Abstract:

Future wireless devices have to support many applications (e.g., remote robotics, wireless automation, and mobile gaming) with extremely low latency and reliability requirements over wireless connections. Optimizing wireless transceivers while switching between wireless connections with different circuit characteristics requires addressing many hardware impairments that have been overlooked previously. For instance, switching between transmission and reception radio functions to facilitate time division duplexing can change the load on the power supply. As the supply voltage changes in response to the sudden change in load, the carrier frequency drifts. Such a drift results in transient carrier frequency offset (CFO) that cannot be estimated by conventional CFO estimators and is typically addressed by inserting or extending guard intervals. In this paper, we explore the modeling and estimation of the transient CFO, which is modeled as the response of an under damped second order system. To compensate for the transient CFO, we propose a low complexity parametric estimation algorithm, which uses the null space of the Hankel-like matrix constructed from phase difference of the two halves of the repetitive training sequence. Furthermore, to minimize the mean squared error of the estimated parameters in noise, a weighted subspace fitting algorithm is derived with a slight increase in complexity. The Crámer–Rao bound for any unbiased estimator of the transient CFO parameters is derived. The performance of the proposed algorithms is also confirmed by the experimental results obtained from the real wireless transceivers.

Index Terms:

carrier frequency offset (CFO) estimation, damped sinusoid. Transient response, subspace decomposition, weighted subspace fitting (WSF).

1. INTRODUCTION:

In order to satisfy the exponential growing demand of wireless multimedia services, a high speed data access is required. Therefore, various techniques have been proposed in recent years to achieve high system capacities. Among them, we interest to the multiple-input multiple-output (MIMO). The MIMO concept has attracted lot of attention in wireless communications due to its potential to increase the system capacity without extra bandwidth. Multipath propagation usually causes selective frequency channels. To combat the effect of frequency selective fading, MIMO is associated with orthogonal frequency-division multiplexing (OFDM) technique. OFDM is a modulation technique which transforms frequency selective channel into a set of parallel flat fading channels. A cyclic prefix CP is added at the beginning of each OFDM symbol to eliminate ICI and ISI. The inserted cyclic prefix is equal to or longer than to the channel. The 3GPP Long Term Evolution (LTE) is defining the next generation radio access network. LTE Downlink systems adopt Orthogonal Frequency Division Multiple Access (OFDMA) and MIMO to provide up to 100 Mbps.

Optimizing wireless physical layer for capacity and reliability has led to many improvements from air interface design to signal processing techniques that mitigate radio frequency impairments. Validating these improvements in wireless testbeds has become an important step in fine-tuning the algorithms and identifying any unforeseen modeling errors. For example, the carrier frequency offset (CFO) is well understood radio frequency (RF) impairment in wireless systems. While experimentally validating the CFO estimators in our wireless testbed, we observed an unexpected CFO behavior in time division duplexing (TDD) operation, i.e., a transient CFO waveform caused by the switching between TX and RX in RF front-end hardware.
The impact of the transient CFO can be more pronounced in wireless systems where time division duplex (TDD) is deployed as the duplexing scheme. It must be taken into account in applications such as remote robotics, wireless automation, and mobile gaming where extremely low latency and reliability requirements have to be met over connections with future 5G or legacy wireless systems. Such systems include the next generation mobile communication systems, Long-Term Evolution (LTE) with TDD mode, and cooperative communications where TDD mode is operated in cooperative relay transmissions. For instance, TDD LTE is designed with guard period between the switching from the downlink to the uplink. However, the purpose of the guard period is to guarantee that user equipment (UE) can switch between reception and transmission with no overlap of signals, instead of dealing with the transient effect. In addition, the length of the guard period is designed to handle the propagation delay in the cells. Thus, the hardware design is more challenging for UE away from the base station due to the shorter switch time allowed. Another example is wireless sensor networks (WSN), which require robust wireless communication protocols with low latency and power consumption, switching between transmission and reception, and low-duty-cycle are extensively designed in the MAC layer protocols of WSNs. Therefore, any RF transient will have serious impact on the system performance, which is indexed by signal quality, latency and power consumption.

Here, we propose a parametric estimator based on the null space of the Hankel-like matrix constructed from the phase difference vector. The proposed method is motivated by the linear prediction problems. While the covariance matrix of noise in the subspace based method is not a scaled identity matrix, and, as a result, the proposed subspace based estimator may not be optimized. Then, with the perturbation analysis, we further propose a weighted subspace fitting (WSF) algorithm in order to improve the estimation accuracy. In the proposed WSF algorithm, the Hankel-like matrix is multiplied by a weighting matrix form left such that the estimation error of the objective vector is minimized when the noise is present. The objective vector is the right singular vector of the Hankel-like matrix corresponding to its least singular value. We show that the WSF approach is optimal in terms of estimation accuracy. The Crámer–Rao bound (CRB) for the estimation of the parameters of exponentially damped sinusoid with additive colored noise is presented and compared with the proposed estimators as well as the existing algorithms. Furthermore, the proposed algorithms come with much lower implementation complexity and are suitable for real-time calibration of the transient profile in compact wireless radios.

2. PROBLEM FORMULATION:

A. The Traditional CFO Estimation:
We assume that the receiver estimates the CFO through the preamble OFDM symbol, whose first and second halves are identical. Then, we can express the transmitted preamble signal. Where T is the time interval between the two halves. Thus, the received signal impaired by CFO, CFO between the transmitter and receiver. \( n(t) \) is much less than \( n(t) \). Hence, we drop this noise term thereafter. For the traditional CFO estimation, we assume that the CFO is constant.

B. Experimental Characterization of CFO:
The traditional CFO estimation algorithm presented above has been implemented on the testbed that we have set up using two terminals. As shown in Fig. 1(a), each terminal combines National Instruments (NI) FlexRIO family modules and the Ettus USRP RF front-ends. The terminals are configured as a master or a slave to operate in TDD mode for comprehensive real-time wireless experiments. We use orthogonal frequency division multiplexing (OFDM) as the modulation technique for our physical layer design. The time division duplexing (TDD) is operated with a frame structure defined in Fig. 2. The master terminal transmits the preamble OFDM symbol in the beginning of each frame, followed by four regular OFDM symbols.

![Fig. 1. (a) Hardware architecture of a testbed terminal. (b) System diagram of the testbed in real-time TDD operation.](image-url)
Correspondingly, after TX to RX switching, the slave terminal receives the preamble and four regular OFDM symbols consecutively. The slave terminal updates time and frequency synchronization through the preamble symbol. Table I lists the OFDM system parameters. We compare the estimated CFO and the expected CFO on our testbed running in TDD mode. Using a common reference for both the master and slave terminals, we expect the average CFO estimation in to be 0 Hz. However, as shown in Fig. 3, we observe quite surprising results from this experiment. The traditional CFO estimator has a bias of 1000 Hz over the preamble OFDM symbol contrary to the anticipated 0 Hz CFO. The estimated CFO by the traditional CFO estimator decays with time.

The output of the traditional CFO estimator reduces to 65 Hz in the second OFDM symbol and even reduces to less than 10 Hz in the fourth and fifth OFDM symbols. The estimated CFOs of the second to the fifth OFDM symbols are obtained by computing the phase shift between the received cyclic prefix (CP) and the end part of the OFDM symbol. Interestingly, similar phenomenon has been observed and extensively analyzed on Rice University WARP radios. As observed and extensively documented in this phenomenon can occur in compact and highly integrated RF transceivers that share a common power supply across the transmission and receive functions.

C. Transient CFO Model:

Our experiments and similar findings in Rice WARP boards have motivated us to develop algorithms to estimate and compensate the transient CFO introduced in compact RF transceivers. The voltage response of a dc–dc converter due to step change in load current can be approximated by a second order control system. We experimentally confirmed that the VCO&PLL power supply voltage waveform captured from oscilloscope matches exponentially damped sinusoid.

The relation between the VCO power supply voltage and the output frequency is approximately linear [26]. Then, we claim that the transient CFO, denoted by $\Delta \omega_T(t)$, can be modeled as the step response of a second order underdamped system, which can be expressed as an exponentially damped sinusoid. To model the transient CFO, we define the overall CFO in terms of two parts: a transient CFO and a steady state CFO. We assume that the steady state CFO, denoted by $\Delta \omega_S$, is constant during the preamble OFDM symbol. Then, the overall CFO can be written as

$$\Delta \omega(t) = \Delta \omega_T(t) + \Delta \omega_S.$$ 

In this paper, we assume that the steady state CFO is estimated and removed from the baseband signal to simplify the estimation of the transient CFO. This assumption is reasonable in practice since a terminal first detects the downlink preamble signal to obtain the time synchronization, the steady state CFO, and other necessary information to access to the wireless network during the initialization stage. The terminal stays in RX state during this stage and does not have to be running in TDD mode.

D. Noise Model of $n(t)$:

We can see from (7) that $u(t)$ is the phase difference between two vectors perturbed by white Gaussian noise. We assume that $n_1(t)$ and $n_2(t)$ are independent stationary circular Gaussian processes with zero mean and the variance of $\sigma^2_n$. We now scrutinize the multiplication of the conjugated first half and the second half of the received preamble OFDM symbol. With the statistical knowledge of noise in the phase difference vector. In addition, the performance of the proposed estimators are compared with the CRB in Section 4.

3. PARAMETRIC ESTIMATION:

A. Subspace Based Estimation of $a$ and $\omega_s$:

Estimating the parameters of damped sinusoid has been extensively studied in literature. The existing algorithms are, however, rather generic and can be generally put into four categories: direct fitting in time domain using signal samples, direct fitting in frequency domain with DFT based algorithms, covariance based linear prediction, and subspace (SVD) based linear prediction. The linear prediction problems first build a Hankel matrix $\Psi$ (also called prediction matrix) with signal samples.
Any nonzero vector in the null space of the noise free prediction matrix constructs the coefficients of signal self-prediction (auto-regression) model. Then, the complex roots of the prediction polynomial are found to compute the damping factor and frequency. The prediction filter order, \( L \), which is also the number of columns of the Hankel matrix, is chosen to optimize the estimation performance in the subspace based linear prediction problems, such as the Kumaresan–Tuft (KT) method and matrix pencil method. The authors of gave the first order perturbation analysis for the KT method as

\[
V \ar(\omega_s) \approx \frac{4}{3} \gamma L(N - L) 2 1 \leq L \leq N - L
\]

where \( \gamma \) is the signal-to-noise ratio, and \( N \) and \( L \) are the number of signal samples and columns in the Hankel matrix, respectively. An optimal \( L \approx N/3 \) can be obtained to minimize the estimation error. The optimal \( L \) can be a quite large number given a large data size of \( N \). Increasing \( L \) does not change the rank of the noise free prediction matrix because the rank is determined by the signal order. Meanwhile, when using a large \( L \), the columns of the prediction matrix become less correlated, which leads to a well conditioned matrix. As a result, the estimated signal space as well as the null space is more accurate. This insight motivates us to build the prediction matrix by a Hankel-like matrix structure with less correlated columns. We introduce a time lag \( r \) to build the Hankel-like matrix. If there is one damped sinusoid (single mode) in the phase difference vector, the time lag \( r \) can be chosen to guarantee that the columns of the prediction matrix are less correlated.

Thus, the proposed estimator can still output good estimation accuracy. Meanwhile, the prediction matrix is limited to only three columns (minimum needed for single mode real signal), reducing the computational complexity significantly. We present the selection of \( r \) in terms of minimizing the estimation mean square error (MSE) in Section 3-C. Any non zero vector in the null space of the noise free prediction matrix is the vector of prediction coefficients. When processing the samples in noise, the right singular vector of \( \Psi_r \) corresponding to its least singular value is tan estimate of \( v_0 \). Furthermore, \( \hat{a} \) and \( \hat{\omega}_s \) can be computed from the zeros of the prediction polynomial. The existing subspace based methods, like the KT method and the matrix pencil method [19], improve the estimation accuracy by the truncated SVD approximation. For instance, the KT method (\( L < N - L - 1 \)) decomposes the noise into \( L \) components by using the truncated SVD approximation, and most of the noise components are removed when \( L \) is large. The proposed estimator will not have this benefit because the prediction matrix is limited to three columns. However, keeping the prediction matrix with only three columns lowers the computational complexity with a slight degeneration in performance compared to the methods using low-rank approximation.

### B. Performance Analysis and Weighted Subspace Fitting:

The existing algorithms in literature do not take the subspace structure of the noise matrix into account, and the estimation error may still be substantial. Here, we propose a weighted subspace fitting (WSF) algorithm, in which the subspace structure of the noise matrix is exploited to minimize the mean square error (MSE) of the estimated \( v_0 \). The WSF algorithm is proposed based on the perturbation analysis of the subspace based estimator. We use the first order subspace perturbation analysis presented. The Hankel like matrix is \( \Psi_r = H_r + W_r \), where \( \Psi_r \) and \( W_r \) are defined in (16) and (17), respectively.

we have compared the proposed and existing methods in terms of the estimation accuracy. On the other hand, an advantage of the proposed methods is the lowered computational complexity without sacrificing the estimation accuracy. The reduced computational complexity of the proposed method without WSF is obvious because the Hankel-like matrix has only three columns. We compare the computational complexity of the proposed algorithm without WSF and the KT method. For the SVD operation, the computational cost is reduced from \( O(\text{mL}^2) \) to \( O(\text{m} \cdot 32) \), where \( \text{m} \) is the number of rows of the prediction matrix, \( L \) and \( 3 \) are the number of columns of the prediction matrix in the KT and the proposed method, respectively. \( L \) is set to be \( N/3 \), which is much greater than 3, to obtain optimal performance in the KT method. The number of rows of the prediction matrix in the proposed and the KT methods are \( N - r - 1 \) and \( N - L - 1 \), respectively. The two values are comparable and impact little on making the conclusion of the computational complexity. Thus, we use \( m \) as the number of rows for both methods. Furthermore, to find the roots of the polynomial, the computational cost of the proposed method is much reduced because the orders of the polynomials are 2 and \( L - 1 \) for the proposed and KT methods, respectively.
There is a limited increase in the computational complexity for the proposed method with WSF. The extra operation needed for the WSF algorithm is to compute the weighting matrix D. We are able to exploit the sparsity of Γ to reduce the computational cost. First, the Cholesky decomposition of Γ, which is a lower triangular matrix, denoted by P, has a similar matrix structure as Γ: the main, −rth, and −2rth diagonals of P are non-zero. The number of multiplication operations to obtain each row of P is at most six. Thus, the total number of multiplication operation of the Cholesky decomposition is O(6m) instead of O(m^3). Since D = P^(-1), D is also a lower triangular matrix with non-zero entries on the main, −rth, . . . , −(N − 1)/r rth diagonals. The number of multiplication operations on each non-zero entry is at most three. Thus, the total number of multiplication to compute P^(-1) is approximately O(3m^2/r) instead of O(m^3). Therefore, we conclude that the computational complexity of the proposed method with WSF has a slight increase comparing to the algorithm without WSF.

<table>
<thead>
<tr>
<th>Method</th>
<th>Normalized Computational Time</th>
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<tr>
<td>Proposed Subspace Method</td>
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<tr>
<td>Proposed Subspace Method with WSF</td>
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<tr>
<td>Matrix Pencil Method</td>
<td>35.56</td>
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<tr>
<td>KT Method</td>
<td>103.52</td>
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### IV. NUMERICAL EXAMPLES AND EXPERIMENTAL RESULTS;

In this section, we first present numerical examples of the parametric estimation algorithms and their performance comparison. Then, we verify the proposed algorithm using experimental results collected from our real-time wireless testbed described in Section II-B. For the numerical simulations, the received signal is impaired by the transient CFO as shown in (6).

The transient CFO is determined by a set of parameters of a, ωs, φψ, αψ with typical values. The typical values are picked according to the parametric estimation results with the collected data from our hardware testbed. In our experimental setup, the number of the observations, N is set to be 128.

Fig. 6 shows the original ψ(t) versus the estimated ˆψ(t). ψ(t) is obtained from the collected samples. In addition, ˆψ(t) is estimated by the proposed algorithm with WSF.

Fig. 7 compares the mean square error (MSE) of the estimated damping factor and frequency with different methods. The methods compared include the proposed subspace methods with WSF, the KT method and the matrix pencil method. The CRB for the estimated damping factor and frequency with the colored noise, which is modeled in (13), is also given in Fig. 7 for comparison. We can see that the WSF approach clearly outperforms the proposed subspace estimator without WSF and other existing methods. The gap between the WSF algorithm and the CRB is approximately 1–2 dB. The MSEs are averaged over 500 trials at each SNR level.

Fig. 8 shows the comparison of the received preamble OFDM symbol associated with different processing schemes. The QPSK demodulation is used on all subcarriers in the received preamble OFDM symbol. Fig. 8(a) shows the demodulated symbols without removing the transient CFO from the received samples. Fig. 8(b) and (c) shows the symbols after removing the transient CFO from the received samples by the proposed algorithms with and without the WSF. We can see that the transient CFO significantly impacts the received preamble OFDM symbol. Removing the transient CFO on the received preamble OFDM symbol significantly improves the SNR of demodulated symbols.
The proposed WSF approach shows the best performance in terms of the detection SNR.

4. CONCLUSION:

In this paper, we investigate a unique problem in compact wireless transceivers: transient carrier frequency offset (CFO). The transient CFO is observed under TX/RX switching in systems that require time division duplex (TDD) operation. We demonstrated that the transient CFO can be modeled as the step response of an underdamped second order system. To digitally compensate for the transient CFO, we propose algorithms based on the subspace decomposition of the Hankel-like matrix. A weighted subspace fitting algorithm is also proposed to improve the estimation accuracy. The performance analysis is verified based on both numerical simulations and experimental results from the testbed collected samples. The transient impairments arise in devices that need to switch between various radio functions and degrade the system performance through the distorted signal. The transient impairments would cause more concern in future wireless devices, which have to support many applications (e.g., remote robotics, wireless automation, and mobile gaming) with extremely low latency and reliability requirements over wireless connections. This paper details such an example and solves it by advanced digital signal processing.

REFERENCES:


