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Torsional Vibration Analysis of Pre-Twist Cantilever Beam Using FEM

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Abstract:

Structures having the shape of blades are often found in several practical engineering examples such as turbines and aircraft rotary wings. For reliable and economic designs of the structures, it is necessary to estimate the modal characteristics of those structures accurately. To design these components, the dynamic characteristic, especially near resonant condition, need to be well examined to assure a safe operation. Among the dynamic characteristics of these structures, determining the natural frequencies and associated mode shapes are of fundamental importance in the study of resonant responses. An accurate prediction of the forced response is usually very difficult because of the uncertainty of the excitation. A single free standing blade can be considered as a pretwisted cantilever beam with a rectangular cross-section. The torsional vibration of pre twist cantilever beam of rectangular cross section is done so that this resembles to a blade.

The differential equation for the torsional vibration of pre-twisted cantilever beam of rectangular cross section has been obtained. The beam is considered as Timoshenko beam instead of Euler-Bernoulli Beam or Rayleigh Beam because it will consider shear correction factor, rotary inertia, warping constant. The Galerkin's method is used to obtain the frequencies of various modes of vibration. This is again solved by fem software ANSYS and their results are compared.

Introduction

The torsional vibration of a rotating structure can occur in many engineering applications such as turbo-machinery blades, slewing robot arms, aircraft propellers, helicopter rotors, and spinning spacecraft. To design these components, the dynamic characteristic, especially near resonant condition, need to be well examined to assure a safe operation. Among the dynamic characteristics of these structures, determining the natural frequencies and associated mode shapes are of fundamental importance in the study of resonant responses.

It is very important for manufacturers of turbo machinery components to know the natural frequencies of the rotor blades, because they have to make sure that the turbine on which the blade is to be mounted does not have some of the same natural frequencies as the rotor blade. Otherwise, a resonance may occur in the whole structure of the turbine, leading to undammed vibrations, which may eventually wreck the whole turbine. An accurate prediction of the forced response is usually very difficult because of the uncertainty of the excitation. Moreover, under the resonance conditions, what limits the vibration amplitude is the amount of damping available. In most cases, the damping is almost entirely aerodynamic and its assessment is just as uncertain as the excitation.

Thus, classic design practice for such structures has been mainly to rely on the knowledge of the natural frequencies to avoid anticipated resonances.

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Figure 1.1 Schematic view of a part of a steam turbine

A single free standing blade can be considered as a pre-twisted cantilever beam with a rectangular cross-section. Vibration characteristics of such a blade are always coupled between the two bending modes in the flap wise and chord wise directions and the torsion mode. The problem is also complicated by several second order effects such as shear deformations, rotary inertia, and fiber bending in torsion, warping of the cross-section, root fixing and Coriolis accelerations.

Figure 1.2 Pre-Twisted Beam Models

The torsional vibration occurs when the centroid and the shear center of the cross section of the beam do not coincide. This lack of coincidence between the centroid and the shear center occurs when the beam has less than two axis of symmetry or has anisotropy in the material. This makes the torsional axis different from the elastic axis and thus causes torsional vibration when flexural vibration occurs. When the beam is isotropic and the cross-section of it has two axes of symmetry, centroid and shear center coincides and flexural vibrations and torsional vibration become independent. The flexural-torsional coupled vibration can be analyzed by combining one of the beam theories for bending with a torsional theory and a consideration of the various warping effects. The simplest model for the analysis of coupled bending and torsional vibration is combining the classical Bernoulli- Euler theory for bending and St. Venant theory for torsion .Inclusion of a warping effect, Bishop and Miao results in a better approximation, especially for higher modes. Also, for non slender beams, applying the Timoshenko Beam theory instead of the Bernoulli-Euler theory along with the inclusion of a warping effect can improve the accuracy for higher modes.

When obtaining the natural frequencies and the mode shapes, the Galerkin's method is a good candidate both because of its simplicity and its ability to give good results with relatively less efforts. The Galerkin's method is a very powerful technique that can be used to predict the natural frequencies and mode shapes of vibrating structures with less calculation time and effort. The method requires a linear combination of assumed deflection shapes of structures in free harmonic vibration that satisfies at least the geometrical or kinematical boundary conditions of the vibrating structure. Results from the Galerkin's method depend directly on how closely the assumed shape functions resemble the actual mode shapes. When an assumed shape function contributes to several modes, or when some modes are not represented in the assumed shape functions, then it is difficult to draw definite conclusions from the Galerkin's results. It is very important that the assumed shape functions form a complete set so as to represent all the modes of the structure, and they satisfy at least the geometrical boundary conditions. This study presents some insight into the nature of the natural frequency values, as obtained by the Galerkin's method and their dependence on the nature of the assumed shape functions.

The choice of the admissible functions is very important to simplify the calculations and to guarantee convergence to the actual solution. As basis functions for the Galerkin'smethod, orthogonal polynomials

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enable the computation of higher natural frequencies of any order to be accomplished without facing any numerical difficulties arising from the ill-conditioning of the matrices like the ones one encounters when using simple polynomials as the basis functions. Main objective of the present work is to study the analysis of the torsional vibration of beams and obtain the natural frequencies of structures experiencing torsional vibration.

Types of vibrations:

Vibration can be defined as regularly repeated movement of a physical object about a fixed point. Vibrations can be classified based on various factors like

A) Nature of excitation (usually the excitation will be periodic).

B) Nature of displacement.

Nature of vibration:

Here the vibration depends on the nature of deformation of the beam, when the external forces acts on the system. These are of two types namely:-

A) Flexural or transverse vibration.

B) Torsional vibration.

Literature survey

Many researchers analyzed uniform and twisted Timoshenko beams using different techniques: Exact solutions of Timoshenko's equation for simple supported uniform beams were given by Anderson [1]. The general equations of motion of a pre-twisted cantilever blade were derived by Carnegie [2]. Then Carnegie [3] extended his study for the general equations of motion of a pre-twisted cantilever blade allowing for torsion bending, rotary inertia and deflections due to shear. Dawson et al. [4] found the natural frequencies of pre-twisted cantilever beams of uniform rectangular cross-section allowing for shear deformation and rotary inertia by the numerical integration of a set of first order simultaneous differential equations. They also made some experiments in order to obtain the natural frequencies for beams of various breadths to depth ratios and lengths ranging from 3 to 20 in and pre-twist angle in the range 0°-90°. Gupta and Rao [5] used the finite element method to determine the natural frequencies of uniformly pre-twisted tapered cantilever beams. Subrahmanyam et al. [6] applied the Reissner method and the total potential energy approach to calculate the natural frequencies and mode shapes of pre-twisted cantilever blading including shear deformation and rotary inertia. Rosen [7] presented a survey paper as an extensive bibliography on the structural and dynamic aspects of pre-twisted beams.

Chen and Keer [8] studied the transverse vibration problems of a rotating twisted Timoshenko beam under axial loading and spinning about axial axis, and 4 investigated the effects of the twist angle, rotational speed, and axial force on natural frequencies by finite element method. Chen and Ho [9] introduced the differential transform to solve the free vibration problems of a rotating twisted Timoshenko beam under axial loading. Lin et al. [10] derived the coupled governing differential equations and the general elastic boundary conditions for the coupled bending-bending forced vibration of a non-uniform pre-twisted Timoshenko beam by Hamilton's principle. They used a modified transfer matrix method to study the dynamic behavior of a Timoshenko beam with arbitrary pre-twist. Banerjee [11] developed a dynamic stiffness matrix and used for free vibration analysis of a twisted beam. Rao and Gupta [12] derived the stiffness and mass matrices of a rotating twisted and tapered Timoshenko beam element, and calculated the first four natural frequencies and mode shapes in bending-bending mode for cantilever beams.

Narayanaswami and Adelman [13] showed that a straightforward energy minimization yields the correct stiffness matrix in displacement formulations when transverse shear effects are included. They also stated that in any finite element displacement formulation where transverse shear deformations are to be included, it is essential that the rotation of the normal

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

(and not the derivative of transverse displacement) be retained as a nodal degree of freedom. Dawe [14] presented a Timoshenko beam finite element that has three nodes and two degrees of freedom per node, which are the lateral deflection and the cross-sectional rotation. The element properties were based on a coupled displacement field; the lateral deflection was interpolated as a quintic polynomialfunction and the cross-sectional rotation was linked to the deflection by specifying satisfaction of the moment equilibrium equation within the element. The effect of rotary inertia was included in "lumped" form at the nodes. Subrahmanyam et al. [15] analysed the lateral vibrations of a uniform rotating blade using Reissner and the total potential energy methods. Another vibration analysis of rotating pre-twisted blades have been done by Yoo et al. [16]

1 Beam theory:

The study of the torsional vibration start from the basic beam theories, it is important to review and study the derivation of the various basic beam theories prior to the study of torsional vibration of beams. Here, basic beam theories of Euler-Bernoulli, Rayleigh, shear and Timoshenko beam theories are reviewed from their derivation. The assumptions made by all models are as follows.

1. One dimension (the axial direction) is considerably larger than the other two.
2. The material is linear elastic (Hookean).
3. The Poisson effect is neglected.
4. The cross-sectional area is symmetric so that the neutral and centroidal axes coincide.
5. The angle of rotation is small so that the small angle assumption can be used.

The Euler-Bernoulli Beam Theory

The simplest beam theory is the Euler-Bernoulli Beam theory which relies on bending effect only. The Euler-Bernoulli beam theory does not consider the rotatory inertia and the shear effects. The strain energy of a uniform beam due to bending is

$$PE = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2 dx \quad \text{------------ (3.1)}$$

where E is the modulus of elasticity, I the area moment of inertia of the cross-section about the neutral axis, v(x; t) the transverse deflection at the axial location x and time t, and L the length of the beam. The kinetic energy is

$$KE = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial v(x,t)}{\partial t}\right)^2 dx \quad \text{------------------(3.2)}$$

where . is the density of the beam and A, the cross-sectional area. The Lagrangian is given by

$$\begin{aligned} L &= KE - PE \\ &= \int_0^L \left[\frac{\rho A}{2}\left(\frac{\partial v(x,t)}{\partial t}\right)^2 - \frac{EI}{2}\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2\right] dx \quad \text{---------------- (3.3)} \end{aligned}$$

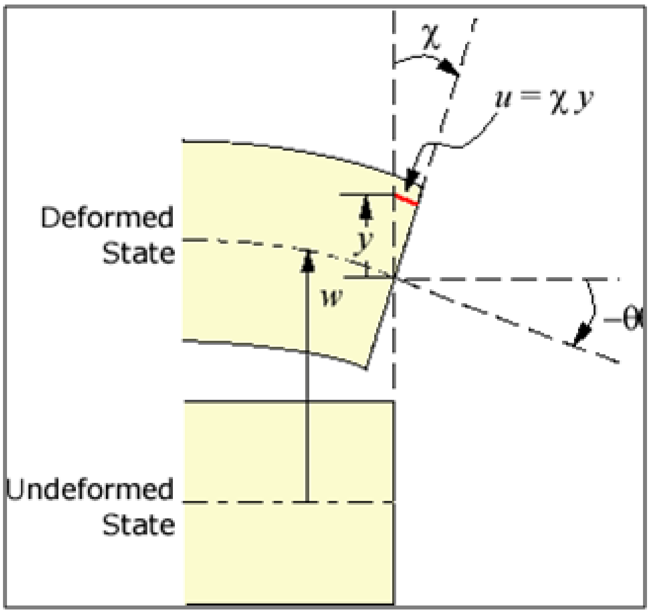

Fig 3.1 The Rayleigh Beam Model

The Rayleigh beam theory provides a slight improvement on the Euler-Bernoulli theory by including the effect of rotations of the cross-section. It partially corrects the overestimation of natural frequencies in the Euler-Bernoulli model. However, the natural frequencies are still overestimated with this method especially in the case of non-slender beams because shear effect is not considered in this method

The Rayleigh beam adds the rotary inertia effects to the Euler-Bernoulli beam. The Kinetic energy due to the rotary inertia is

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$$KE_{rot} = \frac{1}{2}\int_0^L \rho I \left(\frac{\partial^2 v(x,t)}{\partial t \partial x}\right)^2 dx \text{ ---------------------- } (3.4)$$

Therefore, the Lagrangian becomes the addition of the rotating effect to the Lagrangian of Euler-Bernoulli beam and to be

$$L = \frac{1}{2}\int_0^L \left[\rho A \left(\frac{\partial v(x,t)}{\partial t}\right)^2 + \rho I \left(\frac{\partial^2 v(x,t)}{\partial t \partial x}\right)^2 - EI \left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2\right] \text{ ------------------ } (3.5)$$

The Shear Beam Model

The shear beam theory adds shear distortion to the Euler-Bernoulli model. By adding shear distortion to the Euler-Bernoulli beam, the estimate of the natural frequencies improves considerably. This theory does not take into account effects of rotation i.e. rotary inertia.

The Shear Beam Model adds the effect of shear distortion to the Euler-Bernoulli model. The total rotation is the sum of the rotation of the cross-section due to the bending moment, s, and the angle of distortion due to shear, and is approximated by the first derivative of the deflection

$$s(x,t) + \alpha(x,t) = \frac{\partial v(x,t)}{\partial z} \text{ ---------------------- } (3.6)$$

Therefore, the strain energy resulting in bending becomes:

$$PE_{bending} = \frac{1}{2}\int_0^L \left(EI \frac{\partial s(x,t)}{\partial x}\right)^2 dx \text{ ----------------------- } (3.7)$$

and the strain energy resulting from shear becomes:

$$PE_{shear} = \frac{1}{2}\int_0^L \kappa GA \left(\frac{\partial v(x,t)}{\partial x} - s(x,t)\right)^2 dx \text{ --------------------- } (3.8)$$

where .k is the shear factor.

The Lagrangian of the shear beam model is

$$L = \frac{1}{2}\int_0^L \left[\rho A \left(\frac{\partial v(x,t)}{\partial t}\right)^2 - EI \left(\frac{\partial s(x,t)}{\partial x}\right)^2 - \kappa GA \left(\frac{\partial v(x,t)}{\partial x} - s(x,t)\right)^2\right] dx. \text{ ---} (3.9)$$

The Timoshenko Beam Model

As mentioned earlier that the Euler Bernoulli beam theory can approximate the natural frequency in case of higher frequency modes and slender beam, hence Timoshenko beam theory can be implemented in such situations. In this theory deformation due to transverse shear and kinetic energy due to rotation of the cross-section become important. Energy expressions include both shear deformation and rotary inertia.

The assumption made in the previous theory that the plane sections which are normal to the undeformed centroidal axis remain plane after bending, will be retained. However, it will no longer be assumed that these sections remain normal to the deformed axis

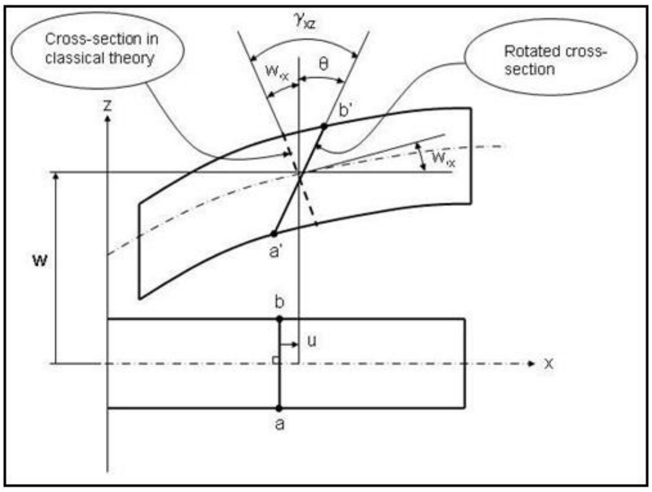

Fig 3.2 Timoshenko Beam

Assumptions made in Timoshenko Beam theory:-

a)Plane sections such as 'ab', originally normal to the centerline of the beam in the undeformed geometry, remain plane but not necessarily normal to the centerline in the deformed state.

b) The cross-sections do not stretch or shorten, i.e., they are assumed to act like rigid surfaces.

Comparison among beam theory:-

	Euler Bernoulli beam theory	Shear beam theory	Rayleigh Beam theory	Timoshenko beam theory
Shear Correction factor	X	√	X	√
Rotary Inertia	X	X	√	√

As it can be seen from the above comparison that the Timoshenko beam considers few factors which Euler Bernoulli beam theory does not consider. Hence it can be predicted that Timoshenko beam can give accurate results while considering higher frequencies and slender beams. Hence Timoshenko beam is considered in the present project.

ISSN No: 2348-4845
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Torsional equation of pre-twist beams

Considering a normal rectangular beam of length "L" and cross section area "bxh". Suppose at a distance of "R" from the axis of the fibre exists. Before it gets twisted its will appear as shown in figure.

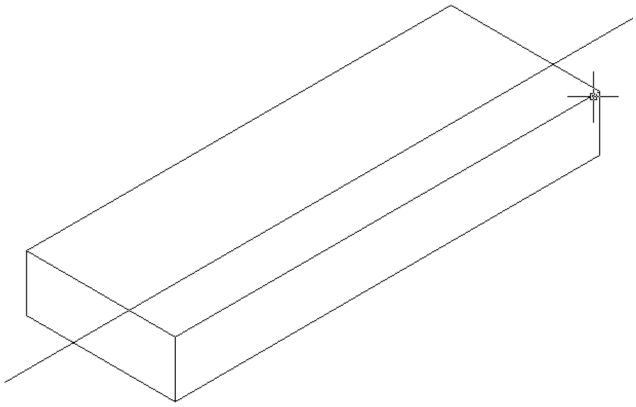


Fig 4.1 Simple rectangular cross section beam

If the beam is twisted through an angle of "β" it will appear as shown below. Where β is twist in radians. Let "θ" angle of twist in degrees.

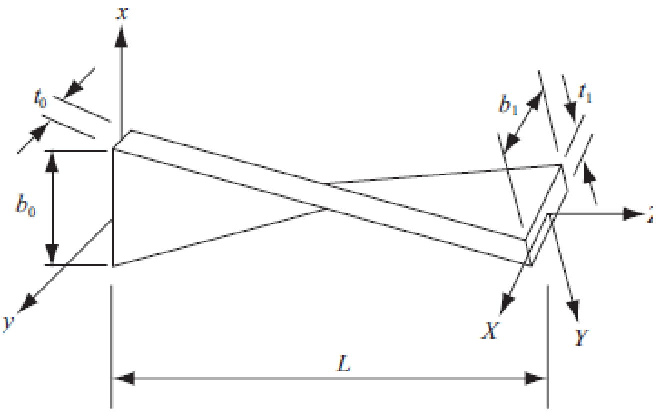
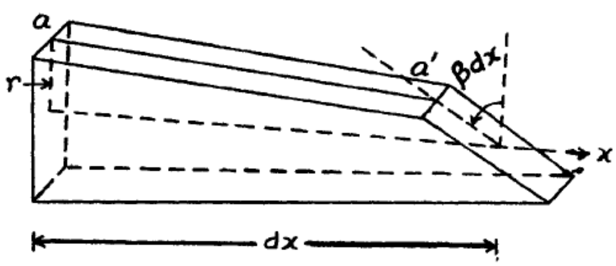


Fig 4.2 Twisted Beam

Considering a small element of blade of length dx



It is well known that when a thin bar is under torsion there is a slight decrease in distance between cross sections. As the overall length is getting affected due to bending it can be said that a strain is developed and hence a stress called normal stress is developed. A normal stress will be on each longitudinal fibre and this in not parallel to the axis of bar, and hence there is a stress component which produces an additional torque. The normal stress developed will be acting in the following way.

Resolving the components of stress along x-direction and along y-direction.

1)Along x-direction $\sigma(\cos R\beta) = \sigma.1$------4.1

2)Along y-direction $\sigma(\sin R\beta) = \sigma R\beta$-----4.2

Since Rβ value is very low

The stress component $\sigma(\cos R\beta)$ is parallel to the axis of the beam and The stress component $\sigma(\sin R\beta)$ is parallel to the axis of the beam.

A torque is generated because of angle of twist θ which results in deformation and this deformation is not uniform.let the torque be Mt.

For easy considering a shaft of circular cross section. The cross-sections of a circular shaft in torsion rotate as if they were rigid in-plane. That is, there is no relative displacement of any two, arbitrarily chosen points of a cross section when the shaft is subjected to a torque about its longitudinal, z, axis. We prove this assertion relying on rotational symmetry and upon the constancy of the internal torque as we move down the axis of the shaft.

Let the points along a radius take the shape of a curve in plane in the deformed state: Now consider the same set of points but from the perspective of someone who has the portion and the shaft to the right to observe.
It is still possible that, while radial lines remain radial, there could be some sort of accordion effect as we march around the axis of the shaft – some radial lines coming closer together, others widening the angle

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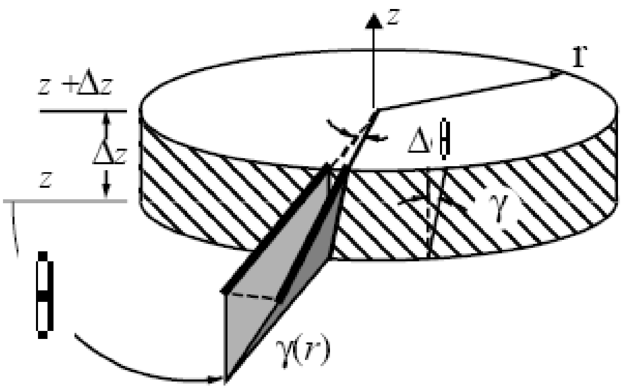
between them. But no, this is not possible since we have complete rotational symmetry. Whatever happens at one angular position must happen at every other angular position.

One further fact follows from the uniformity of torque at each section, namely, the relative rotation of two cross-sections is the same for any two sections separated by the same distance along the axis of the shaft.



If we let 'θ' be the rotation of any section, then this is equivalent to saying dθ/dz is a constant.

Consider now the strains due to the rotation of one section relative to another. The figure shows the rotation of a section located along the axis at z+Δ z relative to a section at z just below it. Of course the section at z has rotated too, most likely. But it is the relative rotation of the two sections which gives rise to a strain, a shear strain. γ, which measures the decrease in right angle, originally formed by two line segments, one circumferential, the other axially directed as shown. From the geometry we can state:

$$\gamma = \lim_{\Delta z \to 0} (r\Delta\theta/\Delta z) = r\frac{d\theta}{dz} \text{ ------------------------------}(4.3)$$

Note that this relationship shows that the shear strain is a linear function of radius - zero at the axis, maximum at the shaft's outer radius. Note too that, with d θ /dz a constant, the shear strain does not vary with z with position along the axis of the shaft.

There are no other strains, with no deformation in plane and no bulging-out or dishing in, there are no other strains. If there existed some asymmetry like that of the truss bay structure with one diagonal number removed from each bay, then we would not be able to rule out a contraction (or extension) in the z direction.

Because we have but one strain component, this will be a very short section. The corresponding stress is the shear stress τ and is related to the shear strain according to:

$$\tau = G\gamma = G\cdot r\left(\frac{d\phi}{dz}\right) \text{ --}(4.4)$$

The resultant torque about the axis of a circular shaft due to a shear stress distribution τ(r), can be obtained as

$$M_T = 2\pi\int_0^R \tau(r)r^2dr \text{ --}(4.5)$$

Substituting the value of τ for equation 4.4 in equation 4.5

$$M_T = 2\pi\int_0^R G\cdot r\left(\frac{d\phi}{dz}\right)r^2dr \text{ ------------------------}(4.6)$$

since dθ/dz and G are constants, we are left with the integral of r3 and can write

$$M_T = GJ\left(\frac{d\phi}{dz}\right) \text{ --}(4.7)$$

where J is a function only of the geometry of the cross-section - its radius R. You may have encountered it as the polar moment of inertia. Similarly for rectangular cross-section bar it can be written as:

$$M_t = GJ \text{ --} (4.8)$$

Considering warping constant the equation can be modified as:

$$M_t = GJC_1 \text{ --}(4.9)$$

Where GJ together forms torsional rigidity for uniform torsion and c1 is the warping rigidity.

It is well known that when a thin bar is under torsion there is a slight decrease in distance between cross sections. As the overall length is getting affected due to bending it can be said that a strain is developed and hence a stress called normal stress is developed. As there are 2 stress component there will 2 forces acting due to pre-twist. One force will be parallel to the bar

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and other force will be perpendicular to bar. The parallel will not have any effect of torque but the perpendicular force will create torque. The value of that force is given by: σ.Rβ.
Where σ- is the stress, R- radius of fibre form the axis and β-twist in radians.

Finite Element Solution

The Finite Element Method (FEM) is a numerical procedure that can be used to obtain solutions to a large class of engineering problems involving stress analysis, heat transfer, electromagnetism, fluid flow and vibration and acoustics.

In FEM, a complex region defining a continuum is discretized into simple geometric shapes called finite elements .The material properties and the governing relationships are considered over these elements and expressed in terms of unknown values at element corners, called nodes. An assembly process, duly considering the loading and constraints, results in a set of equations. Solution of these equations gives us the approximate behaviour of the continuum.

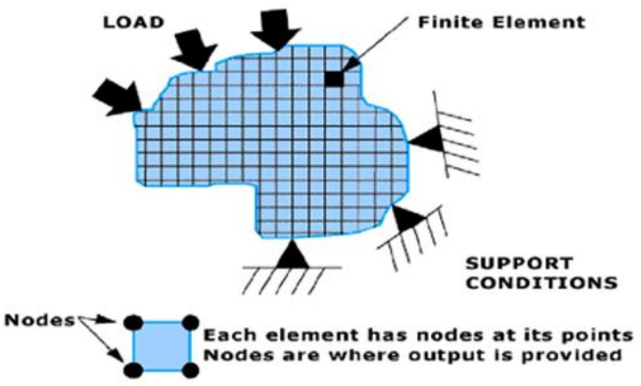

Figure 5.1 Description of the “finite element”

Basic ideas of the FEM originated from advances in aircraft structural analysis. The origin of the modern FEM may be traced back to the early 20th century, when some investigators approximated and modeled elastic continua using discrete equivalent elastic bars. However, Courant has been credited with being the first person to develop the FEM. He used piecewise polynomial interpolation over triangular sub regions to investigate torsion problems in a paper published in 1943. The next significant step in the utilization of Finite Element Method was taken by Boeing. In the 1950’s Boeing, followed by others, used triangular stress elements to model airplane wings. But the term finite element was first coined and used by Clough in 1960. And since its inception, the literature on finite element applications has grown exponentially, and today there are numerous journals that are primarily devoted to the theory and application of the method.

RESULTS AND DISCUSSION

In order to validate the proposed finite element model for the vibration analysis of pretwisted Timoshenko beam, various numerical results are obtained and compared with available solutions in the published literature.

Comparison between ansys results and mathematical modeling results

For b/h = 1/2 and for 0.1 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	1.5800E+04	1.6048e+004	1.55E+00
1	1.3140E+05	1.33961e+005	1.91E+00
2	3.6424E+05	3.69699e+005	1.48E+00
3	5.77110E+05	7.23303e+005	1.99E+00
4	1.10012E+06	11.194771e+06	1.73E+00

For b/h = 1/2 and for 0.2 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	1.74220E+04	1.76686e+004	1.40E+00
1	5.28392E+05	5.33729e+005	1.00E+00
2	8.89298E+05	9.01652e+005	1.37E+00
3	1.27386E+06	1.287315e+006	1.05E+00
4	1.67913E+06	1.697029e+006	1.05E+00

For b/h = 1/2 and for 0.3 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	1.73716E+05	1.76716e+005	1.70E+00
1	5.34540E+05	5.39939e+005	1.00E+00
2	9.00645E+05	9.11880e+005	1.23E+00
3	1.28831E+06	1.301387e+006	1.00E+00
4	1.69016E+06	1.714733e+006	1.43E+00

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For b/h = 1/2 and for 0.4 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	1.84138E+05	1.76686e+004	1.45E+00
1	5.55870E+05	5.33729e+005	1.47E+00
2	9.46607E+05	9.01652e+005	-1.66E+00
3	1.33438E+06	1.287315e+006	-5.49E-01
4	1.75094E+06	1.697029e+006	-1.41E+00

For b/h = 1/3 and for 0.1 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	3.03660E+05	3.0781e+004	6.50E-01
1	1.54246E+05	1.55899e+005	1.06E+00
2	3.89346E+05	3.93908e+005	1.16E+00
3	7.41086E+05	7.49543e+005	1.13E+00
4	1.22098E+06	1.223396e+006	1.97E-01

For b/h = 1/3 and for 0.2 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	3.03130E+04	3.0787e+004	1.54E+00
1	1.54250E+05	1.55909e+005	1.06E+00
2	3.89350E+05	3.93920e+005	1.16E+00
3	7.42060E+05	7.49555e+005	1.00E+00
4	1.22018E+06	1.223340e+006	2.58E-01

For b/h = 1/3 and for 0.3 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	3.06780E+04	3.0797e+004	3.86E-01
1	1.54827E+05	1.55927e+005	7.05E-01
2	3.89372E+05	3.93940e+005	1.16E+00
3	7.41119E+05	7.49576e+005	1.13E+00
4	1.20986E+06	1.223429e+006	1.11E+00

For b/h = 1/3 and for 0.4 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	3.03460E+04	3.0811e+004	1.51E+00
1	1.54394E+05	1.55953e+005	1.00E+00
2	3.93819E+04	3.93968e+005	6.74E-01
3	7.47471E+05	7.49605e+005	1.51E-01
4	1.22155E+06	1.223458e+006	9.77E-01

For b/h = 1/4 and for 0.1 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	0.24404E+05	2.4554e+004	6.11E-01
1	2.10928E+04	2.11097e+005	1.90E+00
2	5.83978E+04	5.84127e+005	2.08E+00
3	1.14180E+05	1.143669e+005	1.63E-01
4	1.88826E+05	1.889724e+006	1.67E+00

For b/h = 1/4 and for 0.2 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	0.24460E+03	2.4560e+004	1.69E+00
1	2.10963E+04	2.11103e+005	1.51E+00
2	5.65719E+05	5.84173e+005	1.00E+01
3	1.11360E+06	1.143673e+006	2.63E+00
4	1.87087E+06	1.889730e+006	9.98E-01

For b/h = 1/4 and for 0.3 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	2.37990E+05	2.42690E+05	1.94E+00
1	2.03112E+05	2.07112E+05	1.93E+00
2	5.69142E+05	5.76142E+05	1.21E+00
3	1.120680E+06	1.13068E+06	8.84E-01
4	1.845740E+06	1.86574E+06	1.07E+00

For b/h = 1/4 and for 0.4 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	2.4097 E+04	2.43970E+04	1.23E+00
1	2.09927E+05	2.10927E+05	4.74E-01
2	5.70624E+05	5.77624E+05	1.21E+00
3	1.11913E+05	1.14113E+05	1.93E+00
4	1.865250E+06	1.88725E+06	1.17E+00

For b/h = 1/5 and for 0.1 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	1.58080E+04	1.6048e+004	1.50E+00
1	1.31461E+05	1.33961e+05	1.87E+00
2	3.65199E+05	3.69699e+05	1.22E+00
3	7.13303E+05	7.23303e+05	1.38E+00
4	1.58080E+06	1.194771e+06	1.84E+00

For b/h = 1/5 and for 0.2 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	1.34660E+05	1.3698e+004	1.69E+00
1	8.76100E+05	8.9110e+004	1.68E+00
2	2.35304E+05	2.38204e+005	1.22E+00
3	4.53860E+05	4.61971e+005	1.76E+00
4	7.58970E+05	7.60167e+005	1.57E-01

For b/h = 1/5 and for 0.3 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	3.24000E+03	1.09342e+003	-4.26E+00
1	1.25550E+04	1.2682e+004	1.00E+00
2	2.78190E+04	2.81725e+004	1.25E+00
3	4.99060E+04	5.0749e+004	1.66E+00
4	7.95820E+04	8.0655e+004	1.33E+00

For b/h = 1/5 and for 0.4 radian twist

Mode no	FEM solution	Mathematical solution	Difference between the solutions in %
0	4.28600E+03	4.3301e+003	1.02E+00
1	2.78130E+04	2.8163e+004	1.24E+00
2	7.43800E+04	7.5313e+004	1.24E+00
3	1.44000E+05	1.46000e+005	1.37E+00
4	2.37206E+05	2.40240e+005	1.26E+00

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The various examples are considered to evaluate the present finite element formulation for the effects of related parameters (e.g. twist angle, length, breadth to depth ratio) on the natural frequencies of the pre-twisted cantilever Timoshenko beams. The natural frequency ratios for the first five modes of vibration are obtained for different breadth to height ratio and in each case with different twist angles. Graphs are plotted with frequency and twist and it can easily be checked out from the Figures that the natural frequencies increase as the twist angle increases.

Conclusion

The equations of motion for the torsional vibration analysis of blades, which have a pre-twisted cross-section, arbitrary orientation are derived. The equations of motion are transformed into a dimensionless form by employing dimensionless variables and several dimensionless parameters representing area moment of inertia ratio, the pre-twist angle, are identified. The Garlekin's method used here to solve the equation gives an upper bound of frequencies. The resultant obtained for various frequencies of torsional vibrations shows that it increases with the amount of pre-twisted and the thinness of the beam.

Numerical results in different cases validated the applicability of the proposed method for solving such an engineering problem. The pre-twisted angles influence the natural frequencies of the beams. The natural frequencies found were compared with the simulated results. The simulation was carried in ANSYS 10.0 software. The simulation software provides the investigator with different mode shape and frequency for all the beam geometry and the resonant data obtained is compared with the mathematical models. It has been noted form graphs that for all the cases twist with 0.5 radians has maximum natural frequency.

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