ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

The Optimization of Two-Stage Planetary Gear Train Based on Genetic Algorithm

A. Sri Harsha
PG Scholar,
Dept. of Mechanical Engineering,
UCEK, JNTUK University,
Kakinada, E. G. Dt., AP, India.

Dr. K. Mallikarjuna Rao
Professor
Dept. of Mechanical Engineering,
JNTUK University,
Kakinada, E. G. Dt., A, India.

Abstract:
Planetary Gear Trains are extensively used for the power transmission. Large reduction in a small volume is possible with planetary gear trains. High efficiency – 95% per stage is common in planetary gear trains. The optimization of the planetary gear train makes the minimum volume (as well as minimum weight). In this paper a two stage planetary gear train is optimized using Genetic algorithm in Matlab used for analysis to satisfy the strength and geometric constraints. The mathematical model and source code are presented.

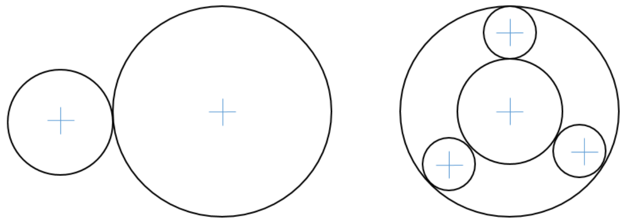
Keywords: ***Optimization; Planetary gear train; Genetic Algorithm; MatLab;***

I Introduction

Planetary gear train normally consists of a centrally mounted sun gear, ring gear and several planet gears that are found in between sun gear and ring gear. The planets are connected by a carrier.

The main advantage of planetary gear train is that it can maintain high torque density. This torque density can be increased by adding more number of planets thus making multiple mesh points. That means a planetary gear with three planet gears can transfer there times torque of a similar sized, fixed axis standard spur gear set.

The Load distribution on multiple gear mesh points implies that the load is supported by N contacts (Where N is the number of planets) increases the torsional stiffness of gearbox by N.



The optimization of multi stage planetary gear train is a complex problem as it involves design variables which are integer type and real type. So genetic algorithm is used to solve this optimization problem. This is an evolutionary optimization technique rather than a traditional optimization technique.

II Literature review

A literature review on planetary gear train design, optimization and analysis was carried out and a brief report of the same is presented as below:

Kalyanmoy Deb et. al. [1] optimized a multi-speed gearbox involving different types of decision variables and objective functions. They demonstrated the use of multi-objective evolutionary algorithm which is capable of solving the multi-objective optimization problem. Tianpei Chen et. al. [2] used differential evolution algorithm based on mathmatica to optimize a 2 stage planetary gear train. They have proved that the optimum design of the planetary gear train could make the volume minimum (as well as the weight) under the conditions of carrying capacity. JelenStefanovicet. al. [3] presented a paper on optimization possibilities of planetary gear trains. They has given original model original model for multi criteria optimization of planetary gear train. This mathematical model for

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

optimization is defined by the variables, objective functions and conditions required for the proper functioning of the planetary gear train. K Akhilaet. al. [4] designed modeled and analyzed a 3 stage planetary reduction gear unit which is being used for a flight vehicle. It is designed to meet the output specifications and modeled using CATIA to check the interference.

The modelled components are checked using ANSYS to check for their strength. Anjali Gupta et. al. [5] optimized a spur gear set using genetic algorithm.

They has taken central distance as a objective function and module, number of teeth on pinion are taken design parameters. The bending stress limit and surface stress limit are considered as constraints.

III Problem Formulation:

Given the total transmission ratio of the gear train is 56.46 and the input torque of the first stage is $T_1 = 5.69 \times 10^4$ N.mm and the input torque of the second stage $T_2 = 4.78 \times 10^5$ N.mm. The remaining parameters are

Stage	Type	Tooth Number	Module	Width
First Stage	Sun Gear	13	3.5	73
	Planetary Gear	43	3.5	73
	Annular Gear	99	3.5	73
Second stage	Sun Gear	18	5.5	139
	Planetary Gear	32	5.5	139
	Annular Gear	82	5.5	139
Total Volume (mm^3)		14415405.59		

2.1 Design Variables:

The design variables for optimizing the gear train are as follows

$$X = \{x1,x2,x3,x4,x5,x6,x7,x8,x9\} = \{t1,m1,b1,c1,r,t2,m2,b2,c2\}$$

Where t1 is the number of teeth on sun gear of the of the first stage and t2 number of teeth on the second stage respectively, m1 and m2 are the modules of first and second stages, b1 and b2 are the tooth width of the first and second stage, c1 and c2 are the modification coefficients of the sun gears in the two stages, r is the reduction ratio of the first stage.

2.2Objective function:

Here minimization of the volume is considered as the objective function. So the volume of a sun and volume of 3 planets of each stage of the gear train should be calculated. Minimizing the volume of the respective gears intern minimize the volume of them. So the objective function is

$$V = \frac{\pi}{4} * [(b1*(d_{s1}^2 + 3d_{p1}^2) + b2*(d_{s2}^2 + 3d_{p2}^2)] \quad \text{----} (1)$$

Here d_{s1} and d_{s2} are the addendum circle diameters of the sun gears and d_{p1} and d_{p2} are the addendum circle diameter of the planet gears in the both stages respectively.

The addendum circle diameter can be written in terms of design variables as

$$d_a = (t + 2(1+c))*m \quad \text{----} (2)$$

Where c is the addendum modification co-efficient of the gear.

And also the number of teeth on the planet gear can be written as

$$r = 2*(1 + \frac{t_p}{t_s})$$

$$t_p = \frac{1}{2} * (r - 2) * t_s \quad \text{-----} (3)$$

From (2) and (3) equation (1) can be written as

$$V = \frac{\pi}{4} *[b1*\{((t1+2*(1+c1))*m1)^2+3*((0.5*t1*(r-2) + 2*(1+c1))*m1)^2\} +b2*\{((t2+2*(1+c2))*m2)^2+3*((0.5*t2*(\frac{R}{r}-2) + 2*(1+c2))*m2)^2\}]$$

Where R = Overall gear train reduction ratio

2.3 Gear Design Parameters:

Design parameters and constants are necessary for the formulation of constraints. Here the gear material chosen is 20CrMnMo and heat treatment is quenching surface and its rigidity is HRC57-61. So the gear design parameters are taken form the mechanical design hand book and are listed below in the table1 and table 2 for 1st and 2nd stages:

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

Table 1. Parameters of 1st stage.

Parameter	Description	Value	Units
Z_{H1}	Node region coefficient	2.22	None
Z_{E1}	Elastic coefficient	189.98	
$Z_{\varepsilon 1}$	Contact ratio coefficient	0.95	None
K_1	Load Factor	2.89	None
Y_{Fa1}	Tooth form factor	2.29	None
Y_{Sa1}	Stress concentration coefficient	1.73	None
$Y_{\varepsilon 1}$	Contact ratio coefficient	1.12	None
σ_{c1}	Allowable contact stress	1033.41	MPa
σ_{b1}	Allowable bending stress	499.39	MPa

Table 2. Parameters of 2nd stage.

Parameter	Description	Value	Units
Z_{H2}	Node region coefficient	2.25	None
Z_{E2}	Elastic coefficient	189.98	
$Z_{\varepsilon 2}$	Contact ratio coefficient	0.94	None
K_2	Load Factor	2.95	None
Y_{Fa2}	Tooth form factor	2.32	None
Y_{Sa2}	Stress concentration coefficient	1.73	None
$Y_{\varepsilon 2}$	Contact ratio coefficient	1.08	None
σ_{c2}	Allowable contact stress	1104.47	MPa
σ_{b2}	Allowable bending stress	521.53	MPa

2.4 Constraints:

(a) Contact stress condition:

$$\sigma_1 = Z_{H1} * Z_{E1} * Z_{\varepsilon 1} * \sqrt{\frac{2*K1*T1*(r1+1)}{b1*t1^2*m1^2*r1}} \leq \sigma_{c1}$$

$$\sigma_2 = Z_{H2} * Z_{E2} * Z_{\varepsilon 2} * \sqrt{\frac{5.78*T2*(r2+1)}{b2*t2^2*m2^2*r2}} \leq \sigma_{c2}$$

Where: T1 and T2 are the input torques on the first and second stages, r1 and r2 are the ratio of tooth number respectively of the both stages.

(b) Bending stress conditions:

$$\sigma_3 = Y_{Fa1} * Y_{Sa1} * Y_{\varepsilon 1} * \frac{2*k1*T1}{b1*t1*m1^2} \leq \sigma_{b1}$$

$$\sigma_4 = Y_{Fa1} * Y_{Sa1} * Y_{\varepsilon 1} * \frac{2*k2*T2}{b2*t2*m2^2} \leq \sigma_{b2}$$

(c) Adjacent conditions:

$$d_{p1} \leq 2 * a_{12} \sin 60^0$$

$$d_{p2} \leq 2 * a'_{12} \sin 60^0$$

Where d_{p1} and d_{p2} are the addendum circle diameter of the planet gears and a_{12} and a'_{12} are the center distance of the gear wheels that are sun and planet gears in the both stages.

(d) Tooth width conditions

$$0.6*t1 \leq \frac{b1}{m1} \leq 1.3*t1$$

$$0.6*t2 \leq \frac{b2}{m2} \leq 1.3*t2$$

2.5 The bounds of the Design variables:

Design Variable	t1	m1	b1	c1	r	t2	m2	b2	c2
Lower bound	13	2	55	4/17	5	13	2	110	4/17
Upper bound	50	9	61	1	10	50	9	140	1

MatLab Source code:

For objective function

```
function y = volume(x)
y =
((3.14285714286/4)*(((x(3))*(((((x(1))+(2*(1+x(4))))*(x(2)))^2)+(3*((((0.5*x(1)*(x(5)-2))+(2*(1+x(4))))*(x(2)))^2))))+((x(8))*(((((x(6))+(2*(1+x(9))))*(x(7)))^2)+((3)*((((0.5*x(6)*((55.46/x(5))-2))+(2*(1+x(9))))*(x(7)))^2))))));
end
```

For Constraints

```
function [c,ceq] = constraint(x)
c(1) = ((0.6 * x(1) * x(2)) - x(3));
c(2) = ((x(3)) - (1.3 * x(1) * x(2)));
c(3) = ((0.6 * x(6) * x(7)) - x(8));
c(4) = ((x(8)) - (1.3 * x(6) * x(7)));
c(5) = ((x(5) * 5.69 * 10000)/(x(3) * x(1) * x(1) * x(2) * x(2) * (x(5) - 2))) - 1.15 ;
c(6) = ((55.46 * 4.78 * 100000)/(x(8) * x(6) * x(6) * x(7) * x(7) * (55.46 - (2 * x(5))))) - 1.28;
c(7) = (5.69 * 10000/(x(1) * x(2) * x(2) * x(3))) - 19.4420;
c(8) = (4.78 * 100000/(x(6) * x(7) * x(7) * x(8))) - 20.4021;
c(9) = (((0.5 * (x(5) - 2) * x(1)) + (2 * (1 + x(4))) ) * x(2)) - (0.866 * x(1) * x(5) * x(2));
c(10) = (((0.5 * ((55.46/x(5)) - 2) * x(6)) + (2 * (1 + x(9))) ) * x(7)) - (0.866 * x(6) * (55.46/x(5)) * x(7));
ceq(1) =x(10);
end
```

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

IV Results and Analysis:

After optimization the following results are obtained for the design parameters:

Stage	Type	Tooth Number	Module	Width
First Stage	Sun Gear	13	3	55
	Planetary Gear	52	3	55
	Annular Gear	117	3	55
Second stage	Sun Gear	14	6	110
	Planetary Gear	25	6	110
	Annular Gear	64	6	110
Total Volume (mm^3)		12044594.338		

Percentage reduction when compared to the given gear train

$$= \frac{14415405.59 - 12044594.338}{14415405.59} \text{ x } 100$$

$$= 16\%$$

The same input data and constraints were taken and solved in "The optimization of Two-Stage Planetary gear train based on Mathmatica" [2] and the result obtain is 13494224.3 mm^3. When compared to the solution what obtained from GA is much efficient. The amount of volume reduced by solving with GA rather than the differential evolution algorithm is

$$= \frac{13494224.3 - 12044594.338}{13494224.3} \text{ x } 100$$

$$= 10\%$$

References:

[1] Kalyanmoy Deb and Sachin Jain, "Multi-Speed Gearbox Design Using Multi-Objective Evolutionary Algorithms" Kanpur genetic algorithms laboratory (KanGAL), Report No.2002001

[2] Tianpei Chen, Zhengyan Zhang, Dingfang Chen and Yongzhi Li, "The Optimization of Two-Stage Planetary Gear Train Based on Mathmatica", pp. 122-136, 2013, Springer-Verlag Berlin Heidelberg 2013

[3] JelenaStefanovic-Marinovic and Milos Milovancevic, "The Optimization Possibilities at the Planetary Gear Train", Journal of Mechanics Engineering and Automation 2 (2012) 365-373.

[4] K Akhila and M Amarnath Reddy, "Design, Modelling and Analysis of a 3 Stage Planetary Reduction Gear Unit of A Flight Vehicle" International Journal of Mechanical Engineering and Robotics Research ISSN 2278-0149

[5] Anjali Gupta, "Design Optimization of the Spur Gear set" International Journal of Engineering Research & Technology (IJERT) ISSN: 2278-0181 Vol. 3 Issue 9, September 2014