

Efficient Implementation of Bit Parallel Finite Field Multiplier Using Redundant Basis

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Abstract:

Redundant basis (RB) multipliers over Galois Field have gained huge popularity in elliptic curve cryptography (ECC) mainly because of their negligible hardware cost for squaring and modular reduction. We have proposed anovel recursive decomposition algorithm for RB multiplication to obtain high-throughput digit-serial implementation. Through efficient projection of signal-flow graph (SFG) of the proposed algorithm, a highly regular processor-space flow-graph (PSFG) is derived. By identifying suitable cut-sets, we have modified the PSFG suitably and performed efficient feed-forward cut-set retiming to derive three novel multipliers which not only involve significantly less time-complexity than the existing ones but also require less area and less power consumption compared with the others. Both theoretical analysis and synthesis results confirm the efficiency of proposed multipliers over the existing ones. It is shown that the proposed structures are the best among the corresponding designs, for FPGA implementation. It is shown that the proposed designs can achieve savings of area-delay-power product (ADPP) on FPGA implementation over the best of the existing designs, respectively.

Index Terms— Digit-serial, Finite field multiplication, FPGA, High-throughput, redundant basis.

I. INTRODUCTION

Finite Field multiplication over Galois Field ($GF(2^m)$) is a basic operation frequently encountered in modern cryptographicsystems such as the elliptic curve cryptography (ECC) and error control coding [1]-[3].

Moreover, multiplication over a finite field can be used further to perform other field operations, e.g., division, exponentiation, and inversion. Multiplication over can be implemented on a general purpose machine, but it is expensive to use a general purpose machine to implement cryptographic systems in cost-sensitive consumer products. Besides, a low-end microprocessor cannot meet the real-time requirement of different applications since the word length of these processors is too small compared with the order of typical finite fields used in cryptographic systems. Most of the real-time applications, therefore, need hardware implementation of finite field arithmetic operations for the benefits like low-cost and high-throughput rate. The choice of basis to represent field elements, namely the polynomial basis, normal basis, triangular basis and redundant basis (RB) has a major impact on the performance of the arithmetic circuits [5]. The multipliers based on RB [6] have gained significant attention in recent years due to their several advantages. Not only do they offer free squaring, as normal basis does, but also involve lower computational complexity and can be implemented in highly regular computing structures. Several digit-level serial/parallel structures for RB multiplier over have been reported in the last years after its introduction by Wu *et al.* An efficient serial/parallel multiplier using redundant representation has been presented in [3]. A bit-serial word-parallel (BSWP) architecture for RB multiplier has been reported. Several other RB multipliers also have been developed for reducing the complexity of implementation and for

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high-speed realization. We find that the hardware utilization efficiency and throughput of existing structures can be improved by efficient design of algorithm and architecture. In this paper, we aim at presenting efficient digit-level serial/parallel designs for high-throughput finite field multiplication over based on RB. We have proposed an efficient recursive decomposition scheme for digit-level RB multiplication, and based on that we have derived parallel algorithms for high throughput digit-serial multiplication. We have mapped the algorithm to three different high-speed architectures by mapping the parallel algorithm to a regular 2-dimensional signal-flow graph (SFG) array, followed by suitable projection of SFG to 1-dimensional processor-space flow graph (PSFG), and the choice of feed-forward cut-set to enhance the throughput rate. Our proposed digit-serial multipliers involve significantly less area-time-power complexities than the corresponding existing designs. Field programmable gate array (FPGA) has evolved as a mainstream dedicated computing platform. FPGAs however do not have abundant number of registers to be used in the multiplier.

Therefore, we have modified the proposed algorithm and architecture for reduction of register-complexity particularly for the implementation of RB multipliers on FPGA platform. Apart from these we also present a low critical-path digit-serial RB multiplier for very high throughput applications.

II. DERIVATION OF PROPOSED HIGH-THROUGHPUT STRUCTURES FOR RB MULTIPLIERS

In this section, we derive the proposed multipliers from the SFG of the proposed Algorithm 1.

A. Proposed Structure-I

The RB multiplication can be represented by the 2-dimensional SFG (shown in Fig. 1) consisting of parallel arrays, where each array consists of bit-shifting nodes (S node), multiplication nodes (M nodes) and addition nodes (A nodes). There are two types of S nodes (S-I node and S-II node). Function of

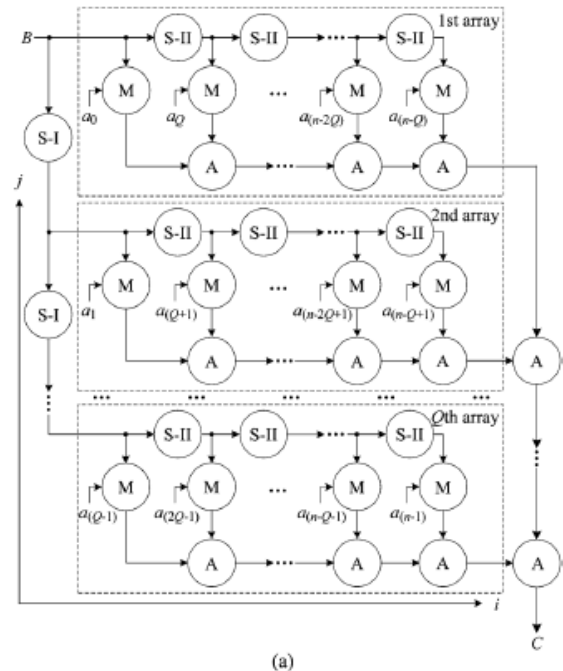


Fig. 1. Signal-flow graph (SFG) for parallel realization of RB multiplication over based on (2) and (3). (a) The proposed SFG. (b) Functional description of S node, where S-I node performs circular bit-shifting of one position and S-II node performs circular bit-shifting by positions. (c) Functional description of M node. (d) Functional description of A node.

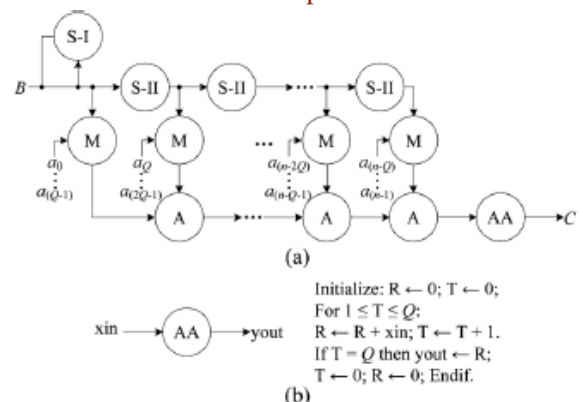


Fig. 2. Processor-space flow graph (PSFG) of digit-serial realization of finite field RB multiplication over . (a) The proposed PSFG. (b) Functional description of add-accumulation (AA) node.

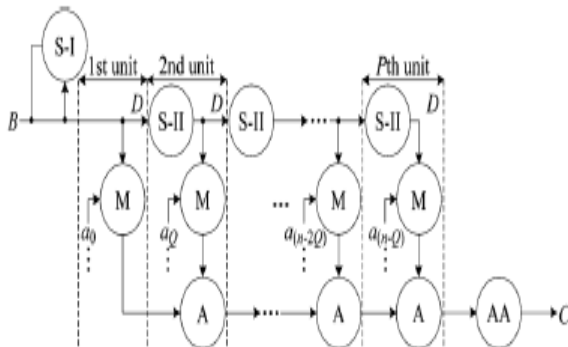


Fig. 3. Cut-set retiming of PSFG of finite field RB multiplication over, where D denotes delay.

S nodes is depicted in Fig. 1(b), where S-I node performs circular bit-shifting by one position and S-II node performs circular bit-shifting by positions for the degree reduction requirement. Functions of M nodes and A nodes are depicted in Fig. 1(c) and 1(d), respectively.

Each of the M nodes performs an AND operation of a bit of serial-input operand with bit-shifted form of operand while each of the A nodes performs an XOR operation. The final addition of the output of arrays of Fig. 1 can be performed by bit-by-bit XOR of the operands in number of A nodes as depicted in Fig. 1. The desired product word is obtained after the addition of parallel output of the arrays. For digit-serial realization of RB multiplier, the SFG of Fig. 1 can be projected along $-$ direction to obtain a PSFG as shown in Fig. 2, where input bits are loaded in parallel to multiplication nodes during each cycle period. The functions of nodes of PSFG are the same as those of corresponding nodes in the SFG of Fig. 1 except an extra add-accumulation (AA) node. The function of the AA node is, as described in Fig. 2(b), to execute the accumulation operation for cycles to yield the desired result thereafter.

For efficient realization of a digit-serial RB multiplier, we can perform feed-forward cut-set retiming in a regular interval in the PSFG as shown in Fig. 3. As a result of cut-set retiming of the Fig. 3, the minimum duration of each clock period is reduced to, where d and x denote the delay of an AND gate and an XOR gate, respectively.

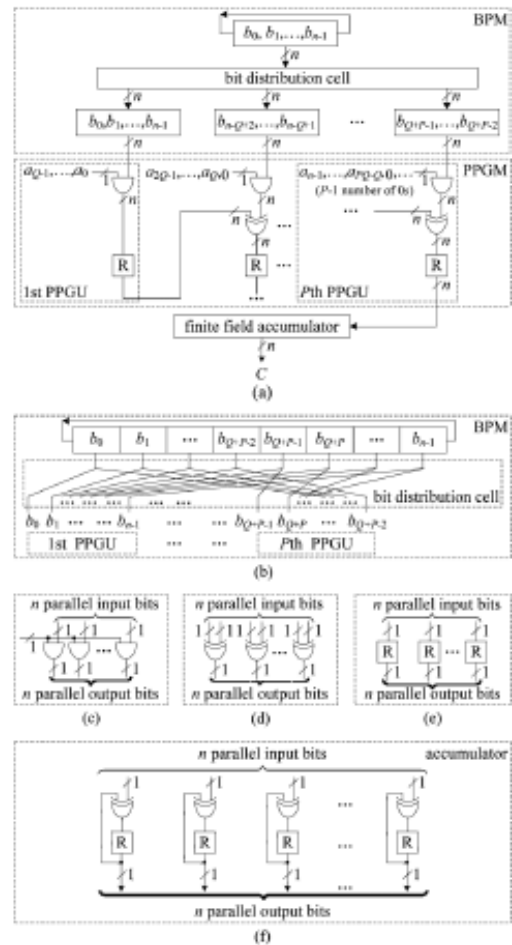


Fig. 4. Proposed structure-I (PS-I) for RB multiplier, where “R” denotes a register cell. (a) Detailed structure of the RB multiplier. (b) Structure of the bit-permutation module (BPM). (c) Structure of the AND cell in the partial product generation module (PPGM). (d) Structure of the XOR cell in the PPGM. (e) Structure of the register cell in the PPGM. (f) Structure of the finite field accumulator.

The PSFG of Fig. 3, is mapped to the high-throughput digit-serial RB multiplier (shown in Fig. 4), referred to as proposed structure-I (PS-I). PS-I contains three modules, namely the bit-permutation module (BPM), partial product generation module (PPGM) and finite field accumulator module. The BPM of Fig. 4 performs rewiring of bits of operand to feed its output to partial product generation units (PPGU)s according to the S nodes of PSFG of Fig. 3, as shown in Fig. 4(b). The AND cell, XOR cell and register cell of PPGM perform

the function of M node, Anode and delay imposed by the retiming of PSFG of Fig. 3, respectively. Structures and functions of AND cell, XOR cell and register cell are shown in Fig. 4(c), (d), and (e), respectively. The input operands are fed to PPGU in staggered manner to meet the timing requirement in systolic pipeline. The accumulator consists of parallel bit-level accumulation cells [as shown in Fig. 4(f)]. The newly received input is then added with the previously accumulated result and the result is stored in the register cell to be used during the next cycle. The duration of minimum cycle period of the PS-I is . The proposed digit-serial design gives the first output of desired product cycles after the pair of operands are fed to the structure, while the successive output are produced at the interval of cycles thereafter.

B. Modification of Proposed Structure-I

For any integer value of P, we can have $(P = kd + l)$, where $0 \leq l < d$ and $1 < P$. Without loss of generality, for simplicity of discussion, we can assume $l = 0$. The approach proposed here for $l = 0$ however can be easily extended to the cases where $l \neq 0$. Define $0 \leq h \leq k - 1$ and $0 \leq f \leq d - 1$ such that (13) can be rewritten as

$$\overline{C}_u = \sum_{h=0}^{k-1} \sum_{f=0}^{d-1} B^{u+fhQ} a_{u+fhQ}$$

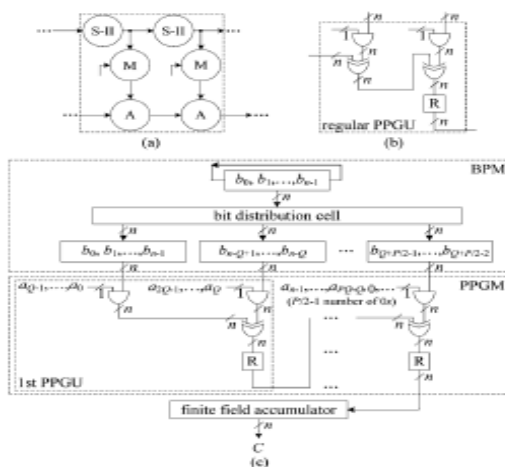


Fig. 5. PS-I for RB multiplier when d=2. (a) Proposed cut-set retiming of PSFG when d=2. (b) Detailed internal structure of merged regular PPGU. (c) Corresponding PS-I for the case d=2.

Based on (14), we can modify the retiming of PSFG of Fig. 3 to derive suitable digit-level architecture for RB multiplier over. For example, to obtain the proposed structure for $d = 2$, a pair of S nodes, a pair of M nodes and a pair of A nodes of the PSFG of Fig. 3 can be merged to form a macro-node as shown within the dashed-lines in Fig. 5. Each of these macro-nodes can be implemented by a new PPGU to obtain a PPGM of PPGUs. Accordingly, two regular PPGUs in the structure of Fig. 4 can be merged into a new regular PPGU as shown in Fig. 5(b), which consists of two AND cells and two XOR cells (the first PPGU requires only one XOR cell). The functions of AND cell, XOR cell and register cell are the same as those described in Fig. 4. The critical path of the structure of Fig. 5(c) amounts to $(T_A + 2T_X)$. The first output of desired product is available from this structure after a latency of $(P/2 + Q)$ cycles, while the successive outputs are available thereafter in each cycle of duration $(T_A + 2T_X)$. The technique used to derive the structure for may be extended for any value of (P/d) , to obtain a structure consisting of PPGUs. The technique based on (4) can significantly reduce the register complexity of the proposed structure, since consecutive PPGUs of the PS-I can be merged together to form units to be processed concurrently. This strategy is quite useful for FPGA-based implementation since the value of can be chosen appropriately, such that the PSFG nodes selected to be processed in a cycle can be mapped to a basic unit of FPGA with low register complexity.

C. Proposed Structure-II

We can further transform the PSFG of Fig. 3 to reduce the latency and hardware complexity of PS-I. To obtain the proposed structure, serially-connected A nodes of the PSFG of Fig. 3 are merged into a pipeline form of A nodes as shown within the dashed-box in Fig. 6(a). These pipelined A nodes can be implemented by a pipelined XOR tree, as shown in Fig. 6(b). Since all the AND cells can be processed in parallel, there is no need of using extra “0”s on the input path to meet the timing requirement in systolic pipeline. The critical path and throughput of PS-II are the same as those of PS-I. Similarly, PS-II can

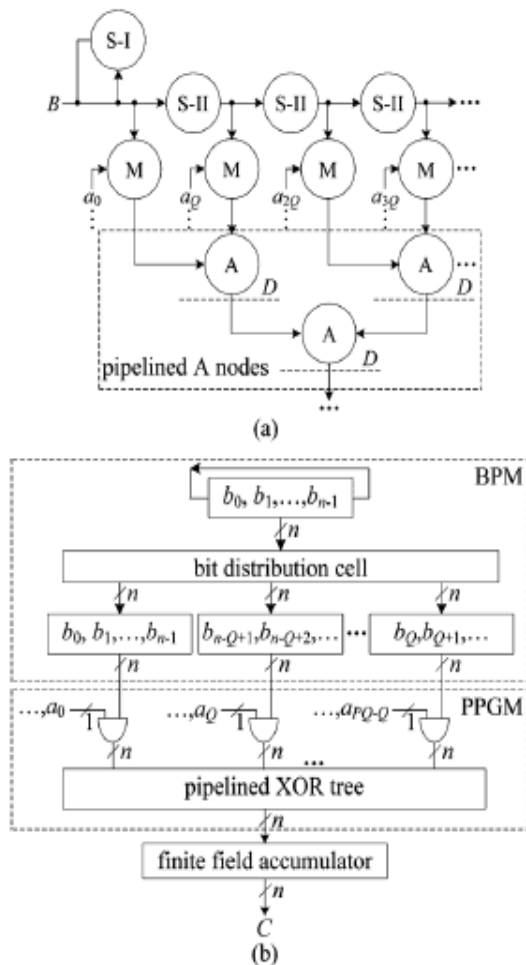


Fig. 6. Proposed structure-II (PS-II) for RB multiplier, where “R” denotes a register cell. (a) Modified PSFG. (b) Structure of RB multiplier be easily extended to larger values of n to have low register-complexity structures.

D. Proposed Structure-III

Since the S nodes of Fig. 3 perform only the bit-shifting operation they do not involve any time consumption. Therefore, we can introduce a novel cut-set retiming to reduce the critical path further, as shown in Fig. 7(a). It can be observed that the cut-set retiming allows to perform the bit-addition and bit-multiplication concurrently, so that the critical-path is reduced to, i.e., the throughput of the design is increased. The proposed high-throughput structure (PS-III) of RB multiplier thus derived is shown in Fig. 7(b). It consists of PPGUs, and each PPGU consists of one AND cell, one XOR cell and two register cells.

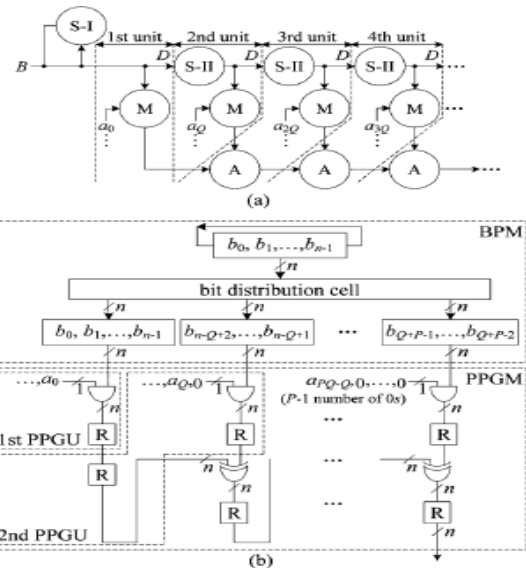


Fig.7. Novel cut-set retiming of PSFG and its corresponding structure: PS-III. (a) Cut-set retiming. (b) BPM and PPGM of PS-III.

The proposed structure yields the first output of desired result cycles after the first input is fed to the structure, while the successive outputs are available in each cycle.

III. RESULTS

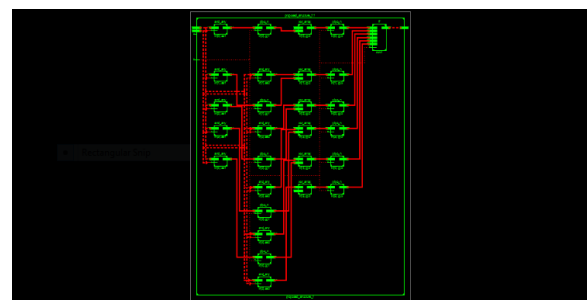


Fig.8. RTL Schematic for cut-set retiming of PSFG of finite field RB multiplication.

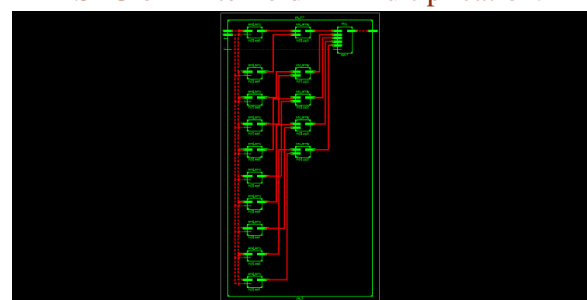


Fig.9. RTL Schematic for pipelined tree of PSFG of finite field RB multiplication.

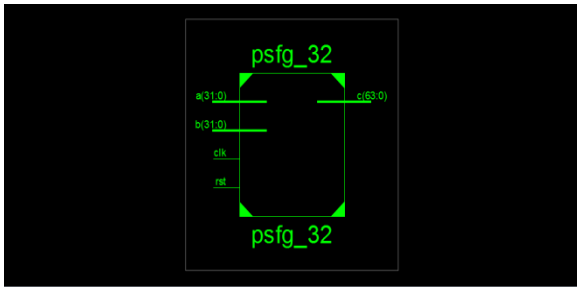


Fig.10. RTL Schematic for novel cut-set retiming of 32bit-PSFG of finite field RB multiplication.

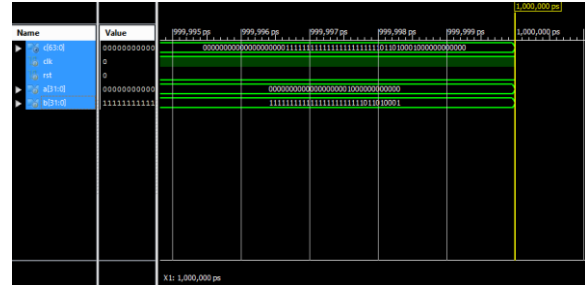


Fig.14. Simulation results for novel cut-set retiming of 32bit-PSFG of finite field RB multiplication.

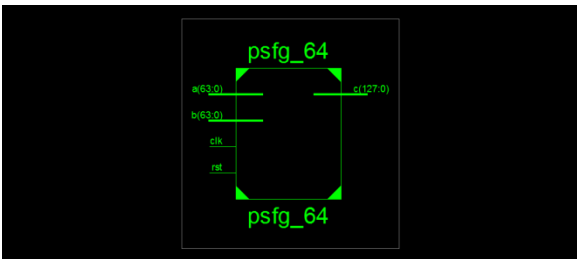


Fig.11. RTL Schematic for novel cut-set retiming of 64bit-PSFG of finite field RB multiplication.

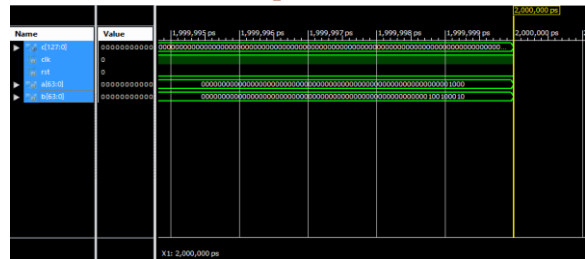


Fig.14. Simulation results for novel cut-set retiming of 64bit-PSFG of finite field RB multiplication.

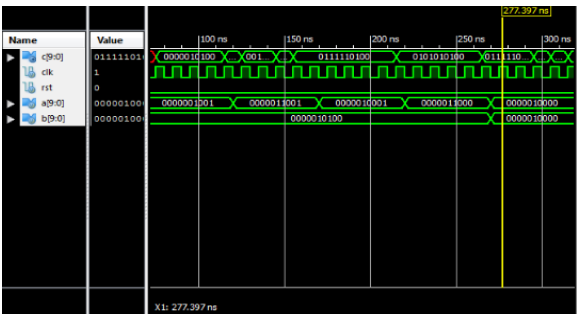


Fig.12. Simulation results for cut-set retiming of PSFG of finite field RB multiplication.

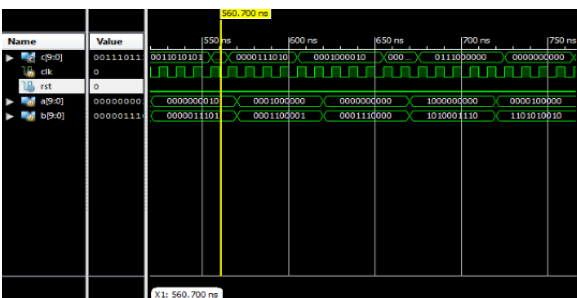


Fig.13. Simulation results for pipelined tree of PSFG of finite field RB multiplication.

V.CONCLUSION

We have proposed a novel recursive decomposition algorithm for RB multiplication to derive high-throughput digit-serial multipliers. By suitable projection of SFG of proposed algorithm and identifying suitable cut-sets for feed-forward cut-set retiming, three novel high-throughput digit-serial RB multipliers are derived to achieve significantly less area-time-power complexity than the existing ones. Moreover, efficient structures with low register-count have been derived for area-constrained implementation; and particularly for implementation in FPGA platform where registers are not abundant. The results of synthesis show that proposed structures can achieve saving of up to 94% and 60%, respectively, of ADPP for FPGA and ASIC implementation, respectively, over the best of the existing designs. The proposed structures have different area-time-power trade-off behavior. Therefore, one out of the three proposed structures can be chosen depending on the requirement of the application environments.

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