ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

Computation of Stabilizing Region Set of Controllers and Design of PI, PID Controllers for Interval Plants

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ABSTRACT

This paper considers first to find out stabilizing region of PI parameters for the control of plant with uncertain parameters.This method presented to its require sweeping over the parameters are required to find stabilizing set of PID controller using the Signature method of synthesis.also,it is need to solve the liner programming of set of inequalities.also,in this paper using results to find the area of parametric robust control and problems of design of stabilization interval plants using PI,PID controllers by kharitonov segments of polynomial theory. some illustrative examples are included to describes the method of approaches and simulation results of kharitonov (interval plants) plants.

Key Words:* *Robust stabilization, linear systems ,PI,PID control, signature method, Boundary locus method, kharitonov plants

1. INTRODUCTION

The set of controllers of a given structure that stabilizes the closed loop is of fundamental importance since every design must belong to the set and any performance specifications that are imposed must be achieved over this set. So, this set is known as Stability Set denoted by

$$\mathbf{S^0}:(\boldsymbol{\delta}(\mathbf{S}, K_p\ K_i\ K_d))$$

The three dimensional set S^o is simply described but not necessarily simple to calculate.

This is new method for the calculation of all stabilizing PI controllers is given. The Basically we use Routh-Hurwitz criterion which will be very difficult due to formation of inequalities. Therefore to simplify the process we use a revolutionary method known as Signature Method. Proposed method is based on plotting the stability boundary locus in the (kp, Ki)-plane and then computing the stabilizing values of the parameters of a PI, PID controller. The technique presented to require sweeping over the parameters and also it's need linear programming to solve a set of inequalities. Thus it offers several important advantages over existing results obtained in this direction. Beyond stabilization, the method is used to shift all poles to a shifted half plane that guarantees a specified settling time of response [1].

Computation of stabilizing set of PI, PID controllers which achieve user specified intervals of plants by using kharitonov polynomial theory is studied. It is shown via an example that the stabilizing region in the (kp, ki)-plane is always a convex set. The proposed method is also used to design of PI,PID controllers for interval plants. The limiting values of a PID controller which stabilize a given system are obtained in the convex set of(kp, ki)-plane, and (ki, kd)plane and 3-D view of stabilizing sets of (kp, ki, kd) observed in the simulation results and. Furthermore, the proposed method is used to compute all the parameters of a PI controller which stabilize a control system with an interval plant family.

3. COMPUTATION OF THE PID STABILIZING SET:

Cite this article as: Ch.V.V.S. Bhaskara Reddy, B.Amarendra Reddy & V.Siva Chaitanya Kumar, "Computation of Stabilizing Region Set of Controllers and Design of PI, PID Controllers for Interval Plants", International Journal & Magazine of Engineering, Technology, Management and Research, Volume 6 Issue 8, 2019, Page 72-82.

ISSN No: 2348-4845
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Consider the plant, with rational transfer function

$$P(s)=\frac{N(S)}{D(S)};$$

With the PID feedback controller

$C(s) = k_p s+k_i+k_d s^2/s(1+sT)$,   T>0 ……….(2)

The closed loop characteristics polynomial is

$\delta(s)=S*D(s)(1+sT)+(k_p s+k_i+k_d s^2)*N(s)$ …….(3)

we form the new polynomial $v(s) =\delta(s)*N(-s)$…..(4)

note that the even odd decomposition of v(s) is of the form $v(s)=v_{even}(s^2(k_i, k_d))+S*v_{odd}(s^2(k_p))$

the polynomial v(s) exhibits the parameter separation property, namely, that k_p appears only in the odd part and (k_i, k_d) only in the even part. By sweeping over the values of (kp, ki) from stable boundary locus (kp vs ki) plane space region. after fixing range of $K_p = K_p^*$,There exists sets of linear inequalities in terms of (K_i, K_d) to satisfying the signature condition.

$$\text{Signature}(V) = n-m+1+2Z^+ ;$$

This will facilitate the computation of the stabilizing set using signature concepts [2].

N(s),D(s) are the numerator, denominator of polynomial degrees 'm', 'n' of Plant P(s) respectively.

The closed loop system is stable if and only if, $\sigma(v)= n-m+2+2z^+$ …………. (5)

closed loop stability is equivalent to the requirement that the n+2 zeros of $\delta(s)$ lie in the open LHP .

this is equivalent to $\sigma(\delta)=n+2$

and to $\sigma(v)=n+2+z^+-z^-$

$n+2+z^+-(m-z^+) =(n-m)+2+2z^+$

z^-, z^+ are denote the no. of roots on the S-plane LHP, RHP of numerator N(s).

$$\text{Sgn}[\ q(w(0^+), K_p\ )] = j;$$

$$j = \text{Sgn}[\ V_{odd}(0^+ ,\ K_p) ;\ \ldots\ldots(6)$$

$$n-m+1+2Z^+ = j(i_0 + 2\sum_{t=1}^{l-1}(-1)^t i_t) \text{ ,if } n + m \text{ is odd,} \quad \ldots\ldots..(7)$$

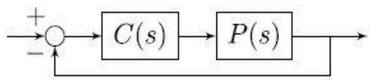
$$= \mathbf{j}(i_0 + 2\sum_{t=1}^{l-1}(-1)^t i_t + (-1)^l\ i_l), \text{ if } n + m \text{ is even} \quad \ldots\ldots.(8)$$

Based on this, we can develop the following procedure to calculate S^0:

the stabilizing set S^0:($\delta(s, k_p, k_i, k_d)$) getting from 3-D graph results.

4. PROCEDURE TO DO SIGNATURE METHOD:

consider unity feedback loop with PID controller c(s) and plant P(s)



$$P(s)=\frac{N(S)}{D(S)};\quad C(s)= K_p + K_d S+\frac{k_i}{S};$$

N(s), D(s) are the numerator, denominator of polynomial degrees 'm', 'n' of Plant P(s) respectively.

closed loop characteristic equation of polynomial is

$$\delta(s, k_p, k_i, k_d) = (1+[P(s)*C(s)])$$

$$\delta(s, k_p, k_i, k_d) = 1+[K_p + K_d S+\frac{k_i}{S}][\frac{N(S)}{D(S)}]$$

$$\delta(s, k_p, k_i, k_d) = S\ D(s) + [K_i + K_d\ s^2]N(s) + [K_p\ S][N(s)].\ \ldots\ldots\ (9)$$

we Assume that N(s) and D(s) are co-prime, that is, they have no common roots and $N(0) \neq 0$.

closed loop characteristic equation of polynomial is

$$\delta(s, k_p, k_i, k_d) = S\ D(s) +[K_p S+K_i+K_d\ s^2]N(s).$$

Assume N(s) has no roots on the imaginary axis.

$v(s) := \delta(S, k_p, k_i, k_d)N(-s).$

The main motto of the V(s) is the achieve a separation of the gains into the Real and Imaginary parts and also the divide into the Even and odd parts of S.

normally S= j w into the V(S) and then divides into k_p, k_i, k_d into the real and imaginary parts and also the Even and odd parts to be divided.

If $\delta(s, k_p, k_i, k_d)$ is multiplied with the N(-s) ,then only divided properly even part section consists k_i, k_d and odd part section includes K_p, otherwise both are not separated. so that

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Even part of V(w) is(k_i, k_d) and odd part is K_p

$$V(w) = P(w) + j\,q(w);$$

V(w) = Even part + odd part (or) real part +imaginary part;

For PI controller (using stability boundary locus method)

Even part and odd parts are equal to zero, then

$$V(w) = [P_1(w)+K_iP_2(w)] + j[\,q_1(w)+ K_p\,q_2(w)].$$

$$K_i = -\frac{P1(w)}{P2(w)};\quad K_p = -\frac{q1(w)}{q2(w)};\ \ldots.(9)$$

For PID controller:

$$V(w)= [P_1(w)+ \{k_i - k_d w^2\}P_2(w)]+ j\,\{q_1(w)+ k_p\,q_2(w)\}\ ;\ \ldots.(10)$$

In v(s), k_p only appears in the odd degree terms of **S**, while k_i and k_d only appears in the even degree terms of **S.** Now equate the odd degree of or imaginary part of S is equal to zero and odd part of k_p terms equal to zero and then by using the (i) R-H criteria, Basically we use Routh-Hurwitz criterion which will be very difficult due to formation of inequalities. Therefore to simplify the process we use a revolutionary method known as Signature Method [3].

(ii) stability boundary locus method or above PI controller technique ,To find out the range of k_p.

for fixed range of $k_p = K_p^*$

There exists sets of linear inequalities in terms of (k_i, k_d) to satisfying the signature condition.

$$\text{Signature}(V) = n-m+1+2Z^+\ ;$$

z^-, z^+ are denote the no. of roots on the S-plane LHP, RHP of numerator N(s).

The range of K_p such that q(w) is the odd part of V(w) and roots of q(w) consider only real and positive roots(w_0,w_1,w_2,w_3..),distinct, finite zeros with odd multiplicity was determined by K_p range.

$$\text{sgn}[\,q(w(0^+), K_p\,)] = j;$$

$$j = \text{sgn}[\,V_{odd}(0^+, K_p)\,];$$

$$I_1 = \{i_0, i_1, i_2, i_3....\};\ I_2= \{i_0, i_1, i_2, i_3....\};\ I_3 = \{i_0, i_1, i_2, i_3....\}.$$

I_1,I_2, I_3 ......are the admissible string sets and must satisfies the signature of V.

$$\sigma(v)\text{-signature } (v)= n +1- m +2z^+.$$

$$n-m+1+2Z^+ = j\left(i_0 + 2\sum_{t=1}^{l-1}(-1)^t i_t\right) \text{,if } n + m \text{ is odd,}$$

$$= j\left(i_0 + 2\sum_{t=1}^{l-1}(-1)^t i_t + (-1)^l\, i_l\right)\text{, if } n + m \text{ is even .}$$

by sweeping over K_p values into the fixed range and then string sets follows that the stabilizing (k_i, k_d) must satisfies the string of inequalities:

$$p1(w_0) +(k_i - k_d\, w_0^2)P_2(w_0) < 0$$

$$p1(w_1) +(k_i - k_d\, w_1^2)P_2(w_1) > 0$$

$$p1(w_2) +(k_i - k_d\, w_2^2)P_2(w_2) > 0$$

Substituting for w_0,w_1,w_2,w_3 in the above expressions, we obtain set of values of (k_i, k_d) form of equations solved by linear programming and denoted by sets S1,S2,S3,S4,S5….S_X.

by sweeping over different K_p values within the interval and repeating above procedure at each stage, we can generate the set of stabilizing (k_p, k_i, k_d) values.

5. Stabilization of interval plants:

First order compensator robustly stabilizes an interval plant family if and only if it stabilizes all of the extreme plants. That is, if the plant is described by a M^{th} order numerator and a N^{th} order denominator with coefficients lying in prescribed intervals, it is necessary and sufficient to stabilize the set of 2^{m+n+1} extreme plants. These extreme plants are by considering all possible combinations for the extreme values of the numerator and denominator coefficients. It is necessary and

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
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sufficient to stabilize only sixteen of the extreme plants. These sixteen plants are generated using the

Kharitonov polynomials associated with the numerator and denominator. Furthermore, when additional a prior information about the compensator is specified (sign of the gain and signs and relative magnitudes of the pole zero), then in some cases, it is necessary and sufficient to stabilize eight critical plants while in other cases, it is necessary and sufficient to stabilize twelve, sixteen, thirty-two critical plants and so on [4]

Consider a SISO interval plant described using

$$G(s)=\frac{N(s)}{D(s)}=\frac{q_m s^m+q_{m-1}s^{m-1}+\ldots+q_0}{p_n s^n+p_{n-1}s^{n-1}+\ldots+p_0} \quad \ldots\ldots\ldots$$

(11)

Where the numerator and denominator coefficients are defined $q_i \in [\underline{q_i}, \overline{q_i}], i=0,1,\ldots,m$

and $p_i \in [\underline{p_i}, \overline{p_i}], j=0,1,\ldots,n.$

Let the Kharitonov polynomials associated with N(s) and D(s) be, respectively:

$$N_1(s)=\underline{q_0}+\underline{q_1}s+\overline{q_2}s^2+\overline{q_3}s^3+\ldots \quad N_2(s)=\underline{q_0}+\overline{q_1}s+\overline{q_2}s^2+\underline{q_3}s^3+\ldots$$

$$N_3(s)=\overline{q_0}+\underline{q_1}s+\underline{q_2}s^2+\overline{q_3}s^3+\ldots \quad N_4(s)=\overline{q_0}+\overline{q_1}s+\underline{q_2}s^2+\underline{q_3}s^3+\ldots$$

and

$$D_1(s)=\underline{p_0}+\underline{p_1}s+\overline{p_2}s^2+\overline{p_3}s^3+\ldots \quad D_2(s)=\underline{p_0}+\overline{p_1}s+\overline{p_2}s^2+\underline{p_3}s^3+\ldots$$

$$D_3(s)=\overline{p_0}+\underline{p_1}s+\underline{p_2}s^2+\overline{p_3}s^3+\ldots \quad D_4(s)=\overline{p_0}+\overline{p_1}s+\underline{p_2}s^2+\underline{p_3}s^3+\ldots$$

By taking all combinations of the $N_i(s)$ and $D_j(s)$ for $i, j=1,2,3,4,$ the sixteen Kharitonov plants family can be obtained.

Define the set $S(C(s)G(s))$ that contains all the values of the parameters of the controller $C(s)$ that stabilize $G(s)$, and then, the set of all the stabilizing values of the parameters of a PI controller that stabilize the interval plant of Eq.(1) can be written as

$$S(C(s)G(s))=S(C(s)G_K(s))=$$

$$S(C(s)G_{11}(s))\cap S(C(s)G_{12}(s))\ldots S(C(s)G_{44}(s))$$

$G_{ij}(s)=\dfrac{N_i(s)}{D_j(s)}$ where $G_{ij}(s)$ represents the sixteen Kharitonov plants family.

6. ILLUSTRATIVE EXAMPLES:

(1).Example for Signature Method

Design the problem of determining stabilizing set of PID gains for the plant $P(s)=\dfrac{N(S)}{D(S)}$;

where $N(s) = s^3-2s^2-s-1$;$D(s) = s^6+2s^5+32s^4+26s^3+65s^2-8s+1$

We use the PID controller with T=0. The closed loop characteristic polynomial is

δ (s, kp, ki, kd) = $s*D(s)+(k_i+k_ds^2)\,N(s)+K_P*S*N(s)$

Here n=6 and m=3

$N_{even}(S^2) = -2S^2 -1$, $N_{odd}(S^2) = S^2 -1$, $D_{even}(S^2) = S^6 + 32S^4+65S^2+1$, $D_{Odd}(S^2) = 2S^4+26S^2-8$

$N(-s) = (-2s^2-1)-s(s^2-1)$

Therefore, we obtain

V(S) = δ (s, k_p, k_I, k_d) N(-s)

$= \{S^2(-S^8 - 35S^6-87S^4+54S^2+9) +(k_i+k_ds^2)\ (-S^6+6S^4+3S^2+1)\}$

$+S*[(-4S^8- 89S^6- 128S^4 - 75S^2 -1) +k_p*(-S^6 + 6S^4 +3S^2 +1)]$

So that

$V(j\omega, kp, ki, kd) = [p_1(w)+(k_i-k_d\omega^2)\ p_2(\omega)] +j[q_1(\omega)+k_pq_2(\omega)]$;

To get the results, we need to separate the even and odd parts equal to zero

For PI CONTROLLER $K_i = -\dfrac{P1(w)}{P2(w)}$; $K_p = -\dfrac{q1(w)}{q2(w)}$;

For PID CONTROLLER:

$V(w)= [P_1(w)+ \{K_i - K_d\ w^2\}P_2(w)]+ j\ \{q_1(w)+ K_p\ q_2(w)\}$;

Where

$P_1(w) = w^{10} - 35w^8 +87 w^6 +54 w^4 -9w^2$

$P_2(w) = w^6+6w^4-3w^2+1$

$q_1(w) = -4w^9+89w^7-128w^5+75w^3-w$

$q_2(w) = w^7+ 6w^5 - 3w^3 +w$

We find that $z^+ = 1$ so that the signature requirement on v(s) for stability is, $\sigma(v) = n-m+1+2z^+= 6$

Since the degree of v(s) is even, we see from the signature formulas that q(ω) must have at least two positive real roots of odd multiplicity. The range of k_p such that q (ω, kp) has at least 2 real, positive, distinct, finite zeros with odd multiplicities was determined to be

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
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(-24.7513, 1) which is the allowable range of k_p. For a fixed $k_p \epsilon$ (-24.7513,1), for instance k_p= -18, we have

$q(\omega, -18) = q_1(\omega)-18q_2(\omega)$

$=-4\omega^9+71\omega^7-236\omega^5+129\omega^3-19\omega$

Then the real, nonnegative, distinct finite zeros of q (ω, -18) with odd multiplicities are

ω_0=o, ω_1=0.5195, ω_2=0.6055, ω_3=1.8804,ω_4=3.6648

Also define $\omega_5=\infty$.since

Sgn [q (0, -18)] = -1

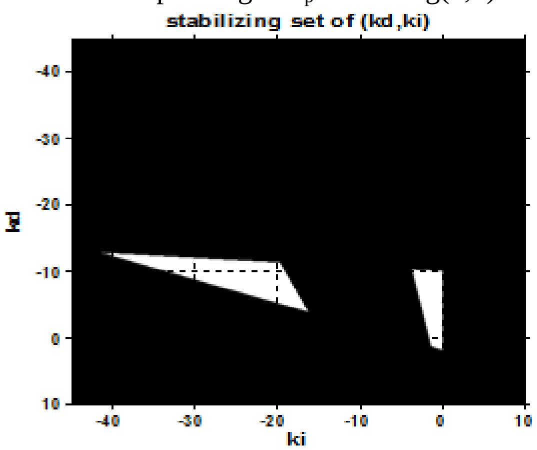
It follows admissible string I= $\{i_0,i_1,i_2,i_3,i_4,i_5\}$ Must satisfy

$\{i_0-2i_1+2i_2-2i_3+2i_4-i_5\}. (-1) =6$

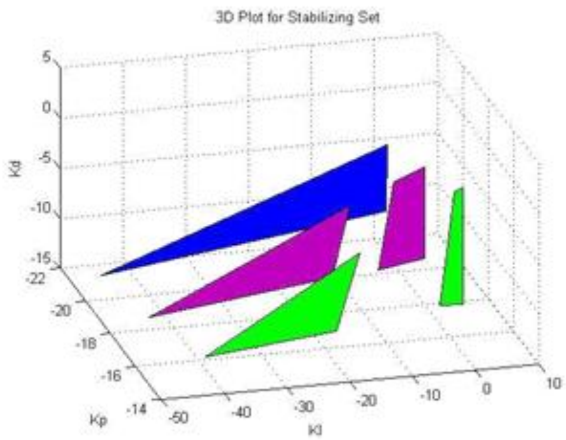
Hence the admissible strings are

I_1= {-1, -1, -1, 1, -1, 1};I_2= {-1, 1, 1, 1, -1, 1};I_3= {-1, 1, -1, -1, -1, 1};I_4= {-1, 1, -1, 1, 1, 1}

I_5= {1, 1, -1, 1, -1, -1}; For I_1 it follows that stabilizing set () values corresponding to k_p=-18 in fig(1,2)



fig(1) 2-D view of stabilizing set of Ki vs Kd



fig(2) fixed Kp value for varying set of (Ki,Kd)

Must satisfy string k_i, kd of inequalities

$P_1(\omega_0) + (k_i-k_d\omega_0^2)\ p_2(\omega_0) <0$

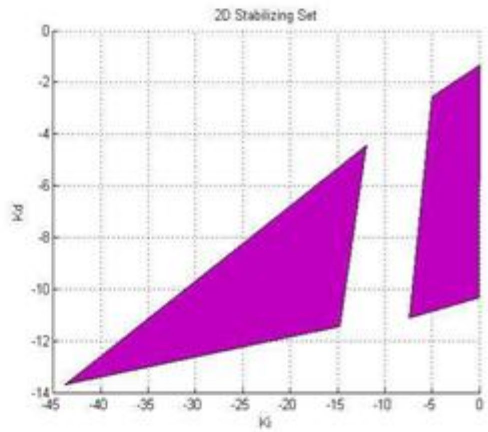
$P_1(\omega_1) + (k_i-k_d\omega_1^2)\ p_2(\omega_0) <0$

$P_1(\omega_2) + (k_i-k_d\omega_2^2)\ p_2(\omega_0) <0$

$P_1(\omega_3) + (k_i-k_d\omega_3^2)\ p_2(\omega_0)>0$

$P_1(\omega_4) + (k_i-k_d\omega_4^2)\ p_2(\omega_0) <0$

$P_1(\omega_5) + (k_i-k_d\omega_5^2)\ p_2(\omega_0)>0$



fig(3) 3-D View of stabilizing set

Substituting forω_0, ω_1, ω_2, ω_3, ω_4 and ω_5 in the above expressions, we obtain

$k_i<0$

$k_i-0.2699k_d<-4.6836$

$k_i-0.3666k_d<-10.0797$

$k_i-3.5358k_d>3.912$

$k_i-13.5777k_d<140.2055$

the set values of (k_i, kd) for which the above equations hold can be solved by linear programming observed in fig(3) and is denoted by S_1.for I_2, we have

$k_i<0$

$k_i-0.2699k_d<-4.6836$

$k_i-0.3666k_d<-10.0797$

$k_i-3.5358k_d>3.912$

$k_i-13.5777k_d<140.2055$

The set values of (k_i, kd) for which the above equations hold can be solved by linear programming and is denoted by S_2. similarly, we obtain

$S_3 = \varphi$ for I_3

$S_4 = \varphi$ for I_4

$S_5 = \varphi$ for I_5

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
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Then the stabilizing set of (k_i, kd) values when k_p=-18 is given by$S_{(-18)} = \bigcup_{x=1,2\ldots3} \ldots s_x$
$= s_1 \bigcup s_2$
From the intersections of plots in space that's 3-dimensional plot fig(3) in which we can select vertex of stabilizing set (K_p, Ki, Kd) values which satisfies the stabilizing criterion.

Example(2).
Design the problem of determining stabilizing set of 3rd order system PID gains using Signature method for the plant

$$P(s)= \frac{N(S)}{D(S)};$$

where $$G(s) = \frac{1.151s + 0.174}{s^3 + 0.739s^2 + 0.921s}$$

$N(S) = 1.151S + 0.1734$; $D(S) = S^3 + 0.739 S^2 + 0.921S$
We use the PID controller with T=0. The closed loop characteristic polynomial is
$\delta(s,k_p,k_i,k_d) = S*D(s)+(k_i+k_d s^2)N(s)+k_p*S*N(s)$
Here n=3 and m=1; $V(S) = \delta(s,k_p,k_i,k_d)$ *N(-s)
$N(-S) = -1.151S+0.1734$
$V(j\omega,k_p,k_i,k_d) = [p_1(w)+(k_i-k_d\omega^2)p_2(\omega)]+j[q_1(\omega)+k_pq_2(\omega)]$;
To get the results, we need to separate the even and odd parts equal to zero

For PI CONTROLLER : $K_i = -\frac{P1(w)}{P2(w)}$; $K_p = -\frac{q1(w)}{q2(w)}$

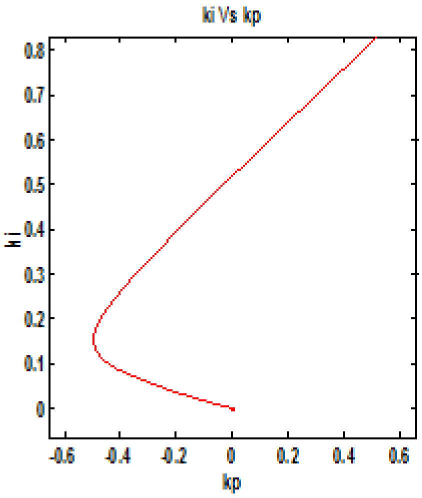
For PID CONTROLLER: $V(j\omega, k_p, k_i, k_d) = [p_1(w)+(k_i-k_d\omega^2)p_2(\omega)]+j[q_1(\omega)+k_pq_2(\omega)]$;

P1(w) = 0.6732w4 -0.1634w2 q1(w) = 1.1510w5-0.9290w3
P2(w) = 1.3248w2+0.0314 q2(w) = 1.3248w3 +0.0314w
We find that $z^+ = 1$ so that the signature requirement on v(s) for stability is,
$\sigma(v) = n-m+1+2z^+ = 4$
Since the degree of v(s) is even, we see from the signature formulas that q(ω) must have at least two positive real roots of odd multiplicity. The range of kp such that q(ω,kp) has at least 2 real, positive, distinct, finite zeros with odd multiplicities was determined (-0.4, 9) which is the allowable range of kp.
For a fixed range of kp ϵ (-0.4,9),
for instance kp= -0.3, we have
$q(\omega,-0.3) = q_1(\omega)-0.3q_2(\omega)$
after the kp value substitution we get polynomial equation is q(w)
Then the real, nonnegative, distinct finite zeros of q(ω,-0.3) with odd multiplicities are w_0,w_1,w_2
w0=0, ω1=0.5195, ω2=∞.
Sgn[q(0,-0.3)] = -1
It follows admissible string $I=\{i_0,i_1,i_2\}$ Must satisfy for I_1 it follows that stabilizing set ()values corresponding to k_p=-0.3
Must satisfy string k_i, k_d of inequalities
$P_1(\omega_0) + (k_i-k_d\omega_0^2)p_2(\omega_0)>0$
$P_1(\omega_1) + (k_i-k_d\omega_1^2)p_2(\omega_0)<0$

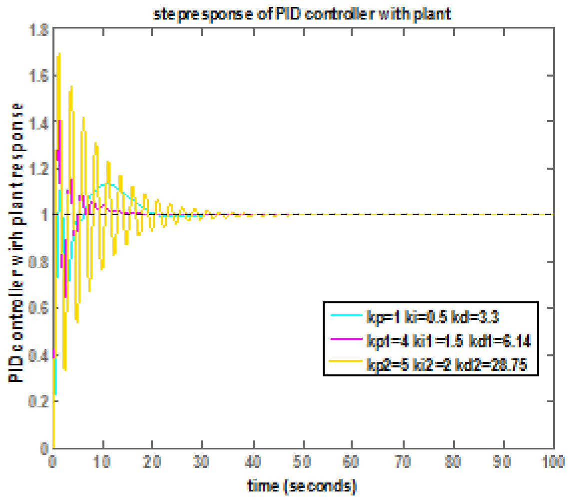
Substituting for$\omega_0=0,\omega_1=0.5195$ and ω_2 in the above expressions, we obtained the graph set of (k_i, k_d).
string of inequalities are
$k_i>0$
$k_i-0.972k_d<0.458$ FOR S1
finding stabilizing set of (k_p, k_i, k_d) constraints ,we fix kp value and find out set of(k_i, k_d)
for varying kp value from (-0.4,9) we find different string set S1,S2,S3,S4.
$k_i>0$
$k_i-1.2699k_d<0.7462$ FOR S2 { KP=1}
$k_i>0$
$k_i-5.8699k_d<3.7462$ FOR S3 { KP=4}

$k_i>0$
$k_i-9.453\ k_d<4.7462$ FOR S3 { KP=8}

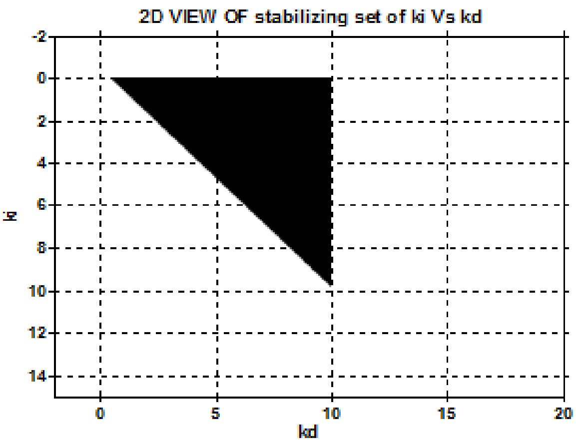
The set values of (k_i, k_d) for which the above equations holds can be solved by linear programming.
We get a 3D plot when we can select vertex set of boundary ends of(k_i, k_d) then we get stabilizing set graph of (K_p ,K_i, K_d) values which satisfies the stabilizing criterion.



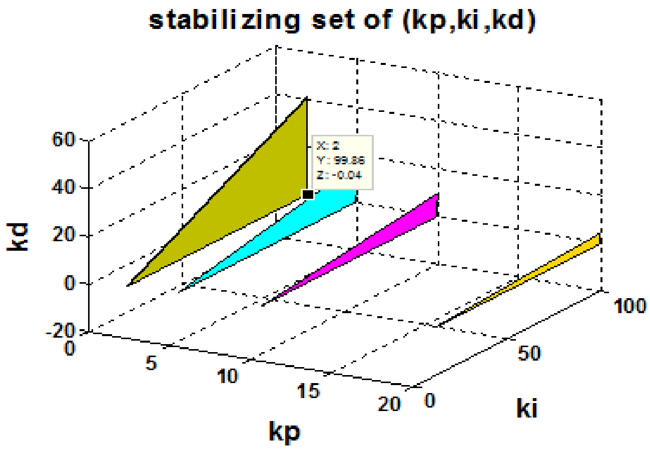
fig(4) boundry region set of Ki vs Kp



fig(7) plant variation with PID controller response

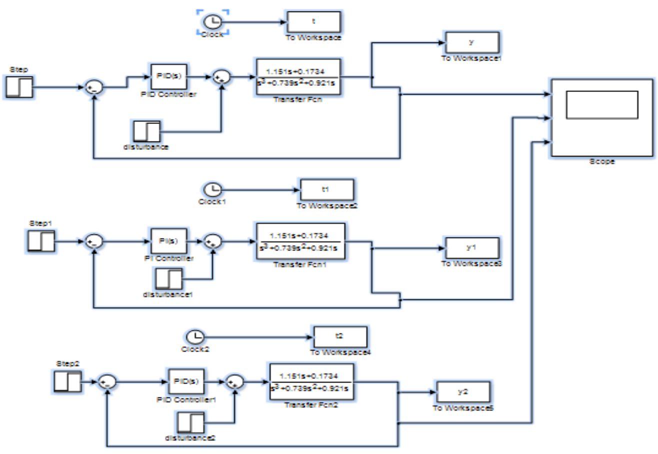


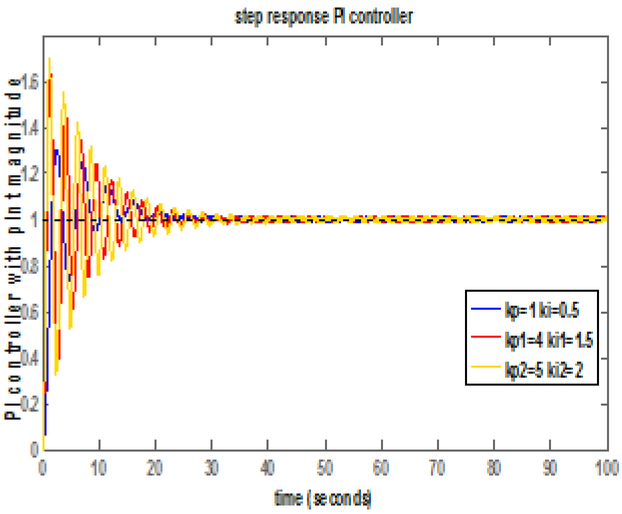
fig(5) stabilizing set space plane Ki vs Kd



fig(8) 3-D view of PID controller stabilizing set

fig(9) simulation block diagram of plant with controllers





fig(6)plant with PI controller response

fig(10) simulation block diagram of plant with controllers

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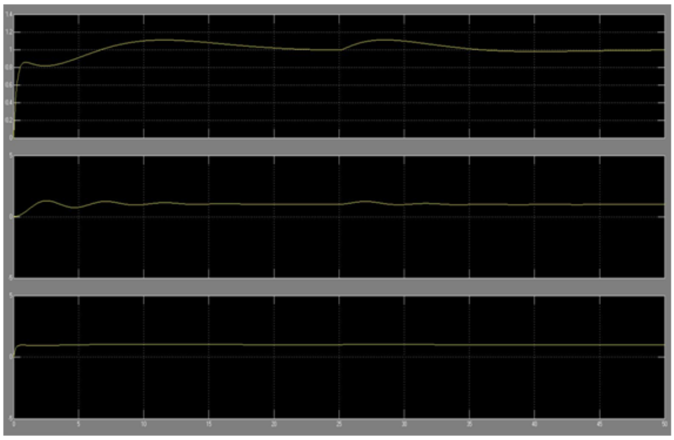

fig(10) simulation results robustness of plant with different (PI,PID) controllers with noise

(3).Example for Design PI,PID controller for interval plants(kharitonov plants) system of 5th order do with stability boundary locus and Signature method .

$$G(s)=\frac{[-6,-3]+[48,50]s+[39,41]s^2+[2,3]s^3+[1,1]s^4}{s^5[1,1]+[2,3]s^4+s^3[31,32]+[35,38]s^2+[49,51]s+[97,101]}$$

$$G_{11}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+2s^4+32s^3+38s^2+49s+97}$$

$$G_{12}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+2s^4+31s^3+38s^2+51s+97}$$

$$G_{13}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+3s^4+32s^3+35s^2+49s+101}$$

$$G_{14}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+3s^4+31s^3+35s^2+51s+101}$$

$$G_{41}(s)=\frac{s^4+2s^3+39s^2+50s-3}{s^5+2s^4+32s^3+38s^2+49s+97}$$

$$G_{42}(s)=\frac{s^4+2s^3+39s^2+50s-3}{s^5+2s^4+31s^3+38s^2+51s+97}$$

$$G_{43}(s)=\frac{s^4+2s^3+38s^2+50s-3}{s^5+3s^4+32s^3+35s^2+49s+101}$$

$$G_{44}(s)=\frac{s^4+2s^3+38s^2+50s-3}{s^5+3s^4+31s^3+35s^2+51s+101}$$

$$G_{21}(s)=\frac{s^4+2s^3+41s^2+50s-6}{s^5+2s^4+32s^3+38s^2+49s+97}$$

$$G_{22}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+2s^4+31s^3+38s^2+51s+97}$$

$$G_{23}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+3s^4+32s^3+35s^2+49s+101}$$

$$G_{24}(s)=\frac{s^4+3s^3+41s^2+48s-6}{s^5+3s^4+31s^3+35s^2+51s+101}$$

$$G_{31}(s)=\frac{s^4+3s^3+39s^2+48s-3}{s^5+2s^4+32s^3+38s^2+49s+97}$$

$$G_{32}(s)=\frac{s^4+3s^3+39s^2+48s-3}{s^5+2s^4+31s^3+38s^2+51s+97}$$

$$G_{33}(s)=\frac{s^4+3s^3+39s^2+48s-3}{s^5+3s^4+32s^3+35s^2+49s+101}$$

$$G_{34}(s)=\frac{s^4+3s^3+39s^2+48s-3}{s^5+3s^4+31s^3+35s^2+51s+101}$$

N1(S)= $s^4+3s^3+41s^2+48s-6$; D1(S) = $s^5+2s^4+32s^3+38s^2+49s+97$

We use the PID controller with T=0. The closed loop characteristic polynomial is

$\delta(s, k_p, k_i, k_d)$ = S*D(s)+(k_i+$k_d s^2$)N(s)+k_p*S*N(s)

Here n=5 and m=4; V(S) = **δ**(s, k_p, k_I, k_d**)** *N(-s)

N(-S) = $s^4-3s^3+41s^2-48s-6$

V(jω, k_p, k_i, k_d) = [p_1(w)+(k_i-$k_d\omega^2$)p_2(ω)]+j[q_1(ω)+$k_p q_2$(ω)];

To using the stability Boundary locus technique, To get the results, we need to separate the even and odd parts equal to zero $K_i=-\frac{P1(w)}{P2(w)}$; $K_p=-\frac{q1(w)}{q2(w)}$;

This method apply for G_{11}toG_{44} up to 16 kharitonov plants that's G_{11}(S),G_{12}(S),G_{13}(S),G_{14}(S),G_{21}(S),G_{22}(S),G_{23}(S),G_{24}(S), G_{31}(S),G_{32}(S),G_{33}(S),G_{34}(S),G_{41}(S),G_{42}(S),G_{43}(S),G_{44}(S). observes in the kharitonov plants boundary locus regions in fig(11)

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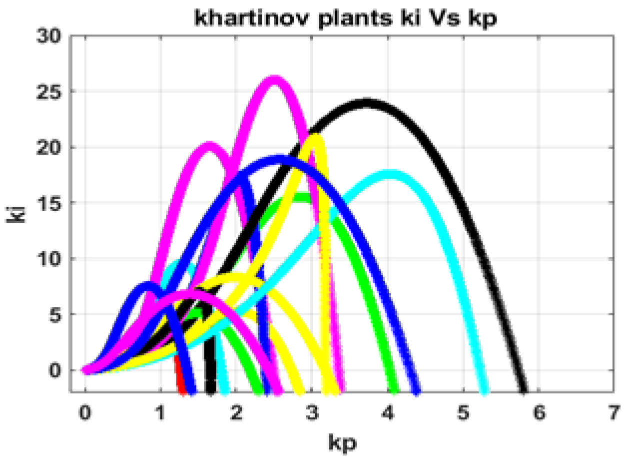
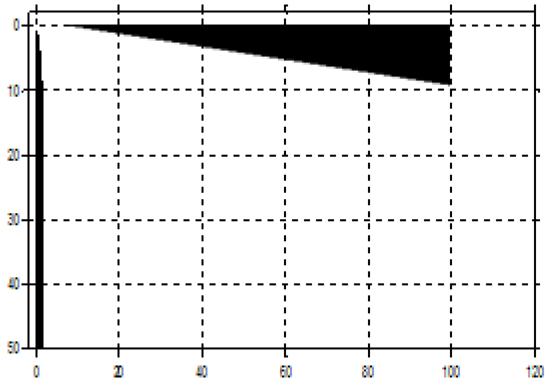


fig(11) boundary locus for fixing Kp vs Ki



fig(12) stabilizing region set of Ki vs Kd

Finally we get the kharitonov 32 plants boundary regions, by using stability boundary locus method , to fixed the stabilizing set of (K_P,K_I) values. From the graph fixed kp=(0,1.549) in the fig(11)

Kp=kp*,now determine string set of (K_i, K_d)values by finding the inequalities constraints ,linear programming technique used.

For PID CONTROLLER:

V(w)= [P_1(w)+ {K_i - K_d w^2}P_2(w)]+ j {q_1(w)+ K_p q_2(w)} ;

There exists sets of linear inequalities in terms of (K_i, K_d) to satisfying the signature condition.

Signature(V) = n-m+1+2Z^+ ;

Here n=5 and m=4

z^-, z^+ are denote the no. of roots on the S-plane LHP, RHP of numerator N(s), σ(v) = n-m+1+2z^+= 4

Since the degree of v(s) is even, we see from the signature formulas that q(ω) must have at least two positive real roots of odd multiplicity. The range of kp such that q(ω, kp) has at least 2 real, positive, distinct, finite zeros with odd multiplicities was determined (0.10,1.50) which is the allowable range of kp. For a fixed kpϵ (0.10,1.50), for instance kp= 0.10, we have q(ω,0.10) = q_1(ω)+0.10q_2(ω)

After the kp value substitution we get polynomial equation is q(w)

Then the real, nonnegative, distinct finite zeros of q(ω,0.10) with odd multiplicities are w_0,w_1,w_2.

Sgn[q (0,0.10)] = -1

It follows admissible string I_1={i_0,i_1,i_2} Must satisfy For I_1 it follows that stabilizing set ()values corresponding to k_p=0.10.Must satisfy string k_i, k_d of inequalities

$P_1(\omega_0)$ + $(k_i-k_d\omega_0^2)*p_2(\omega_0)>0$;$P_1(\omega_1)$ + $(k_i-k_d\omega_1^2)*p_2(\omega_0)<0$

finding stabilizing set of (k_p, k_i, k_d) constraints ,we fix kp value and find out set of (ki, kd)for varying kp value from (0.10,1.50) we find different string set S1, S2, S3, S4, S5, S6, S7...S_X ,by comparing the following inequalities in linear programming, we get 2-D space plane of vertices of (k_i, k_d)in the fig(12)

here x=ki, y=kd

kp=0.10
kp=0.30
condition1=x>0;
condition1=x>0;
condition2=(x-0.9410*y)<0.3507; condition2=(x-0.2744*y)<0.08015;

kp=0.50
kp=0.70

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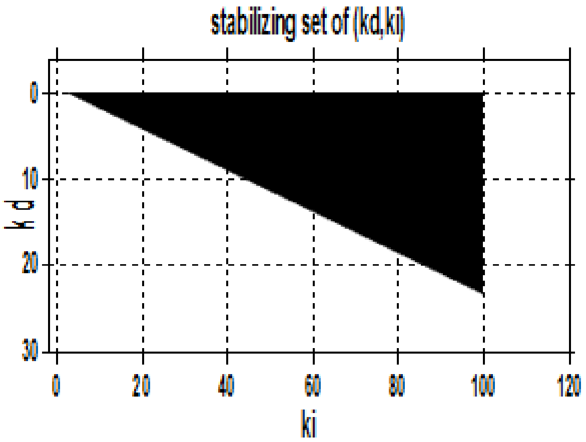
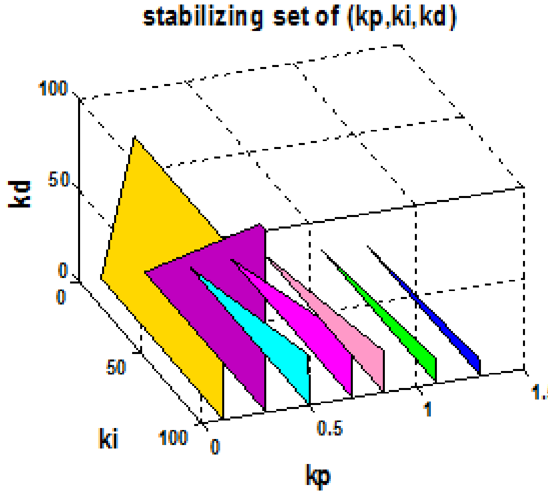
condition1=x>0;
condition1=x>0;
condition2=(x-3.499*y)<1.8620;
condition2=(x-3.010*y)<1.5442;

kp=0.85; kp=1.10
condition1=x>0;
condition1=x>0;
condition2=(x-4.2012*y)<2.3277; condition2=(x-6.9363*y)<4.1572;

kp=1.30
condition1=x>0;
condition2=(x-10.010*y)<6.1250;

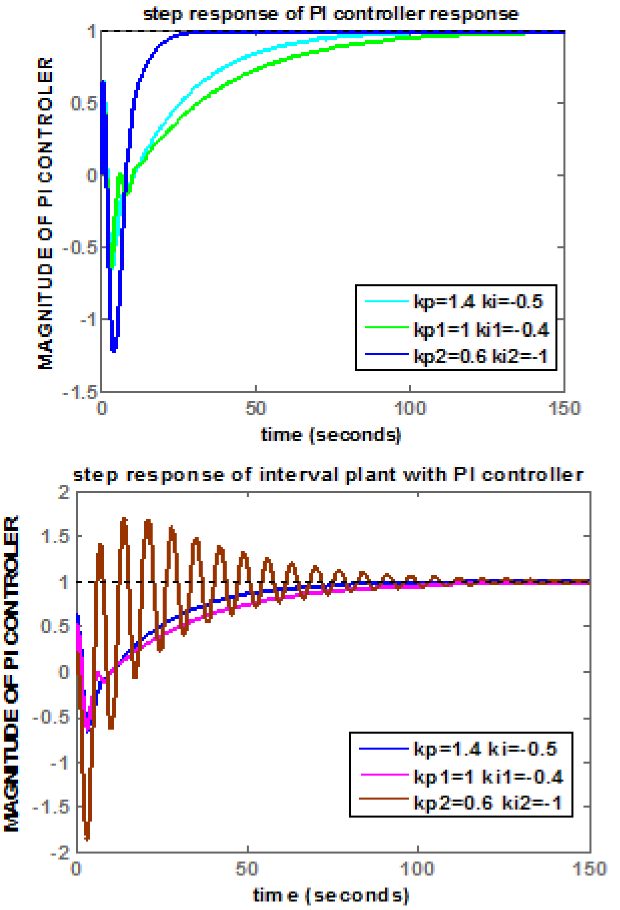


fig(12)2-D view of stabilizing region set of (Kd vs Ki)



fig(13)3-D view of stabilizing region set of (Kd, Ki, Kp)

The set values of (k_i, k_d) for which the above equations holds can be solved by linear programming.
We get a 3D plot observed in fig(13) when we can select vertex set of boundary ends of(k_i, k_d),then we get stabilizing set graph of (K_p ,K_i, K_d) values which satisfies the stabilizing criterion [5].



fig(14,15) Robustness of plants with PI controller response by variation of (kp, ki) with in boundary

7. CONCLUSIONS:

this paper dealt with an approach has been presented for computation of stabilization of PI,PID controllers for robustness of interval plants(kharitonov plants) region of boundary locus and stabilization set of PID gains of (K_p ,K_i, K_d), which can be easily obtained by equating the real and imaginary parts of characteristic equation to zero. The proposed method has further has been used find stabilizing region of PI parameters plant with uncertain parameters and this signature method involves require sweeping over parameters. Also, it needs linear programming to solve set of inequalities used in the signature method for the further solving of region of stabilizing set of PID controller gains in effective method of approach for higher order systems.

8. REFERENCES:

[1]. Shankar P. Bhattacharyya, Aniruddha Datta, L.H.Keel.A." Three Term Controllers" LINEAR CONTROL THEORY STRUCTURE, ROBUSTNESS and optimization

[2]. S. Han and S. P. Bhattacharyya, "PID CONTROLLER SYNTHESIS USING A SIGMA-HURWITZ STABILITY CRITERION," IN IEEE CONTROL SYSTEMSLETTERS, VOL. 2, NO. 3, PP. 525-530, JULY 2018.

[3]. Nusret Tan, Ibrahim Kaya, Celaleddin Yeoglu, Derek P. Athertonet al. "Computation of stabilizing PI and PID controllers using the stability boundary locus." Energy Conversion and Management 47.18 (2006): 3045-3058.

[4]. KiamHeong Ang, Gregory Chong, student member, IEEE, and Yun Li, Member,IEEE et al "PID CONTROL SYSTEM ANALYSIS, DESIGN, AND TECHNOLOGY,IEEE TRANSACTIONS ON CONTROL SYSTEM TECHNOLOGY, VOL 13, NO 4,PP.559-576,JULY 2005.

[5]. S. P. Bhattacharyya, HO MT DATTA,DESIGN OF P,PI,PID CONTROLLERS FOR INTERVAL PLANTS. in: proc of Amer contr conf,philadelpia,june 1998