Speed Tracking of a Linear Induction Motor Enumerative Nonlinear Model Predictive Control

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Abstract:

Direct torque control (DTC) is considered as one of the most efficient techniques for speed and/or position tracking control of induction motor drives. However, this control scheme has several drawbacks: the switching frequency may exceed the maximum allowable switching frequency of the inverters, and the ripples in current and torque, especially at low speed tracking, may be too large. In this brief, we propose a new approach that overcomes these problems. The suggested controller is a model predictive controller, which directly controls the inverter switches. It is easy to implement in real time and it outperforms all previous approaches. Simulation results show that the new approach has as good tracking properties as any other scheme, and that it reduces the average inverter switching frequency about 95% as compared to classical DTC.

Index Terms:

Integer programming, inverter, linear induction motor, nonlinear model predictive control, optimal control, speed tracking control.

I. Introduction:

NOWADAYS, linear induction motors (LIMs) are widely used in a variety of applications, such as transportation, conveyor systems, material handling, pumping of liquid metal, sliding door closers, robot base movers, office automation, drop towers, elevators etc. [1], [2].

This is attributed to several advantages that the latency insertion method (LIM) posses, such as high starting thrust, alleviation of gears between motor and the motion devices, simple mechanical construction, no backlash and small friction, and suitability for both low speed and high speed applications [3]–[5]. The driving principles of the LIM are similar to those of the traditional rotary induction motor. However, the control characteristics of the LIM are more complicated. This is attributed to the change in operating conditions due to mover speed, temperature, and rail configuration. Moreover, there are uncertainties existing in practical applications of the LIM, which are usually composed of unpredictable plant parameter variations, external load disturbances, and unmodeled and nonlinear dynamics. Therefore, the design of LIM drive system should provide high tracking performance, and high dynamic stiffness to overcome the above challenges [6].

Several control techniques have been used to control the speed and/or position of induction motor drives. Among these control techniques, the method of direct torque control (DTC) is considered as one of the most efficient techniques that can be used for induction motors [9]. The basic characteristic of DTC is that the positions of the inverter switches are manipulated directly. The advantages of the DTC strategy are fast transient response, simple configuration, and high robustness against parameter variations. However, classical DTC has inherent drawbacks, such as variable switching frequency, high torque and current ripples, high noise level at low speeds and also
problems with the control of torque and flux at low speeds. Model predictive control (MPC) has been applied to LIM drives for tracking of speed reference trajectories [10]. Based on a linearized model of the LIM, the MPC controller calculates the optimal primary voltages while respecting constraints on flux and current in order to keep them within permissible values. It has been shown that the response is very fast as compared to classical DTC, and with almost no ripples in the current and torque signals. Moreover, it has been shown to be more robust against parameter uncertainty and load disturbance at high speed as well as at low speed. The MPC controller is used in conjunction with a PWM inverter. This often results in a high switching frequency at the inverter switches. Moreover, the computational burden of the on-line optimization and linearization makes real-time implementation impossible.

A performance improvement in terms of a reduction of the switching frequency as compared to classical DTC is shown. However, the approach provides only a feasible solution and no optimal solution. Moreover, the reformulation of the system into MLD-form and computing an explicit solution using a multi-parametric approach is computationally very demanding. Because of this only the case of a fixed operating point is considered. In [12], another MPC scheme is proposed that keeps the motor torque and the stator flux within given hysteresis bounds while minimizing the switching frequency of the inverter. The proposed model predictive DTC (MPDTC) scheme reduces the switching frequency by up to 50% as compared to other techniques, while respecting the torque and flux hysteresis bounds. In this approach, the rotor speed dynamics are neglected and the speed is assumed to remain constant within the prediction horizon.

Moreover, we may consider the non linear dynamics and do not have to linearized the model. We will call our control strategy enumerative nonlinear MPC (ENMPC). Because the optimization is enumerative and over a small number of discrete variables, it is extremely fast, and hence admits real-time implementation. ENMPC is similar to the control scheme presented in [14]. There is a predictive strategy for current control of a three-phase neural-point clamped inverter is presented, where the behavior of the system is predicted for each possible switching position of the inverter, and the position that minimizes a given cost function is selected. Hence this approach is also enumerative.

Several similar approaches can be found in [13]. However, they all consider a prediction horizon of one. In our work, the prediction horizon is longer. This brief is organized as follows. Section II briefly presents the dynamic model of the LIM. In Section III, the ENMPC controller is presented. The system configuration is described in Section IV. Simulation results and general remarks are presented in Section V. Finally, conclusions and suggestions for future work are given in Section VI.

II. LINEAR INDUCTION MOTOR:

A. Dynamic Model of the LIM:

The dynamic model of the LIM is similar to the traditional model of a three phase, Y-connected induction motor in \( \alpha - \beta \) stationary frame, and it can be described by the following differential equations

\[
\frac{di_{as}}{dt} = - \left( \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma T_r} \right) i_{as} + \frac{Lm}{\sigma L_s L_r T_r} \lambda_{ar} + \frac{n_p L_m \pi}{\sigma L_s L_r h} \omega \lambda_{br} + \frac{1}{\sigma L_s} V_{as} \tag{1}
\]

\[
\frac{di_{bs}}{dt} = - \left( \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma T_r} \right) i_{bs} - \frac{n_p L_m \pi}{\sigma L_s L_r h} \omega \lambda_{ar} + \frac{Lm}{\sigma L_s L_r T_r} \lambda_{br} + \frac{1}{\sigma L_s} V_{bs} \tag{2}
\]
where \( T_r = L_r/R_r \), \( \sigma = 1 - L_2^2/m/L_s L_r \), and shown in nomenclature.

The electromagnetic force can be described in the \( \alpha - \beta \) fixed frame as

\[
\frac{d(v)}{dt} = \frac{1}{M} F_e - \frac{D}{M} v - \frac{1}{M} F_L
\]

(5)

where \( T_r = L_r/R_r \), \( \sigma = 1 - L_2^2/m/L_s L_r \), and shown in nomenclature.

The electromagnetic force can be described in the \( \alpha - \beta \) fixed frame as

\[
F_e = k_f \left( \lambda_{ar} i_{\beta s} - \lambda_{br} i_{\alpha s} \right)
\]

(6)

Where \( k_f \) is the force constant which is equal to

\[
k_f = \frac{3n_p L_m \pi}{2L_r h}.
\]

(7)

We will use a forward-Euler discretization of the nonlinear differential equations to obtain a discrete-time model suitable for our purposes of MPC.

**B. DC–AC Inverter:**

The three phase two-level DC–AC inverter used to drive the LIM is shown in Fig. 1. The three switches can be modeled by three binary variables \( u_1, u_2, u_3 \in \{0, 1\} \) representing on/off positions, which imply the following relation:

\[
u_i = \begin{cases} 
1 & \text{if} \quad v_i = \frac{V_d}{2} \\
0 & \text{if} \quad v_i = -\frac{V_d}{2}
\end{cases} \quad i = 1, 2, 3
\]

(8)

Where \( V_{dc} \) is the DC voltage source. The three switches have eight possible different position combinations. The relation between the primary voltage components \( V_{\alpha s}, V_{\beta s} \) and the switching positions is given by the following equation:

\[
\begin{bmatrix}
V_{\alpha s} \\
V_{\beta s}
\end{bmatrix}
= V_{dc} \times \frac{3}{2} \begin{bmatrix}
\frac{2}{3} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
\]

(9)

**C. Control Objectives:**

The main objective is to control the speed of the LIM drive to track the given speed reference. The controller controls the three inverter switching positions to provide the necessary primary voltage to track the speed reference. It is recommended to minimize the average switching frequency of the inverter switches. Constraints over secondary flux and primary current should be considered: the secondary flux should be less than 0.45 Wb, and the primary current should be less than 50 A.

![Fig. 1. Three-phase inverter driving the LIM.](image)

**III. Enumerative Nonlinear MPC Controller:**

The main idea of MPC is to use a model of the plant to predict future outputs of the system. Based on this prediction, at each sampling period, a sequence of future control values is computed through an on-line optimization process, which maximizes the tracking performance while satisfying constraints. Only the first value of this optimal sequence is applied to the plant, and the whole procedure is repeated again at the next sampling period according to what usually is called a “receding” horizon strategy. Applying MPC to an LIM grants a better performance than the classical DTC approach, [10], but the main drawbacks of this technique are the heavy on-line computations that...
make it inapplicable in real-time, and also the high switching frequency that may exceed the maximum allowable frequency. Because of these drawbacks, the following ENMPC controller is proposed. As the three switches of the inverter have only eight different position combinations, an analytical computation of the tracking performance, for the eight possible position combinations can be performed. Then the position of the switches, which are the manipulated variables, that maximizes the tracking performance is selected. The eight different combinations of positions are elements of the set.

The objective function that captures the tracking performance includes the error between the actual speed and the speed reference trajectory. To minimize the inverter switching frequency, a penalty term on the control variations is included in the objective function. The considered objective function is where $\dot{v}$ is the predicted future speed, $w$ is the speed reference, $u$ is the control signal, and where $Q$ and $P_j$ are positive constants. The second term which penalizes the input switching, measures directly the switching number,

$$J = \sum_{k=0}^{N} (v_k - v_{ref})^2 + \sum_{k=0}^{N} P_j |u_{k+1} - u_k|$$

TABLE I: NUMBER OF CONTROL SWITCHES, WHERE INDICES 1, 2, ..., 8 REFERS TO ELEMENTS IN U IN (10)

<table>
<thead>
<tr>
<th>$u(k+j-1)$</th>
<th>$u(k+j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>2</td>
<td>0 2 1 3 1 2</td>
</tr>
<tr>
<td>3</td>
<td>2 0 2 1 3 1 2</td>
</tr>
<tr>
<td>4</td>
<td>1 3 0 2 2 2 1</td>
</tr>
<tr>
<td>5</td>
<td>3 1 1 2 0 2 2 1</td>
</tr>
<tr>
<td>6</td>
<td>1 3 1 2 2 0 2 1</td>
</tr>
<tr>
<td>7</td>
<td>1 1 1 2 2 0 3</td>
</tr>
<tr>
<td>8</td>
<td>2 2 2 1 1 1 3 0</td>
</tr>
</tbody>
</table>

i.e., $T(u(k+j), u(k+j-1))$ is the number of switches as defined in Table I. The value in row i and column j is showing the number of switches when $u(k+j-1)$ has the value of element i and $u(k+j)$ has the value of element j in U in (10). The objective function (11) is minimized subject to constraints that describe the discredited dynamics in (1)-(9). The constants $P_j$ should impose more penalties over the first time-steps than the later steps, to force the transition of the switches to occur as late as possible [21], [22].

This is accomplished by the following constraints: Elimination of small steady-state errors can be accomplished in different ways. The method we have used involves modifying the objective function to not only minimize the tracking error but to also minimize a sum of old tracking errors. Thus, the objective function (11) is redefined as

Here, $E^*(k)$ is a prediction of the sum of the tracking error $E(k)$, where $E(k)$ is defined as follows:

$$E(k) = \sum_{j=0}^{k} (v_j - v_{ref})^2$$

where $v$ is the measured speed, $w$ is the speed reference, and where $K$ is a gain. To avoid that $E$ becomes too large we may replace (14) with $E(k+1) = E(k)$ when $|E(k)|$ is larger than a certain limit. Our method is aiming at providing integral control. There are other approaches to eliminate steady-state offset, see [23]. Notice that many of the traditional integration methods are not possible to use in this application where the control signal is not a continuous valued signal. The concept of control horizon ($N_u < N$) is used to reduce the number of decision variables and thus the computational time. Other methods to reduce the number of optimization variables could also have been used, e.g., blocking of the input variables technique [24]. The objective function (13) is evaluated $s = 8N_u$ times at each time step, and the first control signal in the sequence $u_{opt} = (u(k),..., u(k+Nu−1))$

Algorithm 1 Reducing the Computational Time: Pruning Rule

1) Initializing with $J_{opt} = \infty$, $J'(k) = 0$
2) for $i = 1:s$
3) for $j = 1:N$
4) let
   $$J'(k+j) = J'(k+j-1) + f(\dot{v}(k+j), u'(k+j-1))$$
   where $f(\dot{v}(k+j), u'(k+j-1))$ is the incremental cost at time $k+j$ due to the control signal $u'(k+j-1)$.
5) If $J'(k+j) > J_{opt}$ break and go to step 2
6) At $j = N$
   if $J'(k+N) < J_{opt}$, $J_{opt} := J'(k+N)$
   end if
   end for
7) $J_{opt} = J_{opt}$ the optimal value

Corresponding to the minimum objective function value is then selected and applied to the inverter switches. Increasing the prediction horizon $N$ will lead to more accurate choice of control signals.
However, increasing the prediction horizon will increase the computational time. To account for that, we propose to use different discrete time models with different sampling times as described in . For the first sampling steps, we use a motor model with the true sampling time, and then for later sampling steps we use another model with longer sampling time. This will increase the prediction interval with less number of prediction steps as compared to when using the same sampling time for all predictions. To avoid examining all possible input combinations over the control horizon \( N \), the following incremental algorithm is proposed to compute the optimal control signal sequence. Here, \( u_i \) a candidate optimal control signal sequence that is an element in \( U \times U \times \cdots \times U \), where the number of Cartesian products is \( s - 1 \).

The incremental cost (in step 4 of Algorithm 1) is the predicted cost at time step \( k + j \) due to the control signal \( u_i (k + j - 1) \), and it is given by Algorithm 1 stops the cost function calculations for the control sequence \( u_i \) prematurely if the cost function at prediction step \( j \) is higher than the current upper bound \( J_{opt} \). This saves computational time. The algorithm is similar to one of the pruning rules in the branch and bound (BB) algorithm for solving integer programs. One of the main advantages of MPC is its ability to deal with constraints, i.e., offering optimal control while respecting the given constraints. Including flux and current constraints into our proposed controller is simple. As an example, a maximum flux constraint can be obtained simply by adding the following line to the controller code.

The proposed controller is faster than other standard techniques for solving integer programming problems like for example BB, which is generally considered as one of the most effective techniques. At each step in the BB algorithm a relaxed optimization problem, often a convex quadratic program, is solved where a certain number of integer variables is relaxed to continuous variables with values constrained in \([0,1]\). Solving these relaxed optimization problems takes more time than the analytical computation of the objective function when the number of optimization variables is small, which is the case of the application in this brief.

Moreover, the relaxed problem for the MPC controller we suggest would not be a quadratic program, since we have introduced a penalty term on the number of switches and because the discretized dynamics of the LMI is not linear. Hence they can be expensive to solve. The advantages of the proposed technique besides its simple design and implementation are that there is no complicated on-line optimization to be performed. Furthermore, there is no need to linearize the LIM model as was necessary in [10]. Moreover there is no need to reformulate the system in the hybrid system framework, neither as a piecewise affine model nor as an MLD model as done in [11].

Operating point changes are also easily incorporated in our framework. Even preview control is possible, i.e., in case the future value of the reference value is known, this can be taken into account. There are several applications of LIMs where this is potentially advantageous, e.g., elevators and autonomous trains. The developed technique significantly reduces the computational time. Moreover, one extra dimension of freedom through the choice of the weights \( P_j \) has been added, which enables a tradeoff between the average switching frequency and the speed tracking performance. Note that reducing the torque ripple can only be achieved by increasing the switching frequency and vice versa [25].

IV. System Configuration:

A block diagram of the linear induction motor controlled with the proposed ENMPC controller is shown in Fig. 2. The system consists of the LIM, an inverter, the ENMPC controller, and a flux estimator. The input signals to the ENMPC controller are the speed reference \( w \), the LIM velocity \( v \), the primary currents \( i_{αs} \) and \( i_{βs} \), and estimates of the secondary fluxes \( λ_{αr} \) and \( λ_{βr} \).
A. Control Configuration:

Different values for the control horizon and the prediction horizon, $N_u$, $N$, respectively, have been considered. In experiments, we have seen that for $T_s=100$ $\mu$s the choice of $N_u=2$, $N=10$ provides a good performance at high and low speed tracking, and that there is no need to increase the control horizon. With a shorter control horizon $N_u=1$, we only have a slightly lower performance at low speed.

After successive tuning iterations, the parameters of the MPC controller that gives a good response are: control horizon $N_u=1$, prediction interval $= 10 \times T_s$. The concept of multiple discrete models, as mentioned previously, is used to reduce the number of prediction steps; a model with sampling time $T_s$ is used for the first two steps, and then a model with sampling time equal $4T_s$ is used for the next two steps, i.e., the prediction interval of in total $10T_s$ is covered with four prediction steps.

The weights in the objective function have been chosen as $P_j=1$, $Q=1,000,000$, $P_i=500$, and $k=150$. The considered constraints on fluxes and currents force the controller to keep them within their minimum and maximum limits.
Fig. 6 Simulation wave form of ENMPC response versus DTC response

Fig. 7 Simulation wave form of ENMPC controller at low speed

Fig. 8 Simulation wave form of Secondary flux and primary currents are within the constraints

Fig. 9 Simulation wave form of ENMPC controller at low speed & Secondary flux and primary currents are within the constraints

Fig. 10 Simulation wave form of DTC load decrease

Fig. 11 Simulation wave form of Load increase
VI. Conclusion:

This brief considered speed tracking for a linear induction motor. It presented a new ENMPC controller based on the MPC approach. The developed controller controls directly the inverter switches to track the speed trajectory of the linear induction motor drive. The controller succeeds in tracking the speed trajectory at both high and low speed, and it reduces the switching frequency with about 95% as compared to classical DCT.

The proposed MPC controller response has many advantages; besides being simple to construct and to implement, it has a very fast response, lower ripples over currents and electromagnetic force in comparison to the DTC approach, and robustness against load changes and parameter variations. With this technique, there is no need to use a PWM inverter, and moreover, it reduces significantly the computational time, which is an inherent drawback of classical MPC controllers. Thus real-time implementation is possible. Future work will include experimental works to validate this technique in practice. Finally, the same technique will be examined for other machines, such as rotary induction motors and permanent magnet synchronous motors.

References:


