Reduction of Peak to Mean Power Ratio Using Pre-coder based Hartley Transform for OFDM Systems

Dr. P.V. Naganjaneyulu
Professor & Principal, Department of ECE
PNC & Vijai Institute of Engineering & Technology, Repudi, Phirangipuram, Guntur (Dist), Andra Pradesh.

Abstract- In many wireless network applications Orthogonal Frequency Division Multiplexing (OFDM) plays an important role. But higher peak to mean power ratio, which is well known as peak to average power ratio (PAPR) is a major drawback in OFDM systems, which in results increases the complexity of digital to analog converters (DAC) and analog to digital converters (ADC) and also degrades the efficiency of high power amplifiers (HPA). Here in this letter we introduced a novel PAPR reduction technique using pre-coding matrix based discrete Hartley transform (DHT). The proposed scheme uses M-ary quadrature amplitude modulation (QAM) (where M=4, 8, 16... 256) and it gives the better performance than the companding transform techniques such as exponential companding, conventional companding and non-linear companding OFDM systems, and also original OFDM system.

I INTRODUCTION
Due to the multipath fading effect and receiver complexity of wireless channels, the traditional modulation techniques which are based on single carrier can achieve limited data rates. High data-rate is desirable in many recent wireless multimedia applications [1]. However, as the data-rate in communication system increases, the symbol duration gets reduced. Therefore, the communication systems using single carrier modulation suffer from severe inter symbol interference (ISI) caused by dispersive channel impulse response, thereby needing a complex equalization mechanism. Orthogonal Frequency Division Multiplexing (OFDM) is a special form of multicarrier modulation technique, in which the total frequency selective fading channel will be divided into many orthogonal narrow band flat fading sub channels. In OFDM system high-bit-rate data stream is transmitted in parallel over a number of lower data rate subcarriers and do not undergo ISI due to the long symbol duration[2]. Major advantages of OFDM systems are

- High spectral efficiency
- Simple digital realization by using the FFT operation
- Due to the ISI avoidance, the complexity in the receiver will be reduced
- Various modulation schemes will be used to achieve the best performance of the system

Due to the above mentioned advantages, OFDM has been used in many wireless applications such as Wireless Personal Area Network (WPAN), Wireless Local Area Network (WLAN) [4], Wireless Metropolitan Area Network (WMAN), Digital Audio Broadcasting (DAB) [3] and Digital Video Broadcasting (DVB) [5]. It is also being considered for IEEE 802.20, 802.16 [6], [7] and 3GPP-LTE. With the use of cyclic prefix for eliminating the effect of ISI, there is a need for a simple one tap equalizer at the OFDM receiver. OFDM brings in unparalleled bandwidth savings, which leads to high spectral efficiency.

Despite the widespread acceptance of OFDM, it has its drawbacks:

- OFDM signals with high peak-to-average power ratio (PAPR) leads to the degradation in system performance, which in results the enhanced out-of-band power.
- OFDM systems are more sensitive to Doppler spread when compared to single carrier modulation schemes.
- The system performance will be degraded by the phase noise, which is caused by the imperfections of the transmitter and receiver.
- It requires accurate time and frequency synchronization
- Due to the cyclic prefix (CP) it loses spectral efficiency

As listed above, large envelope fluctuation in OFDM signal is one of the major drawbacks of OFDM. Such fluctuations create difficulties because practical communication systems are peak power limited. Thus, envelope peaks require a system to accommodate an instantaneous signal power that is larger than the signal average power, necessitating either low operating power efficiencies or power amplifier (PA) saturation. In order to amplify the OFDM signal with large envelope fluctuations, PAs with large linear range are required, which makes it very expensive. If PA has limited linear range then its operation in non linear mode introduces out of band radiation and in band distortion. It is also necessary to have D/A and A/D converters with large dynamic range to convert discrete time OFDM signal to analog signal and vice versa.

II LITERATURE SURVEY

From the past decades many PAPR reduction techniques have been proposed to improve the digital communication system efficiency, and still the researchers are focusing on developing extended PAPR reduction schemes with more effective results. Among them, few techniques like block coding schemes [8], Tone Reservation (TR), Tone Injection (TI) [10-12], iterative clipping and filtering [16], Partial Transmit Sequence (PTS) [9], Active Constellation Extension (ACE) [11], Adaptive ACE [12-14], Select Level Mapping (SLM) [19], companding techniques such as linear [10], non-linear, exponential companding [15] are more popular.

The letter proposed in [8] gives a block coding technique for PAPR reduction of multi carrier transmission methods such as OFDM. In this approach the author has used ¾ rate block code and simulated this with an example of four-carrier signal and also with eight-carrier signal. However, block coding scheme has got reduced peak to mean envelop power ratio (PMEPR) but it has the limitation in number of carriers, hence does not suit for longer data sequences. X. Li et al. in [9] proposed a new PAPR reduction scheme which is based on clipping and filtering, which reduces the peak to mean envelop power ratio by clipping the transmitted sequence to certain extent and afterwards filters the clipped data to reduce the PAPR. Although it reduces the PAPR to some extent but while clipping the input data we are losing the original data bits, which in results the poor system efficiency. Wattanasuwakull et al. proposed tone reservation (TR) and tone injection (TI) method [10] to reduce the PAPR of the OFDM signal. Main idea of the approach in [11] is to produce a surplus and linear signal which will reduce the peak-power by reservation of number of subcarriers. However, due to the surplus symbols and the larger signal constellation usage it slows down the data rate and increases transmitter power. Sang et al. in [13] proposed a modified selective mapping scheme (SLM) to reduce the PAPR which performs better than conventional SLM and PTS. Algorithm in [13] proposes a modified SLM in which the subcarriers which are predefined will be inserted by the dummy or complementary sequence taken from flipping method. Partial Transmit Sequences (PTS) is an efficient technique for reduction of Peak-to-Average Power Ratio (PAPR) Orthogonal Frequency Division Multiplexing (OFDM) system. However, higher computational complexity is the major drawback of PTS. In order to overcome this many PTS methods have been proposed. The paper proposed in [14] discusses the optimized PTS (O-PTS) scheme with super imposed training, in which the O-PTS method reduces the PAPR to 7.25 dB from 10 dB of conventional PTS scheme. In [15] transformation based approach for PAPR reduction has been proposed, which uses linear companding transform. Another approach of PAPR reduction proposed in [16], which presents a new hybrid peak-to-average power ratio reduction technique with the combination of ACE and TR. Recently, adaptive active constellation extension (AACE) method has been proposed in [17-18]. In this approach AACE overcomes the drawbacks of the existed ACE and clipping based active constellation extension (CB-ACE). In [20] the author tried to explain the companding OFDM techniques to reduce the PAPR. Companding techniques have got the better performance than the ACE, AACE but it supports limited modulation schemes and it suffers from large degree of companding value However, all the methods which have been discussed above have many drawbacks. To overcome all the drawbacks of conventional schemes here in this paper we
introduced a novel scheme which is based on pre-coding matrix Discrete Hartley Transform (DHT). This technique is efficient, signal independent, distortion less, it does not require any optimization algorithm and PAPR is completely eliminated.

III PROPOSED TECHNIQUE

a. PAPR in OFDM Systems

Input data sequence to be transmitted with $N$ subcarriers in an OFDM symbol is $(0), X(1), \ldots, X(N-1)$. The baseband representation of the OFDM symbol is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{j2\pi n t / N}, \quad 0 \leq t \leq T$$

$T$ is OFDM symbol duration. When $N$ value will be increased then both the real and imaginary parts of $x(t)$ would become Gaussian distributed and each with zero mean with a variance of $E[x(t)^2]/2$, and the OFDM symbol amplitude follows a Rayleigh distribution. Therefore, it is possible that the OFDM symbol amplitude will be exceeds than the maximum amplitude. Practical hardware such as A/D and D/A converters, high power amplifiers (HPA) has finite dynamic range; therefore the maximum amplitude of OFDM signal should be limited.

PAPR of an OFDM signal has been defined as:

$$PAPR = \max \left\{ E[x(t)^2] \right\}$$

In dB value the PAPR can be written as $PAPR = 10 \log_{10}(PAPR)$. It is easy to see from (2) that reduction of PAPR can be achieved by reducing the numerator and increasing the denominator, or both.

Complementary cumulative distributed function (CCDF) will be used as a measurement of PAPR effectiveness, which is the probability that PAPR exceeds some threshold, i.e.,

$$CCDF = \text{Probability} (PAPR > p_0)$$

where $p_0$ is the threshold

b. New Companding Algorithm

Let a non-linear companding function is $f(x)$, and $x(t) = \sin(\omega t)$ be the compander input, then the companded signal $y(t)$ can be written as:

$$y(t) = f[x(t)] = f[\sin(\omega t)]$$

The companding algorithm uses a smooth function named as airy function to reduce the peak value. The companding function is as follows:

$$f(x) = \beta . \text{sign}(x) . [\text{airy}(\alpha) - \text{airy}(\alpha . |x|)]$$

Where $\text{airy}(.)$ is the airy function, $\beta$ is the adjusting factor for the average output power of the compander to the same level of input power.

$$\beta = \frac{E[x(t)^2]}{E[\text{airy}(\alpha) - \text{airy}(\alpha . |x|)]^2}$$

Here $E[.]$ denotes the expectation.

The decompanding function is the inverse of $f(x)$, and it can be written as:

$$f^{-1}(x) = \frac{1}{\alpha} . \text{sign}(x) . \text{airy}^{-1} \left( \text{airy}(\alpha) - \frac{|x|}{\beta} \right)$$

c. Pre-coding Discrete Hartley Transform

Generally, the DHT is an invertible function $H: \mathbf{R}^N \rightarrow \mathbf{R}^N$ (where $\mathbf{R}$ denotes the set of real numbers) and it is a linear function. The $N$ real numbers $x_0, \ldots, x_{N-1}$ are transformed into the $N$ real numbers $H_0, \ldots, H_{N-1}$.

The Pre-coding matrix $P$ can be written as

$$P = \begin{bmatrix}
    p_{00} & p_{01} & \cdots & p_{0(N-1)} \\
    p_{10} & p_{11} & \cdots & p_{1(N-1)} \\
    \vdots & \vdots & & \vdots \\
    p_{(N-1)0} & p_{(N-1)1} & \cdots & p_{(N-1)(N-1)}
\end{bmatrix}$$

Where $P$ is a Pre-coding of size $N \times N$ is shown in above equation. The complex baseband OFDM signal with $N$ sub carriers can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P \cdot X_k e^{j2\pi k f t}, \quad 0 \leq t \leq N T$$
We can express modulated OFDM vector signal with $N$ subcarriers as follows:

$$x_N = \text{IFFT}\{P \cdot X_N\}$$  \hspace{1cm} (9)

The DHT is a linear transform and $N$-point DHT can be defined as:

$$H_k = \sum_{n=0}^{N-1} x_n \left[ \cos \left( \frac{2\pi nk}{N} \right) + \sin \left( \frac{2\pi nk}{N} \right) \right]$$

$$= \sum_{n=0}^{N-1} x_n \cdot \text{cas} \left( \frac{2\pi nk}{N} \right)$$  \hspace{1cm} (10)

Where $\text{cas} \ \theta = \cos \theta + \sin \theta \ \text{and} \ \ k = 0, 1, \ldots, N - 1$

$$p_{mn} = \text{cas} \left( \frac{2\pi mn}{N} \right)$$  \hspace{1cm} (11)

$P$ is pre-coding matrix of size $N \times N$, and both $(m,n)$ are integers from range 0 to $N-1$. The DHT is also invertible transform which allows us to recover the $X_n$ from $H_k$ and inverse can be obtained by simply multiplying DHT of $H_k$ by $1/N$.

**IV SIMULATION RESULTS**

Experimental results have been done in MATLAB 2014a version with 4.00 GB RAM and i3 processor for decreasing the CPU time to execute the MATLAB code for this project. Here, in the fig. 2 (a) we had compared the companding scheme with exponential companding and without companding transform, it shows that the exponential companding transform got 3.85 dB, companding transform got 4.25 dB and 12 dB for Original OFDM system i.e., without companding. In fig.2 (b), we analyzed the bit error rate (BER) performance of companding, exponential companding and original OFDM system, BER for the companding transform is much lower than the exponential and original OFDM systems, which shows that the companding transform is better than the exponential and original OFDM systems in terms of BER but companding method is slide inferior than the exponential in terms of PAPR. In fig. 2 (c) and (d), it shows that the proposed DHT scheme has got the PAPR of 0.9 dB with 16-QAM and 32-QAM for $N=64$ subcarriers, which is much lower than the companding, exponential companding, pre-coding OFDM system and original OFDM system.

![Fig.1 Pre-coding matrix based OFDM system](image_url)
CONCLUSION

In this letter, we analyzed the performance of Pre-coding matrix DHT for the reduction of PAPR in high speed OFDM systems using M-QAM constellation modulation scheme (where M=16, 32, 64, 128 and so on). The simulation results which have been executed in MATLAB shows that the proposed pre-coding DHT OFDM System has got the better performance than the conventional schemes such as Original pre-coded system, Companding scheme, exponential companding and without companding OFDM systems respectively. Thus, it is concluded that proposed system shows better PAPR reduction than conventional reduction techniques. Additionally, it does not require any power increment, complex optimization and side information to be sent for the receiver.

REFERENCES

3. ETSI, “Radio broadcasting systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed


