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# Modeling and Design of Robust LQI Controller for DC-DC Buck Converter

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**Abstract:**

This paper presents the linear quadratic regulator with integral (LQI) action applied to design controller for a DC_ DC buck converter to get desired dynamic response. For design a controller in continuous mode by using LQI small signal average state space model is obtained and for digital controller design discrete modelling is obtained. This mathematical state space models reproduce the converters dynamic behavior. After modelling with the aim to obtain the optimal control law that minimizes the predefined cost function, the compensator design is performed using MATLAB. The controlling validated through simulation in SIMULINK.

**Keywords:**

DC-DC Buck converter, Small Signal Modeling, Discrete Modelling, Lqi Controller.

## I. INTRODUCTION:

The switch-mode DC–DC converter has evolved into an essential component for electronic equipment and is finding widespread applications in computers, battery chargers, and solar cell based power converters used in space power conditioning systems etc.These converters are nonlinear dynamical systems. The nonlinearities arise primarily due to switching, power devices, and passive components, such as inductors, and parasitic. State space averaging method can be used to model the non-linear DC_DC converter. This method proposes linearized model of nonlinear converter. With the linearized models do not predict large-signal stability information, and are only sufficient to predict small-signal stability. Small signal model is derived from the linearization around a nominal point of space state average model.

By using small signal state space modeling we can obtain different transfer functions, and hence different controllers to get desired response of DC_DC converter. By using small signal modeling of converter we can also obtain the state transition matrixes, which useful in finding discrete modelling of converter, and we can also find digital controller. Recently, the growing interest in practical digital control for high frequency DC-DC converters has prompted renewed interest in discrete-time analysis and modeling to facilitate precise direct digital compensator design. Regarding control strategies, linear-quadratic regulators (LQR) offer some interesting properties, such as that they can obtain “optimal” response of the system in accordance to the designer’s specifications, can be methodologically applied with independence of the order of the system, and are intrinsically stable. Furthermore, LQR can be straightforwardly calculated from the matrices of the small signal state-space averaged model of the system.

## II. STATE SPACE LARGE SIGNAL MODELING OF DC_DC CONVERTER:

State space modeling of switching converter can be done by using state space averaging technique. The DC_DC converter circuit shown in figure below

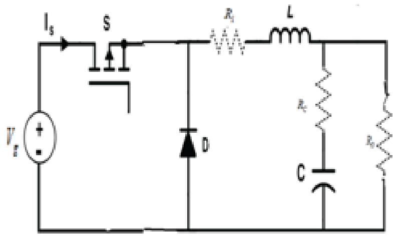


Fig 1. DC-DC Buck Converter

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During switch on condition:

$$\frac{dx}{dt} = A_1X + B_1U$$

$$\frac{dy}{dt} = C_1X + D_1U$$

Where

$$A_1 = \begin{bmatrix} -\frac{1}{(R+R_C)C} & \frac{R}{(R+R_C)C} \\ -\frac{R}{(R+R_C)L} & -((R_L + \frac{RR_C}{R+R_C})\frac{1}{L}) \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} \frac{R}{R+R_C} & \frac{RR_C}{R+R_C} \end{bmatrix},\ D_1 = 0 \cdot$$

During switch off condition:

$$\frac{dx}{dt} = A_2X + B_2U$$

$$\frac{dy}{dt} = C_2X + D_2U$$

Where

$$A_2 = \begin{bmatrix} -\frac{1}{(R+R_C)C} & \frac{R}{(R+R_C)C} \\ -\frac{R}{(R+R_C)L} & -((R_L + \frac{RR_C}{R+R_C})\frac{1}{L}) \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} \frac{R}{R+R_C} & \frac{RR_C}{R+R_C} \end{bmatrix},\ D_2 = 0 .$$

By using state space averaging technique, the converter can be modelled as

$$\frac{dx}{dt} = A\,X + B\,U$$

$$\frac{dy}{dt} = C\,X + D\,U$$

Where

$$A = \begin{bmatrix} -\frac{1}{(R+R_C)C} & \frac{R}{(R+R_C)C} \\ -\frac{R}{(R+R_C)L} & -((R_L + \frac{RR_C}{R+R_C})\frac{1}{L}) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{R}{R+R_C} & \frac{RR_C}{R+R_C} \end{bmatrix}$$

$$D = 0$$

## III. STATE SPACE SMALL SIGNAL MODELING OF DC_DC CONVERTER

$$\frac{d}{dt}\hat{x} = A\hat{x} + B\hat{u} + [(A_1 - A_2)X + (B_1 - B_2)U]\hat{d}$$

$$\hat{y} = C\hat{x} + D\hat{u} + [(C_1 - C_2)X + (D_1 - D_2)U]\hat{d}$$

Let $\hat{u} = 0$, it implies we assume perturbations in input is zero

Let $K = [(A_1 - A_2)X + (B_1 - B_2)U]$

Therefore

$K = B_1V_g$ Because $A_1 = A_2$ and $B_2 = 0$

Small signal modeling

$$\frac{d}{dt}\hat{x} = \tilde{A}\hat{x} + \tilde{B}\hat{u} \qquad \hat{y} = \hat{c}\hat{x}$$

Where $\tilde{A} = A$, $\tilde{B} = B_1V_g$ and $\tilde{C} = C$

$$\tilde{A} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 0 \\ \frac{V_g}{L} \end{bmatrix}$$

$$\tilde{C} = [1 \quad 0]$$

## IV. DISCRETE MODELING OF DC_DC CONVERTER TRAILING EDGE OFF TIME SAMPLING

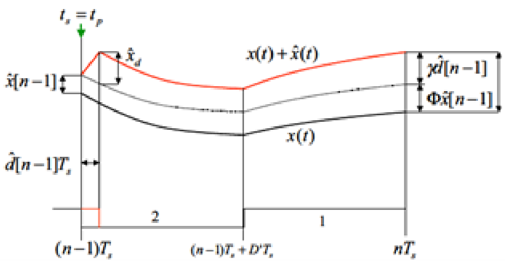


Fig 2. Trailing edge OFF Time Sampling

Above circuit is timing diagram of traling edge off time sampling. From above figure

During interval $(n-1)T_S \le t \le (n-1)T_S + (1-D)T_S$

$$\frac{d}{dt}\hat{x} = A\hat{x} + B\hat{u} + [(A_1 - A_2)X + (B_1 - B_2)U]\hat{d}$$

Assume $\hat{u} = 0$

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After state feedback with integral action, the system is in the form of:

$$\hat{x}(k+1)=\hat{\phi}\hat{x}(k)+\hat{\gamma}\hat{u}(k)$$

Where

$$\varphi=\begin{bmatrix}1-\frac{T_S}{RC} & \frac{T_S}{C}\\ -\frac{T_S}{L} & 1\end{bmatrix} \qquad \phi=\begin{bmatrix}0.8 & 0.2\\ -0.2 & 1\end{bmatrix}$$

$$\hat{\gamma}=\begin{bmatrix}-C\gamma\\ \gamma\end{bmatrix} \qquad \hat{\gamma}=\begin{bmatrix}-0.4\\ 0.4\\ 2\end{bmatrix}$$

On the other hand, the term is, $\Delta v(k)$ in fact, the error **e**(k ) from the references and the output variables:

$$\Delta v(k)=v(k)-v(k-1)$$

$$\Delta v(k)=r(k)-y(k)+v(k-1)-v(k-1)$$

Therefore

$$\Delta v(k)=e(k)$$

Where

$$\phi=e^{AT_S} \text{ and } \gamma=Ke^{AT_S}$$

$$K=[(A_1-A_2)X+(B_1-B_2)U]$$

$$\phi\approx I+AT_S$$

$$\phi\approx\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-\frac{1}{RC} & \frac{1}{C}\\ -\frac{1}{L} & 0\end{bmatrix}T_S$$

$$\varphi=\begin{bmatrix}1-\frac{T_S}{RC} & \frac{T_S}{C}\\ -\frac{T_S}{L} & 1\end{bmatrix}$$

$$K=\begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}[V_g]$$

$$\gamma=\begin{bmatrix}1-\frac{T_S}{RC} & \frac{T_S}{C}\\ -\frac{T_S}{L} & 1\end{bmatrix}\begin{bmatrix}0\\ \frac{V_g}{L}\end{bmatrix}$$

$$\gamma=\begin{bmatrix}\frac{T_S V_g}{CL}\\ \frac{V_g}{L}\end{bmatrix}$$

Hence discrete modeling of DC_DC buck converter with trailing edge ON time sampling is obtained These are useful for finding digital controller.

## PARAMETERS OF DC_DC BUCK CONVERTER:

$L=5\mu H$ , $R_L=25m\Omega$ , $C=5\mu F$ , $R_c=16m\Omega$

$R=1\Omega$, $V_g=10$ volts, $V_{ref}=1$ volt, $d=0.2$,

$f_S=1MH$ $T_S=1\mu$sec

## LQR Control With Integral Action Applied to the Converter:

The control strategy using state-feedback has been applied to allocate the poles of the closed loop system (if the system is a completely controllable state) in any position, chosen to meet design specifications. An advantage of the LQR method when compared to the allocation method is that the first one provides a systematic mode of calculation for the state feedback control gain matrix.

Let us consider a system defined by the following space state equation:

$$\frac{d}{dt}\hat{x}=\hat{A}\hat{x}+\hat{B}\hat{u} \qquad \hat{y}=\hat{C}\hat{x}+\hat{D}\hat{u}$$

The quadratic optimal regulator aims is to find matrix K for the optimal control vector given by so that the cost function in is minimized.

$$\hat{u}=-K\hat{x}$$

Determining the parameters in the above equations is a must to design the LQR controller. Here u is equivalent to the small signal duty cycle d. Analyzing above DC_DC converter model, it can be seen verified that the output voltage dynamic behavior of the buck converter shown in Figure ( ) does not have a pole placed at the state plane origin. Therefore, it is necessary to place an integrator into the controller in order to eliminate the static error between the control reference and the controlled variable, which in this case is the output voltage. In other words, the LQR control must have integral action. The block diagram of the LQR with integral action is shown in Figure ( 2 ) and is named specifically as LQI

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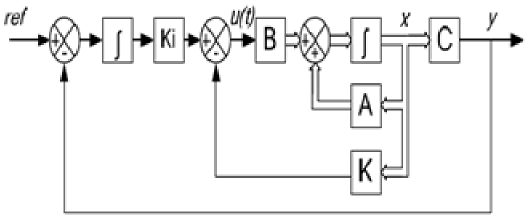


Fig. 2. Equivalent block diagram of the LQR control with integral action

The LQR with integral action (LQI) assumes the following expanded matrix configuration:

$$A_{NEW} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$$

$$B_{NEW} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$C_{NEW} = \begin{bmatrix} C & 0 \end{bmatrix}$$

Whose gain is given by

$$K = [k \quad -k_i]$$

k - State-feedback vector gain.
ki - Integral action error gain.
In this case, the model cost function is defined by

$$J = \int_{t_o}^{\infty} X^T(t)QX(t) + U^T(t)RU(t)dt$$

The gain K is obtained solving Riccati equation for P:

$$0 = Q - PBR^{-1}B^T P + PA + A^T P$$

After obtaing P, K can be obtained as $K = -R^{-1}B^T P^O$

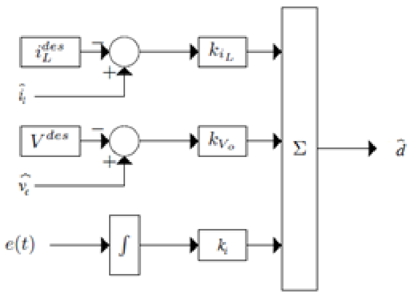


Fig. 3. Linear optimal control

After several MATLAB simulations are carried out, the best Q and R estimated values were found to be:

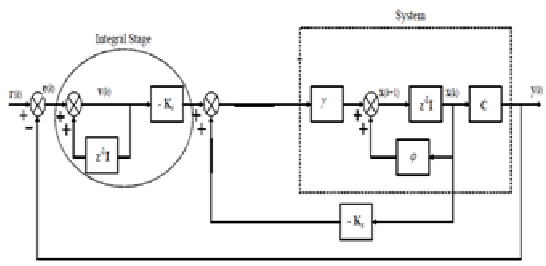
$$Q = \begin{bmatrix} 25\times10^{-3} & 0 & 0 \\ 0 & 75\times10^{-6} & 0 \\ 0 & 0 & 20\times10^{3} \end{bmatrix}$$

R=1;
The physical implement this control system, every state variable must be measurable and available for feedback. In other words, the system must be of controllable. Thus, the controllability test is performed as follows: the controllability matrix is determined using command Co=ctrb(Anew, Bnew) and then, the rank test of this matrix is done with rank (Co). If the rank order is the same one of Anew, then it is a controllable state system. Once the test is finished, the optimal gain of the state feedback vector is determined using instruction [k, P, e] = LQR(Anew, Bnew, Q, R). This instruction determines the solution of riccati equation corresponding to matrix P , determines the optimal gain of feedback. matrix K and the poles location of closed loop system. The gain values are

$$K = [-0.1274 \quad 0.1599 \quad 141.4214]$$

## LQI for digital controller design:

Additionally, due to the introduction of an integral actuation in the direct chain, zero-order errors caused by step references or disturbances can be removed in steady-state condition . This integral-type optimal regulator is called LQR with integral actuation.



LQR diagram with integral actuation.

After state feedback with integral action, the system is in the form of:

$$\hat{x}(k+1) = \hat{\phi}\hat{x}(k) + \hat{\gamma}\hat{u}(k)$$

Where

$$\varphi = \begin{bmatrix} 1 - \frac{T_S}{RC} & \frac{T_S}{C} \\ -\frac{T_S}{L} & 1 \end{bmatrix}$$

$$\phi = \begin{bmatrix} 0.8 & 0.2 \\ -0.2 & 1 \end{bmatrix}$$

$$\hat{\gamma} = \begin{bmatrix} -C\gamma \\ \gamma \end{bmatrix}$$

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$$\hat{\gamma} = \begin{bmatrix} -0.4 \\ 0.4 \\ 2 \end{bmatrix}$$

On the other hand, the term is, $\Delta v(k)$ in fact, the error **e**(k ) from the references and the output variables:

$$\Delta v(k) = v(k) - v(k-1)$$

$$\Delta v(k) = r(k) - y(k) + v(k-1) - v(k-1)$$

Therefore

$$\Delta v(k) = e(k)$$

Restrictions that must be considered when minimizing the cost function J in the LQR problem are:

$$J = \frac{1}{2}\sum_{k=1}^{N-1} X^T(k)QX(k) + U^T(k)RU(k)$$ and

$$\hat{x}(k+1) = \hat{\phi}\hat{x}(k) + \hat{\gamma}\hat{u}(k)$$

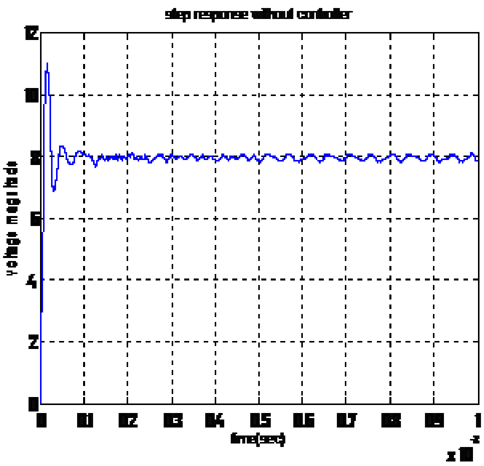
An optimal K matrix is obtained when the LQR problem is solved, which defines all the constants of the control diagram:

$$K = [k_i \quad k_x]$$

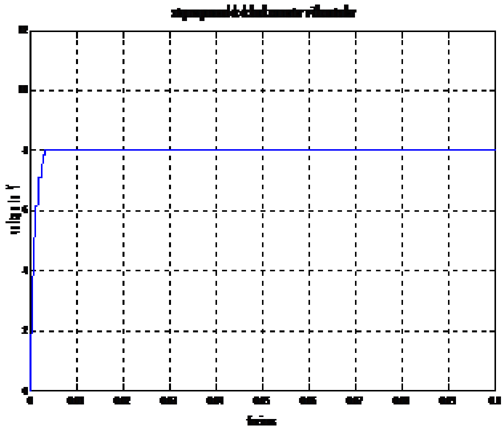
Integral and state feedback gains are

$$K = [340.4 \quad -1319.3 \quad 1993]$$

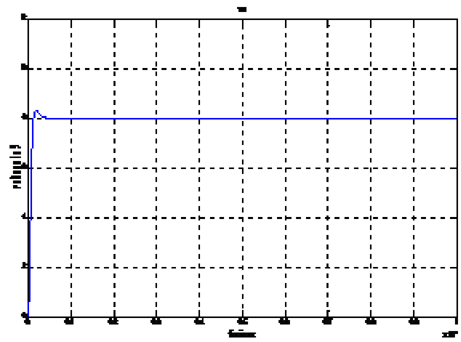
## IV. SIMULATION RESULT'S:
## DC_DC BUCK CONVERTER WITHOUT CONTROLLER



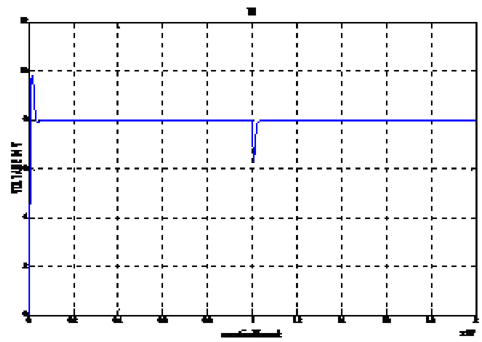

Step resonse of DC DC converter with LQI controller in Continuous mode.



Step response of DC_DC buck converter with LQI based controller in discrete mode



Step response of discrete lqi controller with load perturbations



## V.CONCLUSION:

State space modelling of DC_DC buck converter is obtained in both continuous and discrete modes. Discrete modelling is done for trailing edge off time sampling PWM DC_DC buck converter is obtained. By state space modelling optimal state regulatory problem is formed, with the aim to obtain the optimal control law that minimizes the predefined cost function, the compensator design is performed using

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MATLAB, and controlling is validated through simulation in SIMULINK OBSERVATIONS: Steady state behavior of DC_DC converter is improved by using LQI controller but transient response still same as before (without controller) Designed LQI controller is a robust controller, it is verified under load changes

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