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Random vortex method (RVM) for viscous flow

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Introduction:

There is a possibility of modeling a large class of fluid flows by the equations of the Navier strokes.. The Navier-Stokes equations, which describe viscous fluid flows, are so complicated that analytical and numerical treatments, especially at large Reynolds numbers, are very difficult. CFD (computational fluid dynamics) are suggested for different simulation of fluid flow problems. The traditional sort of CDF consists of partitioning the underlying physical space into a calculating and computational grid. The Navier-Stokes equations are discretized on this grid. Generating the grid consists of particular efforts for complicated geometries. On the other hand, attempts to utilize numerical methods face several important limitation. For example, in the study of unsteady flow of an incompressible fluid past an object a t high Reynolds numbers, R, the crucial region is small in size and, in addition, involves boundaries and vortex sheets. This puts the numerical method based on a grid at a considerable disadvantage because the mesh width must decrease as R increases. Consequently, at very large R, a very fine grid must be imposed or else the numerical viscosity due to the grid will swamp the effects of the true viscosity as represented by the Reynolds number. As the matter of fact, it can be a problematic issue. The computational grid can also introduce numerical difficulties in the simulation. There is some difficulties with grid-based which avoid in Vortex methods .this method also offer an alternative approach.

It used a vorticity-velocity formulation .The vorticity field is discretized into a finite number of vortex elements with the specified strengths (rather than specifying it on a fixed grid). The velocity field is obtained from the vorticity field and track of a finite number of vortex elements are kept in a Lagrangian reference of frame. As, the vorticity is tracked in order to displace of individual particles, There is no necessity for a fixed grid on which the governing differential equations and the unknowns are identified. Rvm is one off simple and completely grid-free vortex method, in which each time step is divided into two fraction steps. In the first fractional time step, the mechanism of diffusion is frozen and displacement of the center of elements which is due to convection of the flow field is calculated using a fourth order Runge-Kutta integrating scheme. In the second fractional time step, the effect of diffusion is considered using a random walk. The random vortex method was introduced for the first time in 1973 by Chorin [2]. Later in 1978, he presented the idea of vortex sheet for boundary layer and improved the accuracy of the results [3].C. Marchioro[7], J. Goodman[5], D-G. Long[6], E. G. Puckett[10],] F. Milinazzo and P. G. Saffman[8], have discussed about convergence of the random vortex methods. They also have studied how the errors in the random walk computations vary with the number of vortices. From the result it is clear that accuracy of method increase with increasing the number of particle. Random vortex method has been used extensively. It used for simulation the flow over cylinders, pitching airfoils, jets and cavities in various Reynolds number by [1,16,15,11,17,13,14].

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Some of the researcher used random vortex method for combustion problem [4,12,211,9].

Governing equation:

The basis of application and governing equations of vortex method is vorticity transfer equation in two-dimensions (3) which is gained by applying Curl Operator on Navier Stocks equations, (2), and then merging it with incompressible continuity equation, (1), in absence of body forces.

$$\frac{D\rho}{Dt}+\rho\nabla.\vec{V}=0\Rightarrow\frac{\partial\rho}{\partial t}+\vec{V}.grad\rho=-\rho\vec{\nabla}.\vec{V} \quad (1)$$

$$\frac{\partial\vec{V}}{\partial t}+\vec{V}.grad\vec{V}=-\frac{1}{\rho}gradP+\nu\nabla^2\vec{V} \quad (2)$$

$$\frac{\partial\vec{\omega}}{\partial t}+\left(\vec{V}.\vec{\nabla}\right)\vec{\omega}=\nu\Delta\vec{\omega} \quad (3)$$

Where V and P are velocity and pressure respectively .What deserves considerable attention in (3) is the absence of pressure as an unknown. This state that vorticity and fluid flow can, with no need for pressure field, is calculated. By removing dimension from (3) with regard to the cylinder's diameter, D, and reference velocity, we get:

$$\frac{\partial\vec{\omega}^*}{\partial t^*}+\left(\vec{V}^*.\vec{\nabla}^*\right)\vec{\omega}^*=\frac{1}{Re}\Delta^*\vec{\omega}^* \quad (4)$$

Where, V^* and t^* are dimensionless velocity and time respectively and υ is kinematic viscosity

Non viscous (Potential) flow throughout the field and no-slip condition due to fluid viscosity is the initial condition and the only boundary condition of the problem respectively

$$\vec{u}.\vec{n}=0,\vec{u}.\vec{t}=0 \quad U(0)=0 \quad (5)$$

u Is velocity vector, $\vec{n}.\vec{t}=0$ are orthogonal and tangent unit vector respectively.

As mentioned before, (3), is solved in two steps. First step corresponds to convection mechanism and the second step deal with diffusion mechanism. The equation relating to the first step (convection mechanism) is in fact Euler equation in the form of vorticity that emphasizes vorticity is a property of fluid and remains constant for particles along the motion direction.

The equation related to convection mechanism by omitting the term from vorticity equation, (3), in order to waive effect of fluid viscosity, the equating relating to convection mechanism is as follows:

$$if\ \nu=0\Rightarrow\frac{\partial\vec{\omega}}{\partial t}+\vec{V}.grad\,\vec{\omega}=0 \rightarrow\begin{cases}\frac{d\vec{r}_p}{dt}=V(\vec{r}_p)\\ \frac{D\vec{\omega}_p}{Dt}=0\end{cases} \quad (6)$$

Second step (diffusion mechanism) is investigated considering the effect of fluid viscosity and Brownian motion of vortex bulbs. Diffusion equation is gained by omitting the term , so (3) will alter as follows:

$$\frac{\partial w}{\partial t}=n\nabla^2 w\rightarrow\begin{cases}\frac{d\vec{r}_p}{dt}=0\\ \frac{d\vec{w}_p}{dt}=n\Delta w(\vec{r}_p)\end{cases} \quad (7)$$

In the first step (convection mechanism) a number of vortexes are in motion and induce velocity on each other and their surrounding as well. Induce velocity by vortex on the point , is as follows

$$\bar{W}_{vortex}(z_i)=\sum_i\frac{-i\Gamma_j|z_i-z_j|}{2p\times\max(|z_i-z_j|,d)}\left(\frac{1}{z_i-z_j}\right) \quad (8)$$

Is the number of vortexes, is radius of each vortex, is circulation of vortex, is the length of segments that should be equal and is conjugation of velocity.

To equal the vertical velocity on boundary of cylinders to zero, we locate sources and sinks on the segments. Induced velocity resulting from sources and sinks located on , on the point is gained as follows:

$$\bar{W}_{source}(z_i)=\frac{a(j)}{2p}\left(\frac{1}{z_i-z_j}\right) \quad (9)$$

Where is the power of source or sink that is positive for sources and negative for sinks.
Simultaneous effect of sinks and sources on each other induces an equal vertical velocity, but in the opposite direction of the induced velocity by vortexes, which finally turn the resultant vertical velocity in each segment to zero. By solving a series of equations and unknowns, power of sink and source is calculate as follows :

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$$A\alpha = b \tag{10}$$

$$\begin{cases} a_{ij} = U_1(ij)n_1 + U_2(ij)n_2 & i \neq j \\ a_{ii} = \frac{1}{2}h^{-1} & i = 1,\dots,N \end{cases} \tag{11}$$

$$U_1(ij) = -\frac{1}{2\pi}\frac{X_j - X_i}{R_{ij}^{\ 2}} \ , \quad U_2(ij) = -\frac{1}{2\pi}\frac{Y_j - Y_i}{R_{ij}^{\ 2}} \ , \quad R_{ij}^{\ 2} = (X_j - X_i)^2 + (Y_j - Y_i)^2 \tag{12}$$

$$b_i = -\vec{W}_{vortex}(i).\vec{n} \tag{13}$$

Where is the coefficients matrix.

In the second step (convection mechanism) random motion of vortices takes place based on Gaussian random variable. Relevant equation, (7), is of thermal type and can be solved using Green's function for two-dimensional heat transfer equation .

$$G\left(\vec{r},t\right) = \frac{1}{4\pi\nu t}\exp\left[-\frac{r^2}{4\nu t}\right] \tag{14}$$

Mentioned Green's function is equal to the probability density function of Gaussian variable with average of zero and variance, σ. So, probability density function of Gaussian variable can be expressed as :

$$p(\eta,t) = \sqrt{\frac{1}{2\pi\sigma^2}}\exp\left[-\frac{\eta^2}{2\sigma^2}\right] \tag{15}$$

By comparing (14),(15) we notice that the above function is a random variable Gaussian function with average of zero and variance of $\sigma = \sqrt{\frac{2t}{R}}$

therefore is a location of vortex in , its location in can be calculated as:

$$Z_j(t+\Delta t) = Z_j(t) + \left[\bar{W}_{pot.}(j) + \bar{W}_{vortex}(j,t) + \bar{W}_{source}(j,t)\right]\Delta t + \eta_j$$

Where T represents time.

Conclusion:

The vortex element method has an extensive application in the numerical simulation of the turbulent flows. Keeping track of the vortex elements in a Lagrangian reference, the formation, growth, convect ion and diffusion of the elements can very well be investigated. Random vortex methods have the significant advantage . Conventional methods must resolve the whole flow but in this method only the portion of the flow in which vorticity occurs must be resolved. The scheme is time-accurate and therefore ideally suited for unsteady flows. It is grid –free and Lagrangian tracking of vorticity is used so if the vorticity separates from the solid surfaces, the vortices follow that motion.The main disadvantage is that it requires a large number of particles to obtain reasonable accuracy, due to its slow convergence rate. It is safe to say that the RVM has been the most widely utilized of viscous vortex methods, especially in engineering applications.

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