

## Efficient method of MMSE Channel Estimation for MIMO-OFDM Using Spatial and Temporal Correlations

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### ABSTRACT

*Channel has been introduced to achieve high data speed and better bit rate. The system becomes more efficient when OFDM (Orthogonal Frequency Division Multiplexing) is combined with MIMO to obtain high transmission rates, good quality of service and minimize the probability of error. Channel estimation is of great importance in MIMO-OFDM system. Channel estimation is used to estimate the transmitted signal using the corresponding receiver signal. This project proposes a parametric sparse multiple input multiple output (MIMO)-OFDM channel estimation scheme based on the finite rate of innovation (FRI) theory, whereby super-resolution estimates of path delays with arbitrary values can be achieved. Meanwhile, both the spatial and temporal correlations of wireless MIMO channels are exploited to improve the accuracy of the channel estimation. For outdoor communication scenarios, where wireless channels are sparse in nature, path delays of different transmit-receive antenna pairs share a common sparse pattern due to the spatial correlation of MIMO channels. Meanwhile, the channel sparse pattern is nearly unchanged during several adjacent OFDM symbols due to the temporal correlation of MIMO channels. By simultaneously exploiting those MIMO channel characteristics, the proposed scheme performs better than existing state-of-the-art schemes. Furthermore, by joint processing of signals associated with different antennas, the pilot overhead can be reduced under the framework of the FRI theory*

### 1.INTRODUCTION

Multiple Input Multiple Output (MIMO) – OFDM is key technology for future wireless communication due to its high spectral efficiency and superior robustness to multipath fading channels [2]. For MIMO-OFDM systems, better channel estimation is essential for system performance [3]. Generally, there are two categories of channel estimation schemes for MIMO-OFDM systems. The first one is nonparametric approach, which utilizes orthogonal frequency domain pilots or time domain training sequence to convert the channel estimation in MIMO-OFDM system to single antenna system [3]. In this paper I proposed Time domain training based orthogonal pilot (TTOP) for example of this channel estimation approach. However, these sort schemes suffers from high pilot overhead when number of transmit antennas increases. The second approach is parametric channel estimation which utilizes sparsity of wireless channels to reduce the pilot overhead [4],[5]. This is much useful for future advancement since it can achieve better higher spectral efficiency. However, path delays of sparse channels are assumed to be located at the integer multiples of sampling period, which is unrealistic in practice. In this paper, a more practical sparse MIMO-OFDM channel estimation scheme based on spatial and temporal correlation of sparse wireless MIMO channels is proposed to deal with arbitrary path delays.

The proposed scheme can achieve super-resolution estimates of arbitrary path delays, which is more suitable for wireless channels in practice. Due to the small scale of the transmit and receive antenna arrays compared to long signal transmission distance in

typical MIMO antenna geometry, channel impulse responses (CIR) of different transmit receive antenna pairs share common path delays [6], which can be translated to as a common sparse pattern of CIRs due to spatial correlation of MIMO channels. Due to temporal correlation of such common sparse pattern doesn't change along several adjacent OFDM symbols previously the MIMO channel estimation schemes were proposed such that they exploit spatial correlation or temporal correlation. But by exploiting both correlations the estimation accuracy will be increases. In this method we reduce pilot overhead by utilizing Finite Rate Innovation (FRI) theory. This technique can recover the analog sparse signal with very low sampling rate; as a result channel sparsity level will decide average pilot overhead length per antenna instead of channel length.

**II. SPARSE MIMO CHANNEL MODEL**

The MIMO channel is shown in Fig.1, its characteristics are

**A) Channel Sparsity:** In typical outdoor communication scenarios, due to several significant characteristics CIR is intrinsically sparse. For an  $N_c \times N_r$  MIMO system, the CIR  $h^{(i,j)}(t)$  between the  $i$ th transmit antenna and  $j$ th receive antenna can be modeled as [1]

$$h^{(i,j)}(t) = \sum_{p=1}^P \alpha_p^{(i,j)} \delta(t - \tau_p^{(i,j)}), 1 \leq i \leq N_t, 1 \leq j \leq N_r \tag{1}$$

Where  $\delta(\bullet)$  is the Dirac function,  $P$  is the total number of resolvable propagation paths, and  $\tau_p^{(i,j)}$  and  $\alpha_p^{(i,j)}$  denote the path delay and path gain of  $p$ th path respectively.

**B) Spatial Correlation**

Because transmitter and receiver antenna array is small compared with the transmitting distance very similar scattering happens in channels of different transmit-receive antenna pairs. Path delays difference from the similar scatters is far less than sampling period for most communication systems. Even though

the path gains are different CIRs of different transmit-receive antenna pairs share common sparse pattern [6].

**C) Temporal Correlation**

For wireless channels, the path delays are not as fast varying as the path gains. And path gains vary continuously. Thus, the channel sparse pattern is nearly unchanged during several adjacent OFDM symbols, and the path gains are also correlated [8].

**III. MMSE ESTIMATION**

A flat block-fading narrow-band MIMO system with  $M_t$  transmit antennas and  $M_r$  receive antennas is considered. Later on,  $M$  value is fixed to 4. The relation between the received signals and the training sequences is given by

$$Y = HP + V \tag{2}$$

Where  $Y$  is the  $M_r \times N$  complex matrix representing the received signals,  $P$  is the  $M_t \times N$  complex training matrix, which include training sequences (pilot signals);  $H$  is the  $M_r \times M_t$  complex channel matrix and  $V$  is the  $M_r \times N$  complex zero mean white noise matrix.

Assuming the training matrix is known, the channel matrix can be estimated using the minimum mean square error (MMSE) method, as

$$\hat{H} = \frac{\rho}{M_r} Y P^H (R_H^{-1} + \frac{\rho}{M_t} P P^H)^{-1} \tag{3}$$

With MSE estimation error given by

$$J_{MMSE} = E \{ \| H - \hat{H}_{MMSE} \|^2 \} = \text{tr} \{ (R_H^{-1} + \frac{\rho}{M_t} P P^H)^{-1} \} \tag{4}$$

$$R_H = Q \Lambda Q^H \tag{5}$$

In (5)  $Q$  is the unitary eigenvector matrix and  $\Lambda$  is the diagonal matrix with nonnegative Eigenvalues. By substituting (5) into (4), one can get

$$J_{MMSE} = \text{tr} \left\{ \left( \Lambda^{-1} + \frac{\rho}{M_t} Q^H P P^H Q \right)^{-1} \right\} \quad (6)$$

To minimize the estimation error (4)  $Q^H Q P^H$  needs to be diagonal [3] [4] [5]. To satisfy this condition, the training sequence developed in [6] [7] can be used. Then using (6), the MSE can be expressed as:

$$J_{MMSE} = \sum_{i=0}^{M_t-1} \sum_{j=0}^{M_r-1} \frac{1}{\frac{\rho}{M_t} \beta_i + (\lambda_i(R_t) \lambda_j(R_r))^{-1}} \quad (7)$$

Where  $R_t$  and  $R_r$  are spatial correlation matrices at transmitter and receiver, respectively;  $\rho$  is the power of training sequence

#### IV. SPARSE MIMO-OFDM CHANNEL ESTIMATION

In this section, the widely used pilot pattern is briefly introduced first, based on which a super-resolution sparse MIMO –OFDM channel estimation method is then applied. Finally, the required number of pilots is discussed under the framework of the FRIT theory.

##### A. pilot Pattern

The pilot pattern widely used in common MIMO –OFDM system is illustrated in Fig 3

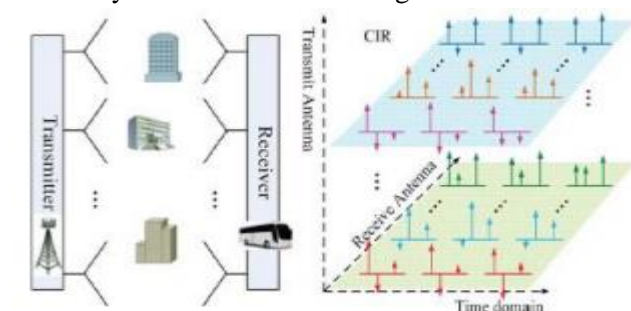


Fig 2. Spatial and temporal correlations of MIMO OFDM channels

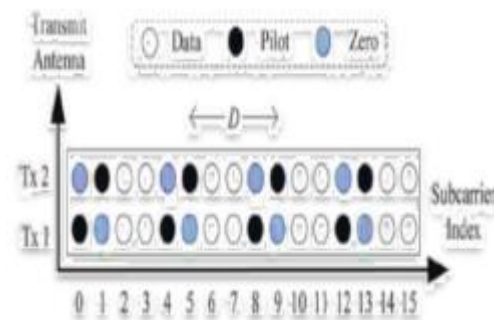


Fig.3 Pilot pattern.

Note that the specific  $N_t=2$ ,  $D=4$ ,  $N_p=4$ , and  $N_{p-total}=8$  are used for illustration purpose.

In frequency domain  $N_p$  pilots are uniformly spaced with pilot interval  $D$  ( e.g.  $D=4$  in Fig. 3 Meanwhile, every pilot is allocated with a pilot index  $l$  for  $0 \leq l \leq N_p-1$ , which is ascending with the increase of the subcarrier index. Each transmit antenna uses  $A$  subcarrier index to distinguish MIMO channels associated with them. Which has initial phase  $\theta_i$  for  $1 \leq i \leq N_t$  and  $(N_t-1)$  zero subcarriers to ensure the orthogonality of pilot. Therefore for  $i$ th transmit antenna, the subcarrier index of the  $l$ th pilot is

$$I_{pilot}^l(i) = \theta_i + lD, \quad 0 \leq l \leq N_p - 1 \quad (8)$$

Consequently, the total overhead per transmit antenna is  $N_{p-total} = N_t N_p$ , and thus,  $N_p$  can be also referred as the average pilot overhead per transmit antenna in the letter.

##### B Super – Resolution Channel Estimation

The equivalent baseband channel frequency response (CFR)  $H(f)$  can be expressed at receiver as

$$H(f) = \sum_{p=1}^P \alpha_p e^{-j2\pi f \tau_p}, \quad -f_s/2 \leq f \leq f_s/2 \quad (9)$$

Where superscript  $i$  and  $j$  in (1) are omitted for convenience is the system bandwidth, and  $T_s$  is the sampling period. Meanwhile, the  $N$ -point discrete Fourier transform (DFT) of the time-domain equivalent baseband channel can be expressed as [5], i.e.,

$$H[k] = H\left(\frac{kf_s}{N}\right), 0 \leq k \leq N-1 \quad (10)$$

Therefore for  $(i, j)$ th transmit-receive antenna pair, according to (8)-(10), the estimated CFRs over pilots can be written as

$$\begin{aligned} \hat{H}^{(i,j)}[l] &= H[l_{pilot}^{(i,j)}] \\ &= H\left(\frac{\theta_i + lD}{N}\right) \\ &= \sum_{p=1}^P \alpha_p^{(i,j)} e^{-j2\pi \frac{(\theta_i + lD)r_p^{(i,j)}}{N}} + W^{(i,j)}(l) \end{aligned} \quad (11)$$

Where  $H^{(i,j)}[l]$  for  $0 \leq l \leq N_p - 1$  can be obtained by using the conventional minimum mean square error (MMSE) or least square (LS) method, and  $W$  is the additive white Gaussian noise (AWGN).

Eq. (5) can be also written in a vector form as

$$\hat{H}^{(i,j)}[l] = (V^{(i,j)}[l])^T a^{(i,j)} + W^{(i,j)}(l)$$

Where  $V^{(i,j)}[l] = [\gamma^{lD r_1^{(i,j)}}, \gamma^{lD r_2^{(i,j)}}, \dots, \gamma^{lD r_p^{(i,j)}}]$  and  $a^{(i,j)} = [\alpha_p^{(i,j)} \gamma^{\theta_i r_1^{(i,j)}}, \alpha_p^{(i,j)} \gamma^{\theta_i r_2^{(i,j)}}, \dots, \alpha_p^{(i,j)} \gamma^{\theta_i r_p^{(i,j)}}]$  and  $\gamma = e^{-j2\pi \frac{f_s}{N}}$ .

Because the wireless channel is inherently sparse and the small scale of multiple transmit or receive antennas is negligible compared to the long signal transmission distance, CIRs of different transmit-receive antenna pairs share common path delays, which is equivalently translated as common sparse pattern of CIRs due to the spatial correlation of MIMO channels.

$$\hat{H}^t = VA^t + W^t, \quad 1 \leq t \leq N_t \quad (12)$$

$$\hat{H}^t = \begin{bmatrix} \hat{H}^{(i,1)}[0] & \hat{H}^{(i,2)}[0] & \dots & \hat{H}^{(i,N_r)}[0] \\ \hat{H}^{(i,1)}[1] & \hat{H}^{(i,2)}[1] & \dots & \hat{H}^{(i,N_r)}[1] \\ \vdots & \vdots & \ddots & \vdots \\ \hat{H}^{(i,1)}[N_p-1] & \hat{H}^{(i,2)}[N_p-1] & \dots & \hat{H}^{(i,N_r)}[N_p-1] \end{bmatrix}$$

When all transmit antennas are considered based on (12), we have

$$\hat{H} = VA + W \quad (13)$$

By Comparing the formulated problem and the classical direction-of-arrival (DOA) problem, I find out that they are mathematically equivalent. Traditional DOA problem is to estimate the DOAs of the  $P$  sources from a set of time-domain measurements, which are obtained from the sensors outputs at distinct time instants (time-domain samples). In this case, we try to estimate the path delays of  $P$  multipath from a set of frequency-domain measurements, which are acquired from pilots of distinct antenna pairs (antenna-domain samples). To efficiently estimate path delays with arbitrary values it has been verified by the total least square estimating signal parameters via rotational invariance techniques (MMSE) algorithm can be applied to (8).

$$\hat{A} = \hat{V}^\dagger \hat{H} = (\hat{V}^H \hat{V})^{-1} \hat{V}^H \hat{H} \quad (14)$$

Furthermore, to improve the accuracy of the channel estimation we can also exploit the temporal correlation of wireless channels. First, path delays of CIRs during several adjacent OFDM symbols are nearly unchanged which is equivalently referred as a common sparse pattern of CIRs due to the temporal correlation of MIMO channels. Thus, the Vander monde matrix  $V$  in

(8) remains unchanged across several adjacent OFDM symbols. Moreover, path gains during adjacent OFDM symbols are also correlated due to the temporal continuity of the CIR, so As in (8) for several adjacent OFDM symbols are also correlated. Therefore, when estimating CIRs of the qth OFDM symbol, we can jointly exploit  $\hat{H}_p$ s of several adjacent OFDM symbols based on (8), i.e.,

$$\frac{\sum_{p=q-R}^{q+R} \hat{H}_p}{2r+1} = V_q \frac{\sum_{p=q-R}^{q+R} A_p}{2r+1} + \frac{\sum_{p=q-R}^{q+R} W_p}{2r+1} \tag{15}$$

Where the subscript  $p$  is used to denote the index of the OFDM symbol, and the common sparse pattern of CIRs is assumed in  $2R+1$  adjacent OFDM symbols, Hence effective noise can be reduced, so the improved channel estimation accuracy is expected. Our proposed scheme exploits the sparsity as well as the spatial and temporal correlations of wireless MIMO channels to first acquire estimations of channel parameters, including path delays and gains, and then obtain the estimation of CFR, which is contrast tonon parametric schemes which estimates the channel by interpolating or predicting based on CFRs over pilots[1].

### C. Discussion on Pilot Overhead

Compared with the model of the multiple filters bank based on the FRI theory, it can be found out that CIRs of transmit receive antenna pairs are equivalent to the semi period sparse subspaces, and the  $N_p$  pilots are equivalent to the  $N_p$  multichannel filters. Theref ore, by using the FRI theory, the smallest required number of pilots for each transmit antenna is  $N_p=2P$  in a noiseless scenario. For practical channels with the maximum delay spread although the normalized chan nel length is usually very large, the sparsely level  $P$  is small, i.e.,  $P \ll L$

Consequently, in contrast to the nonparametric channel estimation method where the required number of pilots heavily dependson  $L$ , our proposed parametric scheme only needs  $2P$  pilots in theory. Note that the number of

pilots in practice is larger than  $2P$  to improve the accuracy of the channel estimation due to AWGN.

### (V) SIMULATION RESULTS

A simulation study was carried out to compare the performance of the proposed scheme with those of the existing state-of-the-art methods for MIMO-OFDM systems. The conventional comb-type pilot and time-domain training based orthogonal pilot (TTOP) [2] schemes were selected as the typical examples of the nonparametric channel estimation scheme, while the recent time-frequency joint (TFJ) channel estimation scheme [4] was selected as an example of the conventional parametric scheme. System parameters were set as follows: the carrier frequency is  $f_c = 1$  GHz, the system bandwidth is  $f_s = 10$  MHz, the size of the OFDM symbol is  $N = 4096$ , and  $N_g = 256$  is the guard interval length  $P$ , which can combat channels whose maximum delay spread is  $25.6 \mu s$ . The International Telecommunication Union Vehicular B (ITU-VB) channel model with the maximum delay spread  $20 \mu s$  and the number of paths  $P = 6$  [4] were considered.

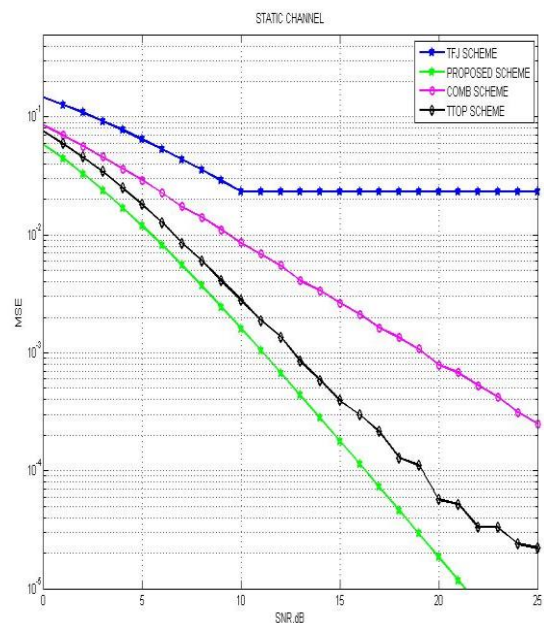


Figure 4 MSE performance comparison of different schemes in a  $4 \times 4$  MIMO System Static channel

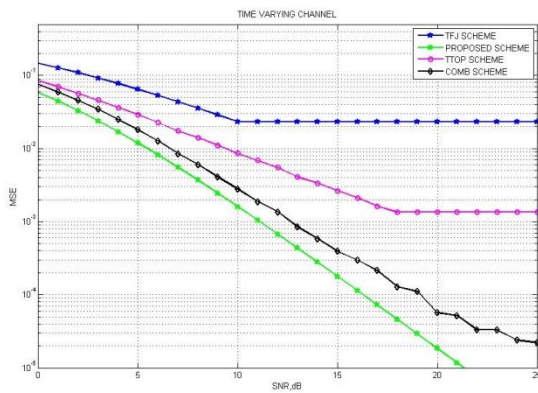


Figure: 5 MSE performance comparisons of different schemes in a  $4 \times 4$  MIMO System time-varying channel with the mobile speed of 90km/h

Fig.4& Fig.5 compares the mean square error (MSE) performance of different channel estimation schemes. Both the static ITUVB channel and the time-varying ITU-VB channel with the mobile speed of 90 km/h in a  $4 \times 4$  MIMO system were considered. The comb-type pilot based scheme used  $N_p=256$  pilots, the TTOP scheme used  $N_p=64$  pilots with  $T$  adjacent OFDM symbols for training, where  $T=4$  for the time-varying channel and  $T=8$  for the static channel to achieve better performance, the TFJ scheme adopted time-domain training sequences of 256-length and  $N_p=64$  pilots, and our proposed scheme used  $N_p=64$  pilots with  $R=4$  for fair comparison. From Fig. 3, we can observe that the conventional parametric TFJ scheme is inferior to the other three schemes obviously. Meanwhile, for static ITU-VB channel, the MSE performance of the proposed parametric scheme is more than 2 dB and 5 dB better than the TTOP and comb-type pilot based schemes, respectively. Moreover, for the time-varying ITU-VB channel, the superior performance of our proposed parametric scheme to conventional nonparametric schemes is more obvious. The existing sparse channel estimation scheme [4] does not work, because path delays may not be located at the integer times of the sampling period for practical channels. The TTOP scheme works well over static channels, but it performs poorly over fast time-varying channels, since it assumes that the

channel is static during the adjacent OFDM symbols. Finally, the comb-type pilot based scheme performs worse than our proposed scheme, and it also suffers from much higher pilot overhead.

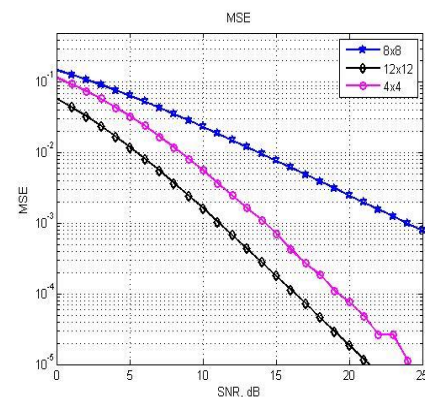


Figure 6 MSE performance of the proposed scheme in  $4 \times 4$ ,  $8 \times 8$ , and  $12 \times 12$  MIMO systems.

The MSE performance of the proposed scheme in  $12 \times 12$  MIMO system is superior to that in  $8 \times 8$  MIMO system by 5 dB with the same  $N_p$  and outperforms that in  $4 \times 4$  MIMO system with the reduced  $N_p$ . These simulations indicate that with the increased number of antennas, the MSE performance improves with the same  $N_p$ . Equivalently, to achieve the same channel estimation accuracy, the required number of pilots  $N_p$  can be reduced.

As a result, the total pilot overhead  $N_{p\_total}$  in our proposed scheme does not increase linearly with the number of transmit antennas  $N_t$ , because the required  $N_p$  reduces when  $N_t$  increases accordingly. The reason is that with the increased number of antennas, the dimension of the measurement matrix [e.g.,  $\hat{\mathbf{H}}$  in (8)] in the TLS-ESPRIT algorithm or the number of the sampling in the model of multiple filters bank [10] increases; thus, the accuracy of the path delay estimate improves accordingly. The superior performance of the proposed scheme is contributed by following reasons. First, the spatially common sparse pattern shared among CIRs of different transmit-receive antenna pairs is exploited in the proposed scheme, such that we can

employ the TLS-ESPRIT algorithm to obtain superresolution estimations of path delays with arbitrary values.

Meanwhile, the FRI theory indicates that the smallest required number of pilots is  $N_p = 2P$  in a noiseless scenario. Therefore, the pilot overhead can be reduced as compared with conventional nonparametric schemes. Second, our scheme exploits the temporal correlation of wireless channels, namely, across several adjacent OFDM symbols, the sparse pattern of the CIR remains unchanged, and path gains are also correlated. Accordingly, by joint processing of signals of adjacent OFDM symbols based on (8), the effective noise can be reduced, and thus, the accuracy of the channel estimation is improved further.

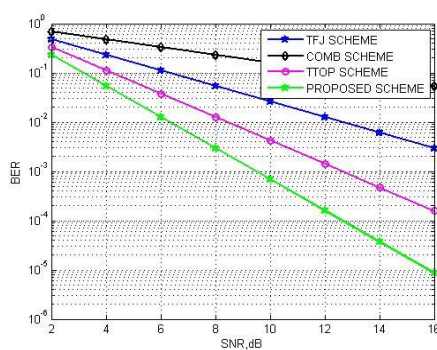


Figure 6 BER performance comparisons of different schemes in a  $4 \times 4$  MIMO System Static channel

### (VI) CONCLUSION

The proposed super-resolution sparse MIMO channel estimation scheme exploits the sparsity as well as the spatial and temporal correlations of wireless MIMO channels. It can achieve super-resolution estimates of path delays with arbitrary values and has higher channel estimation accuracy than conventional schemes. Under the framework of the FRI theory, the required number of pilots in the proposed scheme is obviously less than that in nonparametric channel estimation schemes. Moreover, simulations demonstrate that the average pilot overhead per transmit antenna will be interestingly reduced with the increased number of antennas.

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