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Estimation of Transverse Vibrations in Beams

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Abstract: ***Vibrations can be described as a linear combination of the modes of a structure. An alternative is to describe vibrations as propagating waves travelling in the structure. It was found that the two descriptions often give informative corresponding perspectives. Wave propagation, transmission and reflection in solids have been studied by a number of researchers.. However, it is not practical to solve the full problem because it yields more information than usually needed in applications. Therefore, approximate solutions for transverse displacement are sufficient. The beam theories under consideration all yield the transverse displacement as a solution. Early researchers concluded that the bending effect is the single most important factor in a transversely vibrating beam. The Euler Bernoulli model includes the strain energy due to the bending and the kinetic energy due to the lateral displacement. Jacob Bernoulli in 1654-1705 first discovered that the curvature of an elastic beam at any point is proportional to the bending moment at that point. After this Daniel Bernoulli in 1700-1782, who formulated the differential equation of motion of a vibrating beam Understanding the change in material properties in time is necessary for in-service diagnosis of structures and prevention of accidents Therefore, they presented a new experimental technique for evaluating Young's modulus of a vibrating thin sheet from dynamic considerations. The technique utilizes bending resonance from a remote acoustic excitation equations relating the natural frequencies to the mechanical properties are obtained, and Young's modulus is subsequently determined experimentally using the implemented dynamic measurement method Beam theory which adds the effect of shear as well as the effect of rotation to the Euler-Bernoulli beam. The Timoshenko model is a major improvement for non-slender beams and for high-frequency responses where shear or rotary effects are not negligible. Following Timoshenko, several authors have obtained the frequency equations and the mode shapes for various boundary conditions.***

Keywords: ***transverse vibration, Euler Bernoulli theory, Timoshenko theory, vibrations.***

INTRODUCTION

By doing some mathematical elaborations on the method, transverse vibration analysis of uniform and no uniform Euler-Bernoulli beams will be briefly explained and demonstrated with some examples by using some of these novel approaches[1]. To this aim, the theory and analytical techniques about lateral vibration of Euler-Bernoulli beams will be explained first, and then the methods[2] used in the analysis will be described. Finally, some case studies will be presented by using the proposed techniques and the advantages[3] of those methods will be discussed.

BEAM: A beam is a horizontal or vertical structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.

Types of Beams:

Beams are characterized by their profile (the shape of their cross-section), their length, and their material. In

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contemporary constructions, beams are typically made of steel, reinforced concrete or wood. A simple beam is supported by a pin support at one end and a roller support at the other end. A beam with a laterally and rotationally fixed support at one end with no support at the other end is called a cantilever beam. A beam simply supported at two points and having one end or both ends extended beyond the supports is called an overhanging beam.

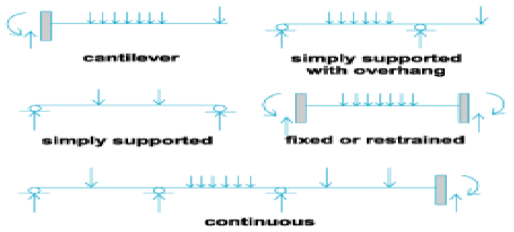

Fig 1: showing the different types of beams.

Boundary considerations:

The beam equation contains a fourth order derivative in *x*. To find a unique solution ω(*x*,*t*),we need four boundary conditions. The boundary conditions usually model supports, but they can also model point loads, distributed loads and moments[12]. The supports or displacement boundary conditions are used to fix values of displacement boundary conditions are also called to fix values of displacement (ω) and rotations dw/*dx* boundary conditions are also called Dirichlet boundary conditions[13-15].

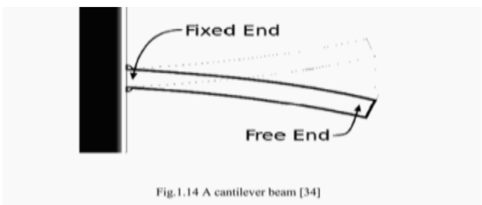

Fig 2: cantilever beam

TRANSVERSE VIBRATION OF EULER BEAM:

It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam[16]. The Euler-Bernoulli model includes the strain energy due to the bending and the kinetic energy due to the lateral displacement. However, the Euler-Bernoulli model tends to slightly overestimate the natural frequencies[17]. This problem is exacerbated for the natural frequencies of the higher modes[18].

Mathematical formulation:

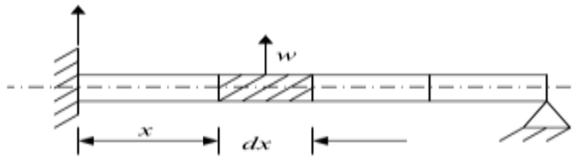

Fig: 3 A beam under transverse vibration

Consider a long slender beam as shown in figure 1 subjected to transverse vibration. The free body diagram of an element of the beam is shown in the figure 2. Here, *M*(*x*, *t*) is the bending moment, *V*(*x*, *t*) is the shear force, and *f*(*x*, *t*) is the external force per unit length of the beam[19-20]. Since the inertia force acting on the element of the beam is

$$\rho A(x)dx\frac{\partial^2 w(x,t)}{\partial t^2}$$

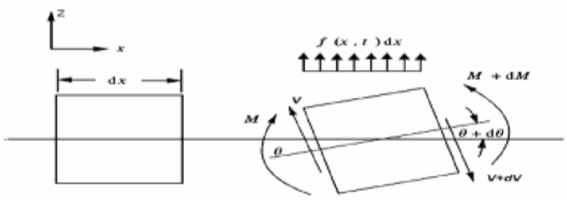

Fig: 4 Free body diagram of a section of a beam under transverse vibration

Balancing the forces in *z* direction gives

$$-(V+dV)+f(x,t)dx+V=\rho A(x)dx\frac{\partial^2 w(x,t)}{\partial t^2},$$

Where ρ is the mass density and*A*(*x*) is the cross-sectional area of the beam. The moment equation about the *y* axis leads to

$$(M+dM)-(V+dV)dx+f(x,t)dx\frac{dx}{2}-M=0$$

By writing

$$dV=\frac{\partial V}{\partial x}dx \qquad \text{and} \qquad dM=\frac{\partial M}{\partial x}dx$$

For a uniform beam above equation reduces to

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$$\left[EI(x)\frac{\partial^4 w(x,t)}{\partial x^4}\right]+\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}=f(x,t)$$

Where

$$c=\sqrt{\frac{EI}{\rho A}}$$

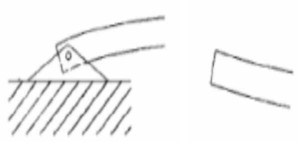
Transverse vibration of beams is an initial-boundary value problem. Hence, both initial and boundary conditions are required to obtain a unique solution w(x,t)[22-24]. Since the equation involves a second order derivative with respect to time and a fourth order derivative with respect to a space coordinate, two initial conditions and four boundary conditions are needed[25].

Initial Conditions:

Since the equation of the motion involves a second order derivative with respect to time and a fourth order derivative with respect to *x*, two initial equations and four boundary conditions are needed for finding a unique solution for *w*(*x*, *t*). Usually, the values of transverse displacement and velocity are specified as $W_0(x)$ and $W_0(x)$ at t=0, so that the initial conditions become:

$$w(x,t=0)=w_0(x)$$
$$\frac{\partial w(x,t=0)}{\partial t}=\dot{w}_0(x)$$

Boundary Conditions:



Hinged, $M(0,t)=0$, $D(0,t)=0$; Free, $M(0,t)=0$, $Q(0,t)=0$

Fig: 5 Boundary conditions

The value of $\beta_n L$is unknown and is determined using the boundary condition of the beam given. For different boundary conditions we get different equations. While the case of simply supported beam admits a closed form solution, the other equation has to be solved numerically. All these equations have a number of solutions each corresponding to different modes of vibration[26]. The first, second and third positive solutions were determined for each case and the solutions obtained are used to determine the frequencies of vibration for three modes[27].

Free End:

Frequency Equation: $\sin\beta_n l=0$,

Mode Shape: $w_n=C_n\sin\beta_n x$.

$$\beta_1 l=\pi$$
$$\beta_2 l=2\pi$$
$$\beta_3 l=3\pi$$

Simply supported (pinned) end:

Frequency Equation: $\cos\beta_n l.\cos\beta_n l=1$

Mode Shape: $w_n=C_n\left[\sin\beta_n x+\sinh\beta_n x+\left(\frac{\sin\beta_n x-\sinh\beta_n x}{\cosh\beta_n x-\cos\beta_n x}\right)(\cos\beta_n x+\cosh\beta_n x)\right]$

$$\beta_1 l=4.730041$$
$$\beta_2 l=7.853205$$
$$\beta_3 l=10.995608$$
$$\beta_4 l=14.137165$$

Step-1

Matlab programming for simply supported pinned-pinned beam-5:

```
function SSbeam(~)
% SSbeam.m Simply-supported or Pinned-pinned
beam evaluations
% This script computes mode shapes and
corresponding natural
% frequencies of the simply-supported beam by user
specified mechanical
% properties and size of the beam.
% Prepare the following data:
% - Material properties of the beam, i.e. density (Ro),
Young's modulus (E)
% - Specify a cross section of the beam, i.e. square,
rectangular, circular
% - Geometry parameters of the beam, i.e. Length,
width, thickness
% - How many natural frequencies and mode shapes to
evaluate.
clear all;
clc;
```

ISSN No: 2348-4845
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```
close all;
display('What is the cross-section of the beam?' )
disp('If circular cross-section, enter 1; If square, enter 2; ' );
disp('If rectangle enter 3' );
disp('To see example #2, enter 5' );
CS=input(' Enter your choice: - ' );
if isempty(CS) || CS==0
disp('Example #1. Rectangular cross-section Aluminum beam' )
disp('Length=0.321 [m], Width=0.05 [m], Thickness=0.006 [m];' )
disp('E=69.9*1e9 [Pa]; Ro=2770 [kg/m^3] ' )
L=.321;
W=.05;
Th=.006;
A=W*Th;
Ix=(1/12)*W*Th^3;
E=69.90e+9;
Ro=2770;
elseif CS==1
R=input('Enter Radius of the cross-section: ' );
L=input('Enter Length: ' );
Ix=(1/4)*pi*R^4;
A=pi*R^2;
disp('Material proprties of the beam' );
display('Do you know your beam''s material properties, viz. Young''s modulus and density ?' );
YA=input('Enter 1, if you do;; enter 0, if you don''t: ' );
if YA==1
   E=input('Enter Young' s modulus in [Pa]: ' );
Ro=input('Enter materials density in [kg/m^3]: ' );
else
display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3] ' );
display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3] ' );
display('Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3] ' );
E=input('Enter Young' s modulus in [Pa]: ' );
Ro=input('Enter materials density in [kg/m^3]: ' );
end
elseif CS==2
W=input('Enter Width of the cross-section: ' );
L=input('Enter Length: - ' );
Ix=(1/12)*W^4;
A=W^2;
disp('Material proprties of the beam' );
display('Do you know your beam's material properties, i.e. Young' s modulus and density ?' );
YA=input('Enter 1, if you do; enter 0, if you don't: ' );
if YA==1
E=input('Enter Young' s modulus in [Pa] ' );
Ro=input('Enter the material density in [kg/m^3] ' );
else
display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3] ' )
display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3] ' )
E=input('Enter Young' s modulus in [Pa]: ' );
Ro=input('Enter the material density in [kg/m^3]: ' );
end
elseif CS==3
W=input('Enter Width of the cross-section in [m]: ' );
Th=input('Enter Thickness of the cross-section in [m]: ' );
L=input('Enter Length in [m]: ' );
Ix=(1/12)*W*Th^3;
A=W*Th;
disp('Material proprties of the beam' )
display('Do you know your beam' s material properties, viz. Young' s modulus and density ?' )
YA=input('Enter 1, if you do; enter 0, if you don' t: ' );
if YA==1
E=input('Enter Young's modulus in [Pa]: ' );
Ro=input('Enter the material density in [kg/m^3]: ' );
else
display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3] ' )
display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3] ' )
display('Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3] ' )
E=input('Enter Young' s modulus in [Pa]: ' );
Ro=input('Enter the material density in [kg/m^3]: ' );
end
elseif CS==4
display('Note: you need to compute Ix (area moment of inertia along x axis) and X-sectional area' )
L=input('Enter Length in [m]: ' );
Ix=('Enter Ix in [m^4]: ' );
```

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```
A=('Enter cross-x-sectional area in [m^2]: ' );
disp('Material properties of the beam' )
display('Do you know your beam''s material
properties, viz. Young''s modulus and density ?' );
YA=input('Enter 1, if you do; enter 0, if you don''t: ' );
if YA==1
E=input('Enter Young''s modulus in [Pa]: ' );
Ro=input('Enter material density in [kg/m^3]: ' );
else
display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3] ' )
display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3] '
)
display('Aluminum: E=0.69e+11 [Pa]; Ro=2700
[Kg/m^3] ' )
E=input('Enter Young''s modulus in [Pa]: ' );
Ro=input('Enter material density in [kg/m^3]: ' );
end
elseif CS==5
display('Example #2' )
display('It is a rectangular X-section Aluminum beam '
)
display('Length=0.03; Width=0.005;
Thickness=0.0005; ' )
L=.03; W=.005; Th=.0005;
A=W*Th;
Ix=(1/12)*W*Th^3;
E=70*1e9; Ro=2.7*1e3;
else
F=warndlg('It is not clear what your choice of X-
section of a beam is. e run so you can enter your
beam''s data!!! ' , '!! Warning !! ' );
waitfor(F)
display('Type in:>> SSbeam' )
pause(3)
return
end
display('How many modes and mode shapes would
you like to evaluate ?' )
HMMS=input('Enter the number of modes and mode
shapes to be computed: ' );
if HMMS>=7
disp(' ' )
end
jj=1;
while jj<=HMMS;
betaNL(jj)=jj*pi;
jj=jj+1;
end
%fprintf('betaNL value is %2.3f\n', betaNLall);
betaN=(betaNL/L)';
display('Mode shape corresponding nat. freq (fn) by
Euler-Bernoulli theory:' )
wn=zeros(1,length(betaN));
fn=ones(1,length(wn));
k=1;
while k<=length(betaN);
wn(k)=betaN(k)^2*sqrt((E*Ix)/(Ro*A));
fn(k)=wn(k)/(2*pi);
fprintf('Mode shape # %2f corresponds to nat. freq
(fn): %3.3f/n',k,fn(k) );
k=k+1;
end
x=linspace(0, L, 720);
xl=x./L;
Xnx=zeros(length(betaN),length(x));
for ii=1:length(betaN)
for jj=1:length(x)
Xnx(ii,jj)=sin((ii*pi*x(jj))/L);
end
end
XnxMAX=max(abs(Xnx(1,1:end)));
Xnx=Xnx./XnxMAX;
% Plot mode shapes that are arbitrarily normalized to
unity;
display('NOTE: Upto 5 mode shapes are displayed via
the script options.' );
disp(' Yet, using evaluated data (Xnx) of the script,
more mode shapes can be plotted' );
MMS=HMMS;
if MMS==1
plot(xl,Xnx(1,:), 'b-' )
title('Mode shapes of the Pinned-pinned beam' )
legend('Mode #1' , 0); xlabel('x/L' ); ylabel('Mode
shape X_n(x)')
grid
hold off
elseif MMS==2
plot(xl,Xnx(1,:), 'b-' ); hold on
```

```
plot(xl,Xnx(2,:), 'r-' );grid
title('Mode shapes of the Pinned-pinned beam' )
legend('Mode #1' , 'Mode #2' , 0)
xlabel('x/L' ); ylabel('Mode shape X_n(x)' )
hold off
elseif MMS==3
plot(xl,Xnx(1,:), 'b-' ); hold on
plot(xl,Xnx(2,:), 'r-' )
plot(xl,Xnx(3,:), 'm-' );grid
title('Mode shapes of the Pinned-pinned beam' )
legend('Mode #1' , 'Mode #2' , 'Mode #3' , 0)
xlabel('x/L' ); ylabel('Mode shape X_n(x) ' )
hold off
elseif MMS==4
plot(xl,Xnx(1,:), 'b-' ); hold on
plot(xl,Xnx(2,:), 'r-' )
plot(xl,Xnx(3,:), 'm-' )
plot(xl,Xnx(4,:), 'c-' ); grid
title('Mode shapes of the Pinned-pinned beam' )
legend('Mode #1' , 'Mode #2' , 'Mode #3' , 'Mode #4' ,
0)
xlabel('x/L' ); ylabel('Mode shape X_n(x) ' )
hold off
elseif MMS==5
plot(xl,Xnx(1,:), 'b-' ); hold on
plot(xl,Xnx(2,:), 'r-' )
plot(xl,Xnx(3,:), 'm-' )
plot(xl,Xnx(4,:), 'g-' )
plot(xl,Xnx(5,:), 'k-' )
grid
title('Mode shapes of the Pinned-pinned beam' );
legend('Mode #1' , 'Mode #2' , 'Mode #3' , 'Mode #4' ,
'Mode #5' , 0);
xlabel('x/L' ); ylabel('Mode shape X_n(x) ' );
hold off
elseif MMS>=6
plot(xl,Xnx(1,:), 'b-' ); hold on
plot(xl,Xnx(2,:), 'r-' )
plot(xl,Xnx(3,:), 'm-' )
plot(xl,Xnx(4,:), 'g-' )
plot(xl,Xnx(5,:), 'k-' )
plot(xl,Xnx(6,:), 'c-' )
grid
title('Mode shapes of the Pinned-pinned beam' )
legend('Mode #1' , 'Mode #2' , 'Mode #3' , 'Mode #4' ,
'Mode #5' , 'Mode#6' , 0)
end
xlabel('x/L' ); ylabel('Mode shape X_n(x) ' )
hold off
%% By Timoshenko beam theory, natural frequency
of simply-supported beam
is
% found to be:
alphaSQ=E*Ix/(Ro*A);
rSQ=Ix/A;
% Inertia (effects) is considred
disp('Nat. freq. by Timoshenko (Inertia alone
considered): ' )
omegaNi=zeros(1,HMMS);
for ii=1:HMMS
omegaNi(ii)=sqrt((alphaSQ*ii^4*pi^4)/(L^4*(1+(ii^2*
pi^2*rSQ)/(L^2))));
fprintf('mode # %2f corresponds to nat. freq. (fn)
%3.3f \n' , ii,omegaNi(ii)/(2*pi));
end
% Shear deformation is considred
EoverKG=[1, 2, 3];
for ii=1:HMMS
for jj=1:3
omegaNs(ii,jj)=sqrt((alphaSQ*ii^4*pi^4)/(L^4*(1+(ii^
2*pi^2*rSQ*EoverKG(jj))/(L^2))));
end
end
display('Nat. freq. by Timoshenko (Shear deformation
alone considered) ' )
disp('for 3 values of E/kG = [1; 2; 3]. ' )
fprintf(' %3.3f\t %3.3f \t%3.3f\t \n' , omegaNs'/(2*pi))
disp('In order to use classical beam or Euler-Bernoulli
beam calc''s. One should ' )
disp('pay attention to Length/width ratio that must be
larger than 10' )
FF=warndlg('pay attention to Length/width ratio that
must be greater than 10 to use Euler-Bernoulli beam
method!!! ' , '!! Warning !!' );
waitfor(FF)
end
```

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STEP-2:

Results and Discussion of output: The mode shapes of a beam were calculated and the analysis was done using the finite element method by calculating the characteristic matrices (mass matrix and stiffness matrix)of the given simply supported beam. The natural frequencies and the mode shape of the given hinged hinged type beam were calculated using MATLAB. The natural frequencies can show the pattern of the resonance that a beam is going to follow and its effect on structures. The mode shapes of a structural steel beam with given slenderness ratio and circular cross section was calculated and the following result was obtained:

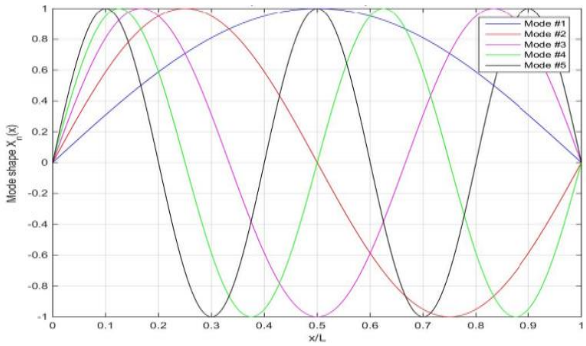
STEP-3:

The output is:

What is the cross-section of the beam?
If circular cross-section, enter 1; If square, enter 2;
If rectangle, enter 3;
If your beam's cross-section is not listed here, enter 4
To see example #2, enter 5
Enter your choice: 1
Enter Radius of the cross-section: 25
Enter Length: 350
Material properties of the beam
Do you know your beam's material properties, i.e. Young's modulus and density ?
Enter 1, if you do; enter 0, if you don't: 0
Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3]
Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3]
Aluminium: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3]
0.824 0.819 0.814
3.236 3.161 3.091
7.071 6.737 6.446
Enter Young's modulus in [Pa]: 2.1e+11
Enter material density in [kg/m^3]: 7850
How many modes and mode shapes would you like to evaluate?
Enter the number of modes and mode shapes to be computed: 6
Mode shape corresponding nat. freq (fn) by Euler-Bernoulli theory:
Mode shape # 1.000000 corresponds to nat. freq (fn): 0.829
Mode shape # 2.000000 corresponds to nat. freq (fn): 3.316
Mode shape # 3.000000 corresponds to nat. freq (fn): 7.461
Mode shape # 4.000000 corresponds to nat. freq (fn): 13.264
Mode shape # 5.000000 corresponds to nat. freq (fn): 20.726
Mode shape # 6.000000 corresponds to nat. freq (fn): 29.845
NOTE: Up to 5 mode shapes are displayed via the script options.
Yet, using evaluated data (Xnx) of the script, more mode shapes can be plotted
Nat. freq. by Timoshenko (Inertia alone considered):
mode # 1.000000 corresponds to nat. freq. (fn) 0.824
mode # 2.000000 corresponds to nat. freq. (fn) 3.236
mode # 3.000000 corresponds to nat. freq. (fn) 7.071
mode # 4.000000 corresponds to nat. freq. (fn) 12.102
mode # 5.000000 corresponds to nat. freq. (fn) 18.076
mode # 6.000000 corresponds to nat. freq. (fn)
24.75812.102 11.199 10.473
18.076 16.236 14.864
24.758 21.615 19.429
In order to use classical beam or Euler-Bernoulli beam calculations. One should
pay attention to Length/width ratio that must be larger than 10 Nat. freq. by Timoshenko (Shear deformation alone considered) for 3 values of E/kG = [1; 2; 3].

STEP-4:

Using Matlab coding for the above equation for simply supported pinned-pinned beam result and plots:



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STEP-1:
5.8 Matlab programming for cantilever beam-6:

```
% input data
%
% rho: density
% E  : Young's modulus
% L  : length of the beam
% b  : width of a rectangular beam
% h  : thickness ofthe beam
%
rho = 2.77e+03;
E = 70e+09;
L = 0.52;
b = 0.0254;
h = 3.175e-03;
%
% vibration of a cantilever beam;
%
%
% input data
%
% rho: density
% E  : Young's modulus
% L  : length of the beam
% b  : width of a rectangular beam
% h  : thickness ofthe beam
%
%
% output
%
% modes(1:3)       : the first three modes in Hertz
% modeshapes(3, 40) : the vibration mode shapes of
the first three modes
%
% constant to be declared
rho = 2.77e+03;
E = 70e+09;
L = 0.52;
b = 0.0254;
h = 3.175e-03;
modes=zeros(3,1);
modeshapes=zeros(3,40);
beta = [1.875  4.694  7.856];
I = b*h^3/12;
A = b*h;
mu = rho*A;
% loop over the three modes
for i=1:3,
%
modes(i) = beta(i)^2 * sqrt(E*I/(mu*L^4));
modes(i) = modes(i)/(2*pi);
% coefficients for computing mode shape
betaL = beta(i);
a1 = sin(betaL) - sinh(betaL);
a2 = cos(betaL) + cosh(betaL);
x=0;
increment = L/40;
% compute modeshapes for j =1:20,
for j =1:40,
y = a1*(sin(x) - sinh(x));
y =y +  a2*(cos(x) - cosh(x));
x=x+(beta(i)/L)*increment;
modeshapes(i,j) = y;
end;
end;
% increment the  beam span by 40 intervals for
plotting
beamspan=(1:40)*(1/40)*L;
%plot the first mode shape;
figure(1);
plot(beamspan, modeshapes(1,:)/ymax);
grid on;
ylabel('modeshape amplitude');
xlabel('beam span');
title('Mode shape of the first mode of a cantilever
beam');
legend([ ' first mode frequency =',
num2str(modes(1))]);
%plot the first mode shape;
figure(2);
ymax =max(abs(modeshapes(2,:)));
plot(beamspan, modeshapes(2,:)/ymax);
grid on;
ylabel('modeshape amplitude');
xlabel('beam span');
title('Mode shape of the second mode of a cantilever
beam');
```

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```
legend([ 'second mode frequency =',
num2str(modes(2))]);
%plot the first mode shape;
figure(3);
ymax =max(abs(modeshapes(3,:)));
grid on;
ylabel('modeshape amplitude');
xlabel('beam span');
title('Mode shape of the third mode of a cantilever
beam');
legend([ ' third mode frequency =',
num2str(modes(3))]);
% end of the program
```

STEP-2:

5.9 Using Matlab coding for the above equation for cantilever beam result and plots:

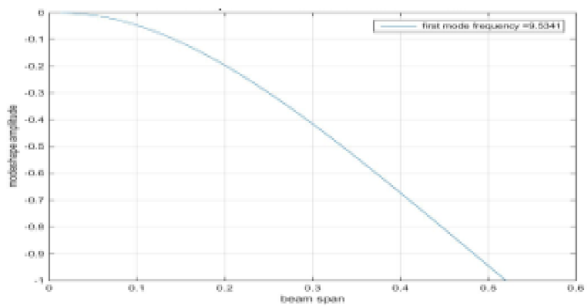

Fig.6.1(a) :mode shape of the first mode

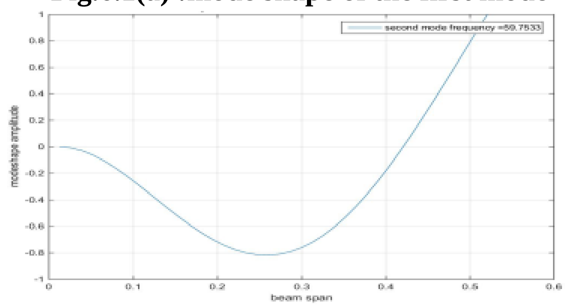

Fig 6.1(b) : mode shape of the second mode

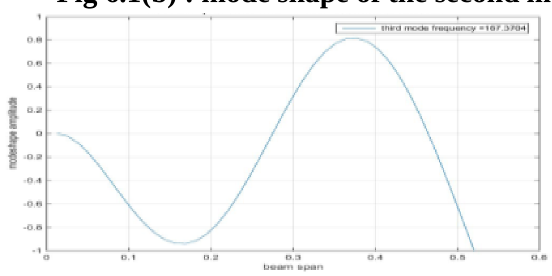

Fig.6.1(c): mode shape of the third mode

5.10 Matlab programming for fixed fixed beam-7:

```
function FF_endBEAM(~)
% Fixed-fixed (Clamped-Clamped) end beam
evaluations
% Mode shapes and natural frequency calculations
% HELP:  This script is to evaluate mode shapes and
corresponding natural
% frequencies of the fixed-fixed end beam by a user
specified mechanical
% properties and geometry size of the beam.
% Prepare the followings:
%  - Material properties of the beam, viz. density (Ro),
Young's modulus (E)
%  - Specify a cross section of the beam, viz.
square,rectangular, circular
%  - Geometry parameters of the beam, viz. Length,
width, thickness
%  -
% Fixed-fixed (Clamped-Clamped)  beam calculations
% E Young's modulus
% Ro density
clear all;
clc;
close all;
%% PART I.
 display('What is the cross section of the beam?')
disp('To see an example #1 (rectangular X-section), hit
enter')
disp('If circular X-section, enter 1; If square, enter 2;')
disp('If rectangle, enter 3; ')
disp('If your beam''s X-section is not listed here, enter
4')
disp('To see an example #2 (rectangular X-section),
enter 5')
disp('To use specific beam''s X-section including, E,
Ix, Ro, enter 6   ')
CS=input(' Enter your choice:   ');
 if isempty(CS) || CS==0
   disp('Example #1. Rectangular X-section Aluminum
beam')
   disp('Length=0.321 [m], Width=0.05 [m],
Thickness=0.006 [m];')
   disp('E=69.9*1e9 [Pa]; Ro=2770 [kg/m^3]')
   L=.321;
```

```
   W=.05;
   Th=.006;
   A=W*Th;
   V=L*W*Th;
   Ix=(1/12)*W*Th^3;
   %Iy=(1/12)*(W^3)*Th;
   E=69.9*1e+9;
   Ro=2770;
elseif CS==1
      R=input('Enter Radius of the X-section= ');
      L=input('Enter Length=  ');
      Ix=(1/4)*pi*R^4;
      %Iy=Ix;
      A=pi*R^2;
      disp('Material proprties of the beam');
      E=input('Enter Young''s modulus in [Pa]=  ');
      Ro=input('Enter material density in [kg/m^3]=  ');
elseif CS==2
         W=input('Enter Width of the X-section: ');
         L=input('Enter Length: ');
         Ix=(1/12)*W^4;
         %Iy=Ix;
         A=W^2;
         disp('Material proprties of the beam');
         E=input('Enter Young''s modulus in [Pa]=  ');
         Ro=input('Enter material density in [kg/m^3]=
');
elseif CS==3
            W=input('Enter Width of the X-section=  ');
            Th=input('Enter Thickness of the X-section=
');
            L=input('Enter Length=  ');
            Ix=(1/12)*W*Th^3;
            %Iy=(1/12)*(W^3)*Th;
            A=W*Th;
            disp('Material proprties of the beam');
            E=input('Enter Young''s modulus in [Pa]= ');
            Ro=input('Enter material density in
[kg/m^3]= ');
elseif CS==4
   display('Note: you need to compute Ix (area moment
of inertia along x axis) and X-sectional area')
   L=input('Enter Length: ');
   Ix=('Enter Ix in [m^4]: ');
   A=('Enter X-sectional area in [m^2]:  ');
   disp('Material proprties of the beam');
   E=input('Enter Young''s modulus in [Pa]= ');
   Ro=input('Enter material density in [kg/m^3]= ');
elseif  CS==5
   display('Given example data are used for
evaluations.');
   display('Given Example is rectangular X-section Al.
beam ');
   L=.3; W=.005; Th=.0005;
   A=W*Th;
   V=L*W*Th;
   Ix=(1/12)*W*Th^3;
   Iy=(1/12)*(W^3)*Th;
   E=70*1e+9; Ro=2.7*1e+3;
 else
   F=warndlg('It is not clear what your choice of X-
section of a beam is. Re-run the script and enter your
beam''s X-section !!!!!!','!! Warning !!');
   waitfor(F)
   display('Type in:>> FF_endBEAM')
   pause(3)
   return
end
%% PART II.
%
display('How many modes and mode shapes would
you like to evaluate ?');
HMMS=input('Enter the number of modes and mode
shapes to be computed: ');
if HMMS>=7
   disp('  ')
 warning('NOTE: Up to 6 mode shapes (plots) are
displayed via the script. Yet, using evaluated data
(Xnx) of the script, more mode shapes can be plotted');
   disp('  ')
end
Nm=4*HMMS;
jj=4;
while jj<=Nm;

betaNL(jj)=fzero(@(betaNL)cosh(betaNL)*cos(betaN
L)-1,jj);
   jj=jj+4;
```

```
end
   index=betaNL~=0;
   betaNLall=betaNL(index);
   %fprintf('betaNL value is %2.3f\n', betaNLall);
   betaN=(betaNLall/L)';
k=1;
wn=zeros(1,length(betaN));
fn=ones(1,length(wn));
while k<=length(betaN);
   wn(k)=betaN(k)^2*sqrt((E*Ix)/(Ro*A));
   fn(k)=wn(k)/(2*pi);
   fprintf('Mode shape # %2f corresponds to nat. freq
(fn): %3.3f\n', k, fn(k) )
   k=k+1;
end
 x=linspace(0, L, 180);
xl=x./L;
sigmaN=(cosh(betaNLall)-
cos(betaNLall))./(sinh(betaNLall)-sin(betaNLall));
 Tc='(cosh(betaN(ii).*x(jj))-cos(betaN(ii).*x(jj)))-
sigmaN(ii).*(sinh(betaN(ii).*x(jj))-
sin(betaN(ii)*x(jj)))';
Xnx=zeros(length(betaN),length(x));
 for ii=1:length(betaN)
   for jj=1:length(x)
      Xnx(ii,jj)=eval(Tc);
   end
end
% Plot mode shapes;
MMS=HMMS;
if MMS==1
   figure;
   plot(xl,Xnx(1,:), 'b-');
   title('Mode shape of the Fixed-fixed (Clamped-
Clamped)  beam');
   legend('Mode #1', 0); xlabel('x/L'); ylabel('Mode
shape Xn(x)'); grid;
   hold off;
elseif MMS==2
      plot(xl,Xnx(1,:), 'b-'); hold on;
      plot(xl,Xnx(2,:), 'r-');
      title('Mode shapes of the Fixed-fixed (Clamped-
Clamped)  beam');
      legend('Mode #1', 'Mode #2', 0);
      xlabel('x/L'); ylabel('Mode shape Xn(x)');
      hold off;
elseif MMS==3
        plot(xl,Xnx(1,:), 'b-'); hold on;
        plot(xl,Xnx(2,:), 'r-');
        plot(xl,Xnx(3,:), 'm-');
        title('Mode shapes of the Fixed-fixed
(Clamped-Clamped)  beam');
        legend('Mode #1', 'Mode #2', 'Mode #3', 0);
        xlabel('x/L'); ylabel('Mode shape Xn(x)');
        hold off;
elseif MMS==4
          plot(xl,Xnx(1,:), 'b-'); hold on;
          plot(xl,Xnx(2,:), 'r-');
          plot(xl,Xnx(3,:), 'm-');
          plot(xl,Xnx(4,:), 'c-'); grid;
          title('Mode shapes of the Fixed-fixed
(Clamped-Clamped)  beam');
          legend('Mode #1', 'Mode #2', 'Mode #3',
'Mode #4', 0);
          xlabel('x/L'); ylabel('Mode shape Xn(x)');
          hold off;
elseif MMS==5 || MMS>5;
            plot(xl,Xnx(1,:), 'b-'); hold on;
            plot(xl,Xnx(2,:), 'r-');
            plot(xl,Xnx(3,:), 'm-');
            plot(xl,Xnx(4,:), 'g-');
            plot(xl,Xnx(5,:), 'k-');
            grid;
            title('Mode shapes of the Fixed-fixed
(Clamped-Clamped)  beam');
            legend('Mode #1', 'Mode #2', 'Mode #3',
'Mode #4', 'Mode #5', 0);
            xlabel('x/L'); ylabel('Mode shape Xn(x)');
            hold off;
elseif MMS==5
  plot(xl,Xnx(1,:), 'b-'); hold on;
  plot(xl,Xnx(2,:), 'r-');
  plot(xl,Xnx(3,:), 'm-');
  plot(xl,Xnx(4,:), 'g-');
  plot(xl,Xnx(5,:), 'k-');
  grid;
  title('Mode shapes of the Fixed-fixed (Clamped-
Clamped)  beam');
```

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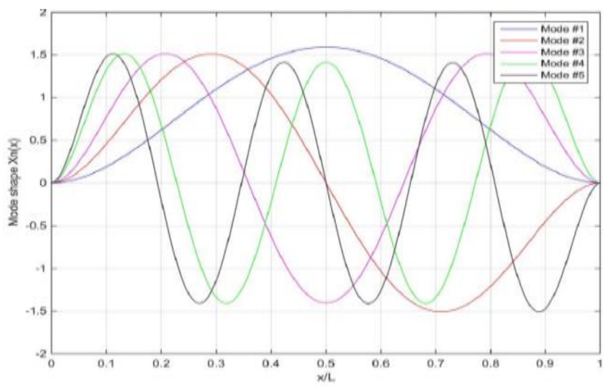
```
   legend('Mode #1', 'Mode #2', 'Mode #3', 'Mode #4',
'Mode #5', 0);
   xlabel('x/L'); ylabel('Mode shape Xn(x)');
   hold off;
elseif MMS>=6
   plot(xl,Xnx(1,:), 'b-'); hold on;
   plot(xl,Xnx(2,:), 'r-');
   plot(xl,Xnx(3,:), 'm-');
   plot(xl,Xnx(4,:), 'g-');
   plot(xl,Xnx(5,:), 'k-');
   plot(xl,Xnx(5,:), 'c-');
   grid;
   title('Mode shapes of the Fixed-fixed (Clamped-
Clamped)  beam');
   legend('Mode #1', 'Mode #2', 'Mode #3', 'Mode #4',
'Mode #5', 0);
   xlabel('x/L'); ylabel('Mode shape Xn(x)');
   hold off;
end
end
```

5.11 mode shape of the third mode Using Matlab coding for the above equation for fixed-fixed beam



6. CONCLUSION:

- In this work, free-free model for a transversely vibrating beam which is Euler-Bernoulli model is examined.
- A governing equation of transverse vibration is derived from Euler-Bernoulli theory. Under the assumption that the system is linear and the material properties are isotropic.
- The rectangular cross section of a free-free beam was analyzed and the natural frequencies and Young's modulus were calculated.
- There is small variation in profiles of beam as well as value of Young's modulus may be due to variation in material properties (material nonlinearity and small amount of damping (contact non-linearity)) and small amount of damping in system during the experimentation.
- Variation In amplitude and value of young's modulus for Theoretical, Numerical and Practical analysis may be due to nonlinearity's in the systems, but effects of these nonlinearity's are not considered in Theoretical and Numerical analysis.
- Using Matlab software different frequency and mode shapes are plotted for different beams with the final equation and the resultant output is given.

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