling inductor. It fluctuates between connections of inductor to source voltage to accumulate energy and then discharges the inductor's energy to the load.

When the switch shown in Fig. 3 is closed (i.e. On-state), the voltage across the inductor is  $V_L = V_i - V_o$ . The current flowing through inductor linearly rises. The diode does not allow the current to flow through it, since it is reverse biased

For off case (i.e. when the switch is opened), diode is forward biased and voltage is  $V_L = -V_o$  (neglecting the drop across diode) across inductor. The inductor current which was rising in ON case now decreases by Voltage  $V_o$ .

Continuous current mode (CCM), in which the inductor current for the converter never reaches to zero.

Discontinuous current mode (DCM), in which the inductor current reaches to zero and remains there for certain period

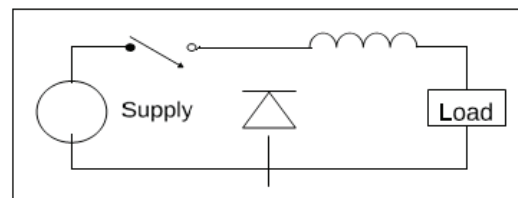


Fig 2: Buck Converter CCM mode

### B. Small-signal model of DC buck converter

Conventionally a converter circuit is modeled using a method of state-space average [3], where in the circuit is modelled by taking an average over a switching cycle, thus obtaining a linearized model over an operating point. In CCM, we have two modes of operation, thus two state-space representations are used in representing the system. The state-space averaging [4] corresponds to the state-space representations of the system; on mode and off mode for continuous current mode (CCM). The other techniques which involve the use of "injected/absorbed currents", treat the switch-diode combination in isolation from the circuit. In discontinuous current mode (DCM) the buck converter has three modes of operation, they are represented by three individual circuits and hence we have three different state-space representations. In the small-signal models of DC-

DC converters, the main focus is to determine the transfer function that corresponds to converter's operation. The buck converter shown in Fig.3 is considered as a nonlinear, time-dependent system, which we linearized through averaging method about a selected operating point, and with respect to the transistor's duty cycle  $k$ [5].

The procedure for state-space averaging can be mentioned point wise as follows:

1. Draw the linear switched circuit model for each state of the switching converter.
2. Write the state equations for each switched circuit model using Kirchoff's voltage and current laws.
3. Average the state-space equations using the duty cycle.
4. Perturb the averaged state equation to yield steady-state(DC) and dynamic(AC) terms and eliminate the product of any AC terms.
5. Transform the AC equations into s-domain to solve for transfer function.

The state-space representation of the system for buck converter's mode<sub>1</sub>, (when the transistor is on), is obtained as,

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + B_1 u(t) \\ y(t) &= C_1 x(t) + D_1 u(t) \end{aligned} \tag{1}$$

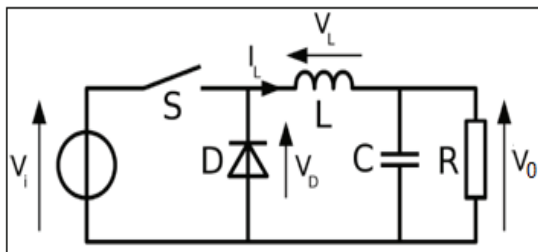


Fig. 3: Buck Converter CCM mode

The state-space variable vector, input vector, output vector and duty cycle respectively are given below as,

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} \\ \mathbf{u}(t) &= [ v_g(t) ] \\ y &= V_{out}(t) \\ k &= ton/T \end{aligned} \tag{2}$$

The corresponding state space matrices are

$$\begin{aligned} \mathbf{A}_1 &= \begin{bmatrix} 0 & -1/L \\ 1/C & -1/R_{load}C \end{bmatrix} \\ \mathbf{B}_1 &= \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \end{aligned}$$

$$C_1 = [0 \ 1] \quad D_1 = [0][0] \tag{3}$$

During off-time operation, the voltage across the diode is zero, so the state-space representation for mode 2 (when the transistor is off), is obtained from mode 1 state-space representation, by setting all the coefficients of  $v_g$  to zero

$$\begin{aligned} \dot{x}(t) &= A_2 x(t) + B_2 u(t) \\ y(t) &= C_2 x(t) + D_2 u(t) \end{aligned} \tag{4}$$

Where;  $A_2 = A_1$ ;  $C_2 = C_1$ ;  $D_2 = D_1$ ; and

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

Now the two linear systems are averaged with respect to their time span during the switching period [6]

$$\begin{aligned} \dot{x}(t) &= [kA_1 + (1-k)A_2]x(t) + [kB_1 + (1-k)B_2]u(t) \\ y(t) &= [kC_1 + (1-k)C_2]x(t) + [kD_1 + (1-k)D_2]u(t) \end{aligned} \tag{6}$$

The duty cycle for an operating point is defined as  $k = K$ , and  $k_{off} = 1 - K$

The averaged system at an operating point is described by

$$\begin{aligned} \dot{x} &= A.X + B.U \\ Y &= C.X + D.U \end{aligned} \tag{8}$$

$$\begin{aligned} A &= K.A_1 + k_{off}.A_2 \\ B &= K.B_1 + k_{off}.B_2 \\ C &= K.C_1 + k_{off}.C_2 \\ D &= K.D_1 + k_{off}.D_2 \end{aligned} \tag{9}$$

The state-space variable vector and state-space input vector for the operating point are:

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} \\ \mathbf{u}(t) &= [ v_g(t) ] \end{aligned}$$

Thus the equations corresponding the DC characteristics over an operating point are,

$$\begin{aligned} 0 &= (-V_c/L) + K(V_g/L) \\ 0 &= (I_L/C) - V_c/(R_{load}C) \\ Y &= V_{out} = V_c \end{aligned} \tag{10}$$

Therefore the voltage across capacitor C and the output voltage  $V_{out}$  are

$$\begin{aligned} V_c &= R_{load} \cdot I_L \\ V_{out} &= K \cdot V_g \end{aligned} \tag{11}$$

Next, the AC characteristics of the system about an operating point are governed by the following equations:

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) \\ y(t) &= C x(t) + D u(t) \end{aligned} \tag{12}$$

This system representation is an approximation of the time varying system, in which a input new variable is introduced, the duty cycle k. Thus the new input vector  $u(t)$  is defined as

$$u(t) = \begin{bmatrix} V_g(t) \\ k \end{bmatrix}$$

Defining the deviations from an operating point, a straight forward linearization is applied

$$\begin{aligned} x(t) &= X + x_o(t) \\ u(t) &= U + u_o(t) \\ y(t) &= Y + y_o(t) \end{aligned} \tag{13}$$

where the respective matrices are

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} I'_L(t) \\ V'_c(t) \end{bmatrix} \\ \mathbf{u}(t) &= \begin{bmatrix} V_g \\ k \end{bmatrix} + \begin{bmatrix} V'_g(t) \\ k' \end{bmatrix} \end{aligned}$$

$$y(t) = V_{out}(t) = V_{out} + V_{out}' \tag{14}$$

$$\begin{aligned} A' &= \left. \frac{\partial \mathcal{F}}{\partial \mathbf{x}} \right|_{\substack{x(t)=X \\ u'(t)=U'}} \quad \text{and} \quad B' = \left. \frac{\partial \mathcal{F}}{\partial \mathbf{u}'} \right|_{\substack{x(t)=X \\ u'(t)=U'}} \\ C' &= \left. \frac{\partial \mathcal{G}}{\partial \mathbf{x}} \right|_{\substack{x(t)=X \\ u'(t)=U'}} \quad \text{and} \quad D' = \left. \frac{\partial \mathcal{G}}{\partial \mathbf{u}'} \right|_{\substack{x(t)=X \\ u'(t)=U'}} \end{aligned}$$

$$\hat{v}_{out}(s) = \hat{v}_C(s) = \frac{(1+K) \cdot V_g}{s \cdot \left( s^2 + s \cdot \left( \frac{1}{R_{load}C} \right) + \frac{1}{LC} \right)}$$

The following variables are defined when the transistor Q is off

$$k = 1 - k \text{ and } K = 1 - K \tag{15}$$

Here the matrices  $A = A$  and  $B = [B \ K]$ ,  $C = C$  and  $D = [D \ K]$  The matrices  $B'$  and  $D'$  are given by,

$$B' = \begin{bmatrix} 1/L & V_g/L \\ 0 & 0 \end{bmatrix} \quad D' = [0 \ 0]$$

Expanding the system representation in the s-domain the following equations are derived:

$$\begin{aligned} x(s) &= ((s.I - A)^{-1} \cdot B) \cdot U(s) \\ y(s) &= C \cdot x(s) + D \cdot U(s) \end{aligned} \tag{16}$$

Here the input vector  $U'(s)$  is the laplace transform of time domain vector  $u'(t)$  and for constant duty cycle its expressions is:

$$U'(s) = \begin{bmatrix} V_g/s \\ K/s \end{bmatrix}$$

Expanding the s-domain and substituting for the variable vector in the second equation, the expression for AC component of the output voltage at a constant duty cycle is: The time-domain expressions of the output voltage AC component is found by the inverse Laplace transform of the above expression. Here all the three coefficients  $G_1, G_2,$  and  $G_3$  are dependent on the values of the converter elements:  $R_{load}, L,$  and  $C$  as well as the solutions of the Characteristic equation.

$$\hat{v}_{out}(s) = \frac{(1+K)V_g}{s \cdot (s+p_1) \cdot (s+p_2)} = \frac{G_1}{s} + \frac{G_2}{(s+p_1)} + \frac{G_3}{(s+p_2)}$$

$$\begin{aligned} v(t) &= C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_1 t + \varphi_n) \\ C_0 &= \frac{1}{T} \int_0^T v(t) dt \\ C_n &= \sqrt{A_n^2 + B_n^2} \\ A_n &= \frac{2}{T} \int_0^T v(t) \cos(n\omega_1 t) dt \\ B_n &= \frac{2}{T} \int_0^T v(t) \sin(n\omega_1 t) dt \\ \varphi_n &= \tan^{-1} \left( \frac{A_n}{B_n} \right) \end{aligned}$$

## FOURIER MODEL OF DC-DC CONVERTERS

Mathematical models of DC-DC converters; buck, boost, and buck-boost, are developed using the Fourier series decomposition method. Each type of DC-DC converter is mathematically reduced to a set of s-domain equations, from which their transfer functions can be derived. At steady-state operation, the Fourier series models are similar to the corresponding large-signal models and small-signal models. Unlike large-signal models and small-signal models, Fourier-series models developed can accurately describe the transient operation of a DC-DC converter, from the first duty cycle until the steady state operation. The DC-DC converter operation consists of sequential switching, generating voltage square waves in some point of the electrical circuit, which is a source of nonlinearity. Power electronics interfaced circuits; DC-DC converters in particular, are inherently nonlinear due to the sequential switching governing the power conversion process. The entire system being nonlinear, the concepts of stability, controllability, and observability, normally characterizing linear, time invariant systems, generally does not apply. However, a good approximation of the sequential switching is to consider any square wave as a sum of sinusoids according to Fourier series theory. Thus, a square wave, of any frequency and duty cycle, can be represented with great accuracy as a sum of sinusoids with variable amplitudes and distinct phase angles:

## FOURIER MODEL OF DC-BUCK CONVERTER

A DC-DC buck converter presented in Fig. 1 has a BJT switching transistor, regulating the flow of current through inductor L and load RC. Since switching transistor Q<sub>switch</sub> can be modeled as a controlled current source, presented in Fig.4(a), the buck converter can be represented as an independent DC-pulse current source, series connected with the energy storing inductor L, and the capacitor-load resistance sub-circuit RC, as presented in Fig. 4(b). The maximum current value of the current-source for the buck converter is determined through Ohm's law; voltage magnitude V<sub>g</sub> is divided by the total resistance of the circuit after the switching transistor has been turned on for a long time. This time ensures that capacitor C is completely charged, and inductor L behaves like a short circuit. Theoretically, no intrinsic resistance in DC mode is associated with either inductor L, capacitor C, voltage source V<sub>g</sub>, and switching transistor Q<sub>switch</sub>. However, real electronic components are characterized by internal resistances.

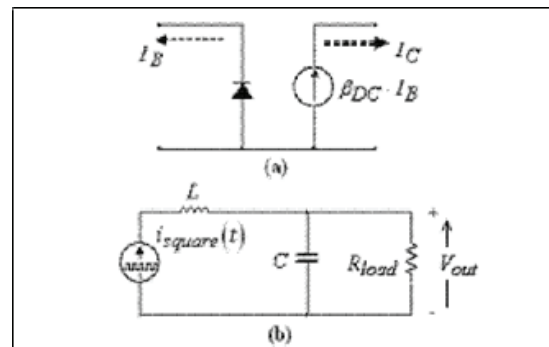


Fig. 4: DC Buck converter circuit topology

Those values have an effect on the current peak magnitude for the equivalent square wave representation approximated by pulse-current used in the buck converter model. Considering the DC bias value of the pulse-current, square wave source, and the first 21 harmonics, the square wave expression for the current pulse is mathematically approximated by the formula :

$$i_{square}(t) = C_0 + C_1 \cdot \sin(\omega_1 t + \phi_1) + C_2 \cdot \sin(\omega_2 t + \phi_2) + C_3 \cdot \sin(\omega_3 t + \phi_3) + \dots \tag{17}$$

By estimating the pulse voltage generated through sequential switching as a sum of sinusoidal waves (i.e., Fourier series representation), the entire system can be considered linear, time-invariant, thus it is possible to determine system's stability, controllability, and observability.

Considering a DC-DC buck converter with an input voltage of 20V and a required duty cycle of 0.2, the voltage applied to the right side of the circuit (right to the diode) has a waveform dictated by the transistor operation: on-off-on-off..., thus resembling a square-wave. The voltage source, switching transistor and the diode are replaced with a DC-bias current source C<sub>0</sub> connected in parallel with a number of sine-current sources (fundamental + harmonics). This is the Fourier series equivalent of a square wave with amplitude of 0.2A and duty cycle of 0.2 and is approximated to the 21st harmonic. The period of one cycle is 1ms, with amplitude.

$$C_0 = k \cdot (V_g = R_{load}) \\ C_0 = 0.2 : 0.2 = 0.04 \tag{18}$$

All the 21 harmonics are found employing the general formulas:

Now,

$$i_L(t) = i_{square}(t) \tag{19}$$

$$i_C(t) = i_{square}(t) - V_C \cdot (1/R_{load})$$

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$A_n = \frac{2}{T} \int_0^T 20 \cdot \cos\left(\frac{n \cdot 2\pi}{T} \cdot t\right) dt + 0 = \frac{20}{n\pi} \sin(k \cdot n \cdot 2\pi)$$

$$B_n = \frac{2}{T} \int_0^T 20 \cdot \sin\left(\frac{n \cdot 2\pi}{T} \cdot t\right) dt + 0 = \frac{20}{n\pi} [1 - \cos(k \cdot n \cdot 2\pi)]$$

$$\phi_n = \tan^{-1}\left(\frac{A_n}{B_n}\right)$$

Fig 5: Formulae to calculate Fourier coefficients

$$\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} 0 & (-1/L) \\ 1/C & (-1/R_{load}C) \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} R/L & Z_1/L & \dots & Z_n/L \\ 0 & 0 & \dots & 0 \end{bmatrix} \cdot [u(t)]$$

$$[v_{out}(t)] = [0 \ 1] \cdot \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + [0 \ 0 \ \dots \ 0] \cdot [u(t)]$$

Fig 6: State-space model of buck converter

The differential equations corresponding to the system operation are

$$\begin{aligned} \dot{i}_L(t) &= (Z/L) \cdot i_{square}(t) - (1/L)v_C(t) \\ \dot{v}_C(t) &= (1/C) \cdot i_L(t) - (1/RC)v_C(t) \end{aligned} \tag{20}$$

Therefore, the state space representation of the Fourier model of buck converter is given in Fig.6  
The input vector is:

$$u(t) = \begin{bmatrix} C_0 \\ C_1 \cdot \sin(\omega_1 t + \phi_1) \\ C_2 \cdot \sin(\omega_2 t + \phi_2) \\ \vdots \\ C_n \cdot \sin(\omega_n t + \phi_n) \end{bmatrix}$$

Elements Zn/L of matrix B are determined for each harmonic from the circuit topology. Now taking Laplace transform of the square wave pulsecurrent we get, Therefore the laplace of the state-space representation is,

$$\begin{aligned} s \cdot Xs &= A \cdot Xs + B \cdot Us \\ Ys &= C \cdot Xs + D \cdot Us \end{aligned} \tag{21}$$

$$I(s) = \frac{C_0}{s} + C_1 \cdot \frac{s \cdot (\sin(\phi_1)) + \omega_1 \cdot (\cos(\phi_1))}{s^2 + \omega_1^2} + C_2 \cdot \frac{s \cdot (\sin(\phi_2)) + \omega_2 \cdot (\cos(\phi_2))}{s^2 + \omega_2^2} + \dots + C_{21} \cdot \frac{s \cdot (\sin(\phi_{21})) + \omega_{12} \cdot (\cos(\phi_{21}))}{s^2 + \omega_{21}^2} = u(s) = U_s$$

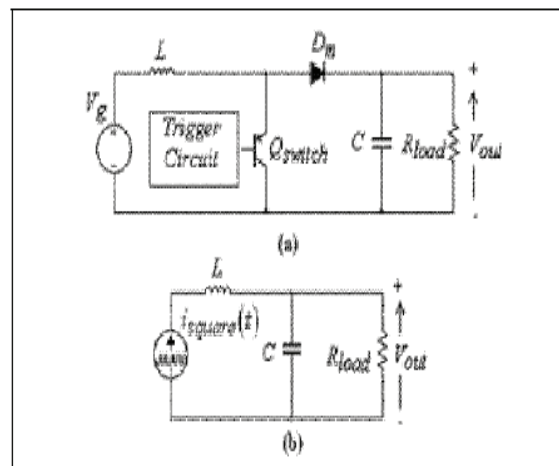


Fig 7: DC Boost converter circuit topology

$$\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} 0 & (-1/L) \\ 1/C & (-1/R_{load}C) \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} R/L & R/L & Z_1/L & \dots & Z_n/L \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \cdot [u(t)]$$

$$[v_{out}(t)] = [0 \ 1] \cdot \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + [0 \ 0 \ 0 \ \dots \ 0] \cdot [u(t)]$$

Fig 8: State-space model of boost converter

The set of equations are rewritten as:

$$\begin{aligned} Xs &= (sI - A)^{-1} \cdot B \cdot Us \\ Ys &= C \cdot Xs + D \cdot Us \end{aligned} \tag{22}$$

### FOURIER MODEL OF DC BOOST CONVERTER

A DC-DC boost converter, shown in Fig. 7(a), has a BJT switching transistor, regulating the flow of current through inductor L, switching transistor, and the voltage source during mode 1. During mode 2 supplementary current for load RC, is generated by the collapsing magnetic field of inductor L. The Fourier-series model for DC-DC boost converter is similar to the buck converter model, with the exception of an additional coefficient  $C_{DC}$  accounting for the continuous current flowing through the inductor L. The expression for the current source, of the Fourier series model of a boost converter is given by:

$$I_{square}(t) = C_0 + C_1 \cdot \sin(\omega_1 t + \phi_1) + C_2 \cdot \sin(\omega_2 t + \phi_2) + C_3 \cdot \sin(\omega_3 t + \phi_3) + \dots \tag{23}$$

The two time independent coefficients are obtained as,

$$C_{DC} = (Vg/R_{load}) \quad \text{and}$$

$$C_o = k.Vg=(r_g + r_L + r_Q) \quad (24)$$

Now the state-space representation for the boost converter is identical with that of the buck converter, only the input vector being  $n + 1$  rows, thus the matrix B has  $n + 1$  columns as shown in Fig.8

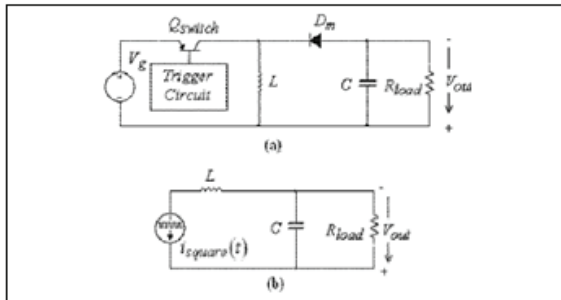


Fig. 9: DC Buck-Boost circuit topology

The input vector for the boost converter is:

$$u(t) = \begin{bmatrix} C_{DC} \\ C_0 \\ C_1 \cdot \sin(\omega_1 t + \phi_1) \\ C_2 \cdot \sin(\omega_2 t + \phi_2) \\ \vdots \\ C_n \cdot \sin(\omega_n t + \phi_n) \end{bmatrix}$$

### FOURIER MODEL OF DC BUCK-BOOST CONVERTER

Dc-DC buck converters combine the performances of both the buck converters and boost converters and they have the ability to vary the output voltage from zero to a maximum value. The Fourier-series model of buck-boost converter is a variation of Fourier-series model of the boost converter as shown below.

- 1.The output voltage polarity is opposite than the polarity of its input voltage, as shown in Fig. 8
- 2.The input current expression of the buck-boost converter has the DC coefficient equal to zero, thus its input current expression is analytically identical to that of the buck converter.

$$I_{square}(t) = C_0 + C_1 \cdot \sin(\omega_1 t + \phi_1) + C_2 \cdot \sin(\omega_2 t + \phi_2) + C_3 \cdot \sin(\omega_3 t + \phi_3) + \dots \quad (25)$$

$$C_o = k.Vg / (r_g + r_L + r_Q) \quad (26)$$

Thus the above modelling has revealed that the state-space representation for buck-boost converter is identical to the state space representation of the buck converter, provided the input coefficients are determined as mentioned above.

### NUMERICAL EVALUATION

This section provides the numerical validation of the three type of converters which have been modelled in the above analysis. Firstly we construct the square wave current pulse using the Fourier-series approximation and then we solve the individual state-space models for the converters in order to obtain the state variable plots and the output voltage plot

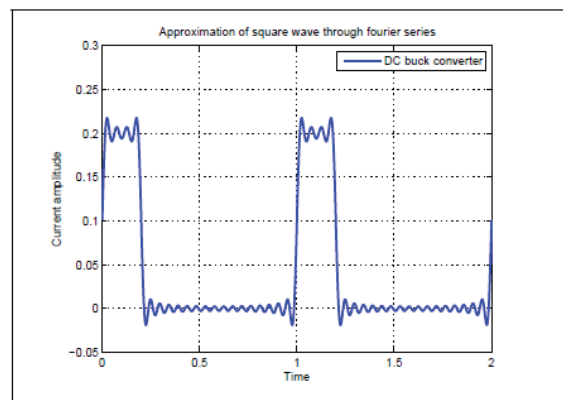


Fig. 10: Fourier-series representation of input current for buck converter

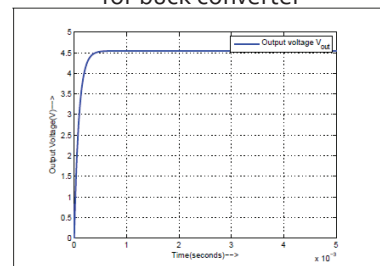


Fig. 11: Output voltage waveform of buck converter

#### A. DC-DC Buck Converter

A square wave current pulse of duty cycle  $k = 0.2$  is represented below. Now it can be represented as a sum of sinusoidal currents and a biased DC current, namely Fourier series.

The DC biased value is  $0.04A$ . The output voltage waveform is shown in Fig.11,

#### B. DC-DC Boost Converter

For a DC-DC boost converter we have a DC coefficient  $CDC$  and DC biased value along with the sinusoidal currents which constitute the fourier series.

The DC coefficient is  $CDC = 2A$  and DC biased value is  $0.1619A$ . The output voltage waveform is Fig.13,

### C. DC-DC Buck-Boost Converter

The square wave current pulse for buck-boost converter is similar to that of the buck converter except the DC bias has a value of  $0.1619A$ . The output voltage waveform is Fig.15,

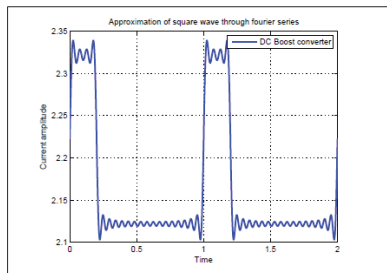


Fig. 12: Fourier-series representation of input current for boost converter

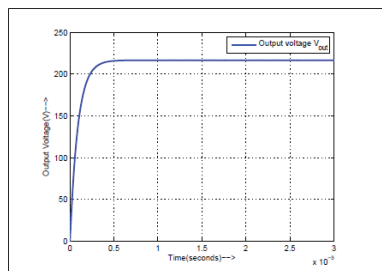


Fig. 13: Output voltage waveform of boost converter

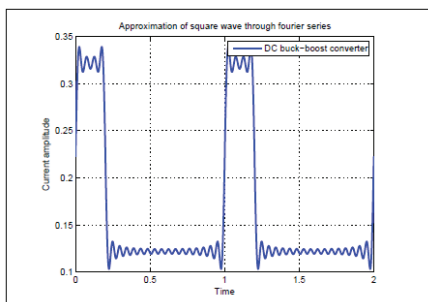


Fig. 14: Fourier-series representation of input current for buckboost Converter

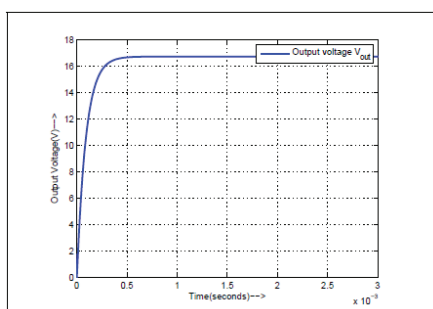


Fig. 15: Output voltage waveform of boost converter

## CONCLUSION

Similar to large-signal and small-signal models, Fourier series models of DC-DC converters are only an approximation of the physical circuit. Its accuracy is dependent on the number of harmonics considered for the input current-source, and on the accurate modelling of each element in the circuit, mainly circuit losses associated with each device. Small-signal models of DC-DC converters do not describe the transient mode of operation. Therefore, the Fourier-series model is a better method of analysis when transient operation is of particular interest in the design and analysis of DC power systems. Fuel-cell systems are characterized by variable cell capacitance, thus susceptible to sub-harmonic voltages and electronic chaos caused by the sequential switching within the DC converter. Those systems are accurately described using Fourier-series models. The Fourier-series approach can be extended to other power electronics circuits, where sequential switching modelling is required, and transient operation description is crucial.

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