ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

Application of Wavelet Transform and Its Advantages Compared To Fourier Transform

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Abstract:

Wavelet analysis is an exciting new method for solving difficult problems in mathematics, physics, and engineering, with modern applications as diverse as wave propagation, data compression, signal processing, image processing, pattern recognition, computer graphics, the detection of aircraft and submarines and other medical image technology. Wavelets allow complex information such as music, speech, images and patterns to be decomposed into elementary forms at different positions and scales and subsequently reconstructed with high precision. Signal transmission is based on transmission of a series of numbers.

The series representation of a function is important in all types of signal transmission. The wavelet representation of a function is a new technique. Wavelet transform of a function is the improved version of Fourier transform. Fourier transform is a powerful tool for analyzing the components of a stationary signal. But it is failed for analyzing the non stationary signal where as wavelet transform allows the components of a non-stationary signal to be analyzed. In this paper, our main goal is to find out the advantages of wavelet transform compared to Fourier transform.

Keywords:

Fourier transform, wavelet, wavelet transform, time-frequency signal analysis 1. Introduction In 1982 Jean Morlet a French geophysicist, introduced the concept of a `wavelet'. The wavelet means small wave and the study of wavelet transform is a new tool for seismic signal analysis. Immediately, Alex Grossmann theoretical physicists studied inverse formula for the wavelet transform.

2. Wavelet:

Wavelet may be seen as a complement to classical Fourier decomposition method. Suppose, a certain class of functions is given and we want to find 'simple functions' $f_{0,} f_1 f_2$............... such that each

$$f(x)=\sum_{n=0}^{\infty} a_n f_n(x)$$

for some coefficients a_n .

Wave let is a mathematical tool leading to representations of the type (1) for a large class of functions f .

Wavelet theory is very new (about 25 years old) but has already proved useful in many contexts.

Definition (Wavelet)

A wavelet means a small wave (the sinusoids used in Fourier analysis are big waves)and in brief, a wavelet is an oscillation that decays quickly. A wavelet means a small wave (the sinusoids used in Fourier analysis are big waves) and in brief, a wavelet is an oscillation that decays quickly.

Equivalent mathematical conditions for wavelet are :

$$\int_{-\infty}^{\infty} |\psi(t)| dt \langle \infty \qquad (2)$$

$$\int_{-\infty}^{\infty} |\psi(t)| dt = 0. \quad (3)$$

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$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega \langle \infty \quad (4)$$

where ψ (ω)) is the Fourier Transform of ψ (t) . Equation (4) is called the admissibility condition.

3. Wavelet Transform Jean Morlet in 1982, introduced the idea of the wavelet transform and provided a new mathematical tool for seismic wave analysis. Morlet first considered wavelets as a family of functions constructed from translations and dilations of a single function called the "mother wavelet" ψ(t). They are defined by

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \Psi\left(\frac{t-b}{a}\right), a, b \in R, a \neq 0 \quad (5)$$

The parameter a is the scaling parameter or scale, and it measures the degree of compression. The parameter b is the translation parameter which determines the time location of the wavelet. If |a| < 1, then the wavelet in (5) is the compressed version (smaller support in time- domain) of the mother wavelet and corresponds mainly to higher frequencies. On the other hand, when |a| > 1, then ψa,b(t) has a larger time-width than ψ (t) and corresponds to lower frequencies. Thus, wavelets have time-widths adapted to their frequencies. This is the main reason for the success of the Morlet wavelets in signal processing and time-frequency signal analysis.

4. Wavelet Series and Wavelet Coefficients

If a function f ∈ L_2(R), the series

$$\sum_{j \in z} \sum_{k \in z} \langle f, \Psi_{j,k} \rangle \Psi_{j,k}(t) \quad (6)$$

is called the wavelet series of f and

$$\langle f, \Psi_{j,k} \rangle = d_{j,k} = \int_{-\infty}^{\infty} f(t) \Psi_{j,k}(t) dt \quad (7)$$

Is called the wavelet coefficient of f

5.Signal

A signal is given as a function f which has a series representation $f(x) = \sum_{-\infty}^{\infty} a_n x^n$

Then all information about the function f is stored in the coefficients $\{a_n\}_{n=0}^{\infty}$

6. Classification of Signals

We can split the class of signals into two classes, namely:

• Continuous signals
• Discrete signals

Continuous Signals:The signals which is described by a continuous function (for example a recording of a speech signal or music signal, which measures the current in the cable to the loudspeaker as a function of time) is called continuous signals Christensen [1], Mallat [6]. Here is a concrete example concerning sound signals: An Example Wavelets are frequently used to remove noise from music recordings. The main idea is to think about a music signal as consisting of the music itself to which some noise is added.

The music signal itself describes how the music changes in time; we can think about the signal as the current through the loudspeaker when we play a recording. Expanding the music piece via wavelets means that we represent this signal via the coefficients $\{d_{j,k}\}$ in (7); the coefficients "tell when something happens in the music". The noise contribution is usually small compared to the music, but irritating for the ears; it also contributes to the coefficients $\{d_{j,k}\}$, butusually less than the music itself. The idea is now to remove the coefficients in (7) which are smaller than a certain threshold value (strictly speaking, this procedure is not applied on the signal itself, but on its so called wavelet transform).

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To be more precise, this means that these coefficients are replaced by zeroes; thus, one lets the remaining coefficients represent the music. The idea is to remove the part of the signal which has no relationship to the music; unfortunately, the above procedure also might cancel smaller parts of the music itself, but still the result is usually considered by the ears as an improvement Christensen [1], Mallat [6].

Discrete Signals:

The signals which is described by a sequence of numbers or pairs of numbers is called discrete signals Christensen [1], Mallat [6]. Here mentioned one example of discrete signals:

An Example A digital black-white photo consists of a splitting of the picture into a large number of small squares, called pixels; to each pixel, the camera associates a light intensity, measured on a scale from, say, 0 (completely white) to 256 (completely black). Put together, this information constitutes the picture. Thus, mathematically a photo consists of a sequence of pairs of numbers, namely, a numbering of the pixels together with the associated light intensity Christensen [1], Mallat [6].

7. Some Application of Wavelets

Wavelets are a powerful statistical tool which can be used for a wide range of applications, namely

- Signal processing
- Data compression
- Smoothing and image denoising
- Fingerprint verification
- Biology for cell membrane recognition, to distinguish the normal from the pathological membranes
- DNA analysis, protein analysis
- Blood-pressure, heart-rate and ECG analyses
- Finance (which is more surprising), for detecting the properties of quick variation of values
- In Internet traffic description, for designing the services size
- Industrial supervision of gear-wheel
- Speech recognition
- Computer graphics and multifractal analysis
- Many areas of physics have seen this paradigm shift, including molecular dynamics, astrophysics, optics, turbulence and quantum mechanics.

Wavelets have been used successfully in other areas of geophysical study. Orthonormal wavelets, for instance, have been applied to the study of atmospheric layer turbulence. In one study by J.F. Howell and L. Mahrt, turbulence measurements were taken over a nine-hour period and analyzed using wavelet decomposition. In another study by Brunet and Collineau, turbulence data recorded over a corn crop was analyzed using the wavelet transform. Wavelets have also been used to analyze seafloor bathymetry or the topography of the ocean floor. In one study by Sarah Little, the use of wavelet analysis revealed patterns, trends, and structures that may be overlooked in raw data. Also, the use of methods like local oracles allowed for separation of data in regions of interest.

Several other geophysical applications such as analysis of marine seismic data and characterization of hydraulic conductivity distributions have also been used. The usefulness of wavelets in data analysis is clear, particularly in the field of geophysics, where large and cumbersome data sets abound. Studies such as the atmospheric layer turbulence and corn crop turbulence have further shown the proficiency of wavelets in the analysis of time-dependent data sets.

8. Data Compression

Consider the following sequence of numbers (which represent a given signal) To these eight numbers we now associate eight new numbers, which appear in the following way. First, we consider the above eight numbers as a series of four pairs

56	40	8	24	48	48	40	16

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First, we consider the above eight numbers as a series of four pairsof numbers, each containing two numbers. We now replace each of these pairs of numbers by two new numbers, namely, their average and the difference between the first number in the pair and the calculated average. The first pair of numbers in the given signal consists of the numbers 56 and 40; our procedure replaces then by the new pair consisting of the numbers 48procedure on all four pairs we obtain the following sequence of numbers:

$\frac{56+40}{2}=48, 56-48=8$ Applying this procedure on all four pairs we obtain the following sequence of numbers

48	8	16	-8	48	0	28	12

In mathematical terms, we have replaced the pair (a,b) by a new pair (c,d) given by

$$c=\frac{a+b}{2}, d=a-\frac{a+b}{2}=\frac{a-b}{2} \quad (8)$$

Let us write the new sequence of numbers under the original sequence:

56	48	8	24	48	48	40	16
48	8	16	-8	48	0	28	12

In some sense, the obtained sequence in the second row contains the same information as the original sequence in the first row: we can always come from the second row and back to the first by inverting the above procedure. That is, we have to solve the equations (8) with respect to a,b and apply the obtained formula to the four pairs of numbers in the second row. Solving the equations (8) with respect to a,b gives

$$a = c + d, b = c - d \quad (9)$$

So the inverse transform is exactly as easy to apply as the transform itself.

When we go back to the original signal from the second row we speak about reconstruction of the original information. If our purpose is to store the information or to send it, we can equally well work with the second row as with the first: we just have to remember to transform back at a certain stage. However, at this moment it is not clear that we gain anything by the transformation. We keep this question open for a moment, and apply the procedure once more; but only on the numbers appearing as averages.

In other words, we let the numbers 8, -8, 0, 12 (Calculated as differences) stay in the table, and repeat the process on the numbers 48, 16, 48, 28 i.e. on the pairs 48, 16 and 48, 28. That is, we calculate average and difference (in the above sense) for each of these pairs.This leads to the numbers 32, 16, 38, 10 which are placed on the free places in the table; this gives us the table

32	8	16	-8	38	0	10	12

These eight numbers also represent the original information: we just have to invertthe above procedure twice, then we are back at the original sequence. However, some bookkeeping is involved: we have to keep track of which numbers we calculated as averages, and which were differences.

35	8	16	-8	-3	0	10	12

So, far we have only argued that the performed operations do not change the information, in the sense that we can come back to the original numbers by repeated application of the inversion formula (9). In order to show that we have gained something, we now apply thresholding on the numbers in the final table. In popular words, this means that we remove the numbers which numerically are smaller than a certain fixed number ; more precisely, we replace them by zeros. If we for example decide to remove all numbers which numerically are smaller than 4, we obtain the table

32	8	16	-8	0	0	10	12

g) Wavelet theory is capable of revealing aspects of data that other signal analysis techniques miss the aspects like trends, breakdown points, and discontinuities in higher derivatives and self-similarity.

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h) It can often compress or de-noise a signal without appreciable degradation.

Results and Discussion:

Fig.1 shows that the original signal and reconstruction signal after thresholding with 4. From Fig.1, we observe that the reconstructed signal are quite close to the original signal. Fig.2 shows that the original signal and reconstruction signal after thresholding with 9. From Fig.2, we observe that even though the rough thresholding is worse, the reconstructed signal still follows the shape of the original signal quite well, except in the neighborhood of points where the signal oscillates heavily.

Hence we can say that data compression is the great achievement of wavelet transform i.e. in wavelet transform, it is often possible to obtain a good approximation of the given signal f by using only a few coefficients. Fig.3 shows the original fingerprint and Fig.4 shows the compressed fingerprint using wavelets and reconstructed. However, manual fingerprint verification is so tedious, time-consuming and expensive in that it is incapable of meeting today's increasing performance requirements. Hence, an automatic fingerprint identification system (AFIS) is widely needed.

Wavelet transform which has wide range of applications such as image compression is used in this paper for Fingerprint verification. It describes the design and implementation of an off-line fingerprint verification system using wavelet transform. In this method, matching can be done between the input image and the stored template without resorting to exhaustive search using the extracted feature. The experimental results show that the wavelet transform based approach is better than the existing minutiae based method and it takes less response time which is more suitable for on-ine verification with high accuracy.

Let us finally mention that compression of fingerprints in principle also can be performed using Fourier transform. However, this classical method is less efficient: at the rate of compression The FBI uses for the wavelet method it would no longer be possible to follow the contours in a reconstructed fingerprint and the result would be useless in this special context. Fourier analysis is a mathematical technique for transforming our view of the signal from time-based to frequency-based. In transforming to the frequency domain, time information is lost which is very important. This is the main drawback of Fourier transform.

In the wavelet transform we do not lose the time information which is useful in many contexts. In Fourier analysis signal properties do not change over time. This drawback is not very important. But most interesting signals contain numerous non- stationary or transitory characteristics like drift, trends, abrupt changes and beginnings and ends Application of Wavelet Transform And Its Advantages Compared to Fourier of event. These characteristics are often the most important part of the signal. The classical Fourier analysis is not suited for detecting them but wavelet analysis is suited for detecting them.

Conclusion:

In this paper, we have discussed about some applications of wavelet such as data compression, recording of a sound signal, music signal, fingerprint verification with the help of a wavelet transform. We have also tried to comparative discussion of Fourier transform and wavelet transform mentioning the drawback of Fourier transform, besides this we have discussed the advantages of wavelet transform. From our above discussion it is clear that the experimental results show that the wavelet transform based approach is better than the existing minutiae based method and it takes less response time which is more suitable for online verification with high accuracy. Finally, we can say that wavelet transform is a reliable and better technique than that of Fourier transform technique. Transform.

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