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Comparative study of different techniques for Time-Frequency Analysis

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Abstract- ***The paper briefly introduces the basic principle of S-transform. With the simulation experiments, the time frequency space characteristics of short-time Fourier transform, Wigner-Ville distribution and S-transform are discussed. As the results suggest, the window of S-transform has a progressive frequency dependent resolution. So the S-transform has a great utility and flexibility in non-stationary signal processing. To compare the time-frequency spectrum of three different analysis methods under various noise conditions, it is obvious that Stransform has better anti-noise performance. It fully proves that S-transform is an even more effective means for non-stationary signal processing.***

Keywords- ***S-transform; Time-Frequnecy Analysis; Short-time Fourier Transform; Continue Wavelet Transform.***

INTRODUNCTION

Mathematical transformations are applied to signals to obtain further information from that signal that is not readily available in the raw signal. I will assume a time-domain signal as a raw signal and a signal that has been transformed by any of the available mathematical transformations as a processed signal. There are a number of transformations that can be applied among which the Fourier transforms are probably by far the most popular. Most of the signals in practice, are time frequency signals in their raw format. That is, whatever that signal is measuring, is a function of time. In other words, when we plot the signal one of the axes is and the other is usually the amplitude. When we plot time-domain signals, we obtain a time-amplitude representation of the signal. This representation is not always the best representation of the signal for most signal processing related applications. In many cases, the most distinguished information is hidden in the frequency content of the signal.

Intuitively, we all know that the frequency is something to do with the change in rate of something. If something a mathematical or physical variable, would be the technically correct term changes rapidly, we say that it is of high frequency, where as if this variable does not change rapidly that is it changes smoothly, we say that it is of low frequency. If this variable does not change at all, then we say it has zero frequency, or no frequency.

The frequency is measured in cycles/second, or with a more common name, in Hertz. For example the electric power we use in our daily life is 60 Hz. This means that if you try to plot the electric current, it will be a sine wave passing through the same point 50 times in 1 second. Now, look at the following figures. The first one is a sine wave at 3 Hz, the second one at 10 Hz, and the third one at 50 Hz. Compare them.

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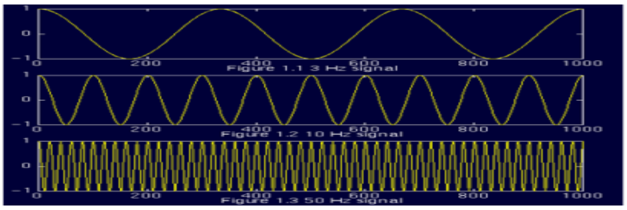

Fig :- Stationary signals

So how do we measure frequency, or how do we find the frequency content of a signal The answer is Fourier transform. If the FT of a signal in time domain is taken, the frequency-amplitude representation of that signal is obtained. In other words, we now have a plot with one axis being the frequency and the other being the amplitude. This plot tells us how much of each frequency exists in our signal. The frequency axis starts from zero, and goes up to infinity. For every frequency, we have an amplitude value. For example, if we take the FT of the electric current that we use in our houses, we will have one spike at 50 Hz, and nothing elsewhere, since that signal has only 50 Hz frequency component.

S-TRANSFORM:-

In non-stationary signal processing field, the time frequency transform is an important method. The Fourier transform is only map the signal from one-dimensional time domain to one-dimensional frequency domain. After the transformation, the signal has a very good frequency resolution, but the time resolution lost completely. This is not for non stationary signal processing. Time-frequency analysis method is one that converses the signal from time domain to time frequency domain and analyzes the non-stationary signals. The short-time Fourier transform, continue Wavelet transform and Wigner-Ville distribution are the most common way to analyze the non-stationary signals. The S-Transom is first proposed by R. G. Stockwell in 1996. It is unique in that it provides frequency-dependent resolution while maintaining a direct relationship with the Fourier spectrum. It is an extension of the ideas of the STFT and is based on a moving and scalable localizing Gaussian window. It is shown to have come desirable characteristics that are absent in continue Wavelet transform. Pinnegeret used it to process the non-stationary signals added Gaussian white noise.

S-Transform is conceptually a hybrid of STF analysis and wavelet analysis, containing elements of both but falling entirely into neither category. S-Transform can compute the magnitude and phase –angle for a given non-stationary signals. The article explores the S-transform from two perspectives, and described the time-frequency analysis features through complex signal. The S-transform has good performance of noise reduction in comparative experiments.

a) S-Transform from Short-Time Fourier Transform:-

The Fourier transform lacks the skill to position time and frequency at the same time. It is not available for the time frequency localization. Gabor first proposed that it could adopt a moving and scalable localizing Gaussian window as the base function. It defined by

$$w_{\tau,\omega}(t) = w(t-\tau)e^{i\omega t}$$

The definition of STFT is

$$STFT(\tau, f) = \int_{-\infty}^{+\infty} x(t)w(t-\tau)e^{-j\omega t}\mathrm{dt}$$

According to the uncertainty principle, the time-bandwidth product could not get smaller without limits. The Gaussian window is able to combine the time domain and frequency domain. The Gaussian window is defined as

$$\omega(t) = \frac{1}{\delta\sqrt{2\pi}}e^{\frac{-t2}{2\delta^2}}$$

According to the nature of Gaussian window, δ is he scale factor to change the width of Gaussian window. In order to make the width of Gaussian window have a better self-adaptability to different frequency components. δ could be defined as a frequency-related function.

$$\delta(f) = \frac{1}{|f|}$$

The new function is

$$\omega(t,F) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}}$$

We can get the expression of S-transform:

$$ST(\tau,f) = \int_{-\infty}^{+\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)}{2}} e^{-i2\pi f t} dt$$

b) S-Transform from Continue Wavelet Transform:-

The definition of Continue Wavelet Transform is

$$W(\tau,d) = \int_{-\infty}^{+\infty} x(t)\, \omega(t-\tau,d) dt$$

ω(t, d) is wavelet mother function. The dilation factor d controls the width of wavelet basis. It also controls the frequency resolution. But the mother wavelet function has to content the admissibility condition on the mother wavelet. The S-transform of function x(t) is defined as a continue Wavelet transform with a special mother wavelet multiplied by the phase factor.

$$S(\tau,f) = e^{j2\pi f t} W(\tau,d)$$

The mother wavelet is defined as

$$\omega(t,f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} e^{-j2\pi f t}$$

The dilation factor d is the inverse of the frequency f. And the wavelet in (9) doesn't satisfy the admissibility condition. So the S-transform is $S(\tau,f) = \int_{-\infty}^{+\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)^2}{2}} e^{-j2\pi f t} dt$ The calculation procedure of S-transform as follows:

1) To calculating the Discrete Fourier Transform of X [kT] with a time sampling interval of T, it can get X(m)
2) To calculating the Discrete Fourier Transform of the Gaussian window at a specific n , which called a voice Gaussian. The voice Gaussian is W (n, m).
3) To making a frequency shifting n, it can get X (m + n)
4) Multiply W (n, m) by X (m + n) to obtain B (n,m) .
5) The S-transform can be restored by the fast Fourier inverse transform of B(n, m)
6) Repeat the step (3) and (4) to finish all frequencies. Then the S-transform can be obtained.

The implementation procedure of discrete S-transform as follows:

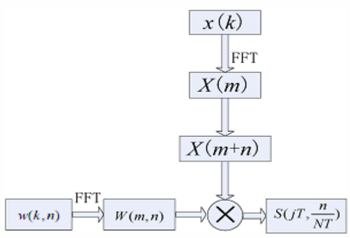

Figure: The S-Transform Schematic diagram

c) Stockwell Transform:-

Given a time series h(t), the local spectrum at time t = τ can be determined by multiplying h(t) with a Gaussian located at t = τ. Thus the 'Stockwell Transform 1' is given by Given a time series h(t), the local spectrum at time t = τ can be determined by multiplying h(t) with a Gaussian located at t = τ . Thus the 'Stockwell Transform 1' is given by

$$S(f,\tau,\sigma) = \int_{-\infty}^{+\infty} h(t) g(t-\tau)\, e^{-i2\pi f t} dt$$

The most convenient way of looking at the integral is to define

$$p1(t,f) = h(t) e^{-i2\pi f t}$$

Substituting in we get

$$S(f,\tau,\sigma) = \int_{-\infty}^{+\infty} p1(t,f) . g(t-\tau)\, dt$$

$$= p1(t,f) * g(t,\sigma)$$

where * denote the convolution operation. Let S(f,τ,σ) $ B(f,α,σ), where B(f,α,σ), is the Fourier transform of S(f,τ,σ). Thus

$$S(f,\tau,\sigma) = \int_{-\infty}^{+\infty} B(f,\alpha,\sigma)\, e^{-i2\pi \alpha \tau} d\alpha$$

From convolution theorem, we know that

$$\{ * g(t,\sigma)\} \leftrightarrow \int_{-\infty}^{+\infty} P1(f,\alpha)\, e^{-2\pi a t} d\alpha$$

since $p1(f,t) = h(t) e^{i2\pi f t}$

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$$p1(f,\alpha) = H(\alpha) * \delta(\alpha - f)$$

$$B(f,\alpha,\sigma) = [H(\alpha) * \delta(\alpha - f)].G(\alpha,\sigma)$$

$$S(f,\tau,\sigma) = \int_{-\infty}^{+\infty}\{[H(\alpha) * \delta(\alpha - f)].G(\alpha,\sigma)\}e^{-i2\pi\alpha\tau}d\alpha$$

$$B(f,\alpha,\sigma) = \int_{-\infty}^{+\infty} S(f,\tau,\sigma)e^{-i2\pi\alpha\tau}d\tau$$

$$B(f,\alpha,\sigma) = [H(\alpha) * \delta(\alpha - f)].G(\alpha,\sigma)$$

$$[H(\alpha) * \delta(\alpha - f)] = \frac{B(f,\alpha,\sigma)}{G(\alpha,\sigma)}$$

since $H(\alpha) * \delta(\alpha - f)$ is the forward translation of $H(\alpha)$, we can perform a backward translation to recover $H(\alpha)$ from $H(\alpha) * \delta(\alpha - f)$.

$$H(\alpha) * \delta(\alpha - f) = H(\alpha - f)$$

$$H(\alpha - f) = \frac{B(f,\alpha,\sigma)}{G(\alpha,\sigma)}$$

$$H(\alpha - f) * \delta(\alpha, f)\left[\frac{B(f,\alpha,\sigma)}{G(\alpha,\sigma)}\right] * \delta(\alpha + f)$$

$$H(\alpha) = \frac{B(f,\alpha+f,\sigma)}{G(\alpha+f,\sigma)}$$

Therefore $S(f,\tau,\sigma)$ is the transform of h(t) at t=τ *and* σ represents the width of the Gaussian g(t). The sequence of operations for the calculations of the S-transform is

1.Determine

H(α)	-------------	h(t)
G(α, σ)	-------------	g(t, σ)

2. Calculate H(α)*δ(α-f), which is H(α) translated to f.
3. Multiply G(α, σ) and shifted H(α)
4. Take the inverse Fourier Transform

$$\sigma = \frac{1}{f}$$

$$g(t,\sigma) = \frac{f}{\sqrt{2\pi\sigma}}e^{-\frac{t^2}{2\sigma^2}}$$

$$g(t,\sigma) = \frac{f}{\sqrt{2\pi\sigma}}e^{-\frac{t^2f^2}{2}}$$

(3.29)

$$G(\alpha,f) = e^{-\frac{2\pi^2\alpha^2}{f^2}}$$

Equation is derived from the Fourier Transform pair,

$$e^{-at^2} \leftrightarrow \sqrt{\frac{\pi}{a}}e^{-\frac{\omega^2}{4a}}$$

which for a Gaussian function

$$g(t,\sigma) = \frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{t^2}{2\sigma^2}}$$ becomes

$$\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow | \frac{1}{\sqrt{2\pi\sigma}}\sqrt{\pi 2\sigma^2}e^{-\frac{\omega^2}{4}2\sigma^2}$$

$$\leftrightarrow e^{-\frac{\omega^2\sigma^2}{2}}$$

The S-Transform can be defined as a CWT with a specific mother wavelet multiplied by a phase factor.

$$S(\tau,f) = e^{i2\pi ft}W(\tau,d)$$

where

$$W(\tau,d) = \int_{-\infty}^{+\infty} h(t)\,\omega(t-\tau)dt$$

is the Wavelet Transform of a function h(t) with a mother wavelet w(t, f), defined as

$$\mathrm{w(t,f)} = \frac{|\mathrm{f}|}{\sqrt{2\pi}}e^{-\frac{t^2f^2}{2}}e^{-i2\pi ft}$$

The S transform separates the mother wavelet into two parts, the slowly varying envelope(the Gaussian function) which localizes in time, and the oscillatory exponential kernel e−2ft which selects the frequency being localized. It is the time localizing Gaussian that is translated while the oscillatory exponential kernel remains stationary. By not translating the oscillatory exponential kernel, the S-Transform localizes the real and the imaginary components of the spectrum independently, localizing the phase spectrum as well as the amplitude spectrum. This is referred to as absolutely referenced phase information. The ST produces a time frequency representation instead of the time scale representation developed by the WT. Figure

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3.1 pictures the time frequency representation of the test time series described in Chapter 2. The time and frequency locations of the time series in the time frequency plane can be directly read out from the plot. The S-transform is a method of spectral localization. It can be applied to fields that require the calculation of event initiation. It has found applications in many fields including Geophysics, Biomedical Engineering, Genomic Signal Processing, Power transformer protection.

RESULTS

a) Input Signal:-

x= 1.5*cos(100*pi*t)+2*sin(400*pi*t + 100*pi*t.^2)+ cos(200*pi*t+ sin(30*pi*t)) is taken as input signal. x is assumed as a non stationary random signal. Below figure shows the input signal ,which I have used in this document.

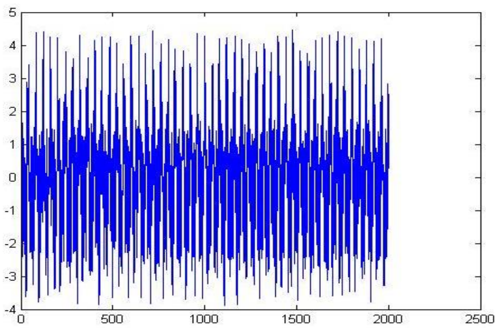

Fig - Input Signal

b) Short Time Fourier Transform:-

When Short time Fourier transform applied to the input signal

x= 1.5*cos(100*pi*t)+2*sin(400*pi*t + 100*pi*t.^2)+ cos(200*pi*t+ sin(30*pi*t)) ,

first it takes one particular window as stationary signal and applies Fourier transform .

Below figure shows the outcome of short time Fourier transform.

The idea of STFT is to break a non stationary signal into sections, in all of which the signal is stationary. Then, the regular Fourier transform can be calculated in each section and the energy density can be determined in each section. Therefore, the original signal can be non stationary, as long as it is stationary within each window. The window function is chosen by the user to be a particular size.

The STFT spectrogram, which plots the energy density contributions from each frequency, is time-dependent. However, the frequencies must be constant within each window. Figure shows the STFT signal.

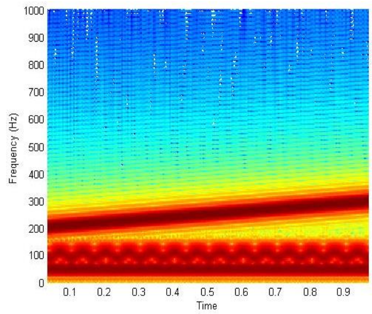

Fig :-Short Time Fourier Transform

c) Wigner Ville Distribution:-

A significant characteristic of WVD, which essentially correlates the signal with a time- and frequency-translated version of itself, is that it does not contain a windowing function as those in the Fourier and wavelet frameworks. This unique feature frees WVD from the smearing effect due to the windowing function, and, as a result, the WVD provides the representation that has the highest possible resolution in the time-frequency plane. . In contrast, the WVD has a better defined representation of both wave packets and a very narrow line for the sharp pulse. The tradeoff is the interference that cannot be removed completely even when the smoothing function is used.

When wigner ville distribution (WVD) applied to the input signal

x= 1.5*cos(100*pi*t)+2*sin(400*pi*t + 100*pi*t.^2)+ cos(200*pi*t+ sin(30*pi*t))

the out of the signal x is much better when compared with the short time Fourier transform. By observing figure 5,3 we can easily identify the difference. In figure 4.4 time and frequeny resolution is much better when compared with figure 4.2.

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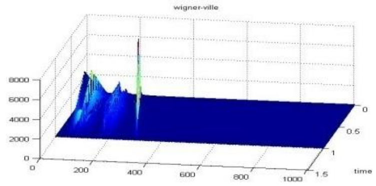

Figure:-Wigner -Ville Distribution

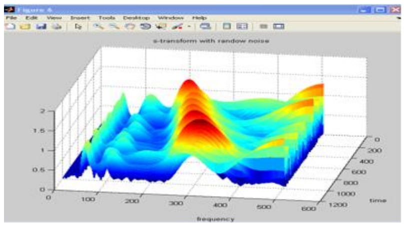

Figure :-S Transform with random Noise

d) Scalogram:-

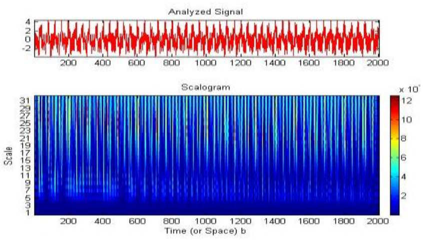

Fig :- Figure Scalogram

e) S-TRANSFORM :-

In non-stationary signal processing field, the time frequency transform is an important method. The Fourier transform is only map the signal from one-dimensional time domain to one-dimensional frequency domain. After the transformation, the signal has a very good frequency resolution, but the time resolution lost completely. This is not for non stationary signal processing. Time-frequency analysis method is one that converses the signal from time domain to time frequency domain and analyzes the non-stationary signals. The short-time Fourier transform, continue Wavelet transform and Wigner-Ville distribution are the most common way to analyze the non-stationary signals. Below figure shows the S-transform of input signal x.

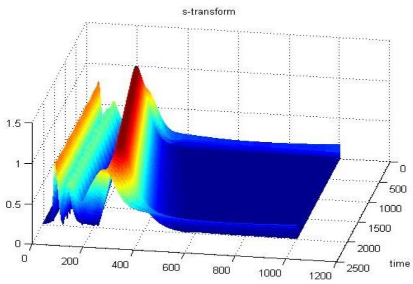

Figure:- S-Transform

CONCLUSION

The S-transform is based on a moving and scalable localizing Gaussian window. In the extension, present the generalized S transform, which its application of windows having frequency dependence in their shape (width and height). For the different applications, we can choose the suitable window and parameters to acquire the good resolution in the part we want to emphasize. From the power quality analysis, the S transform exhibit the ability of identifying the power quality disturbance by noise or transient. This is the wavelet transform cannot achieve because its drawback of sensitive to noise. But the *S* transform still have two drawbacks, first one is in the DC term (frequency = 0), the *S* transform cannot analyze the variation of *S* transform on time. Second, in high frequency, the window will be too narrow, so the points we can practically apply will be too less.

FUTURE SCOPE

Even though the time frequency resolution obtained in S transform and the modified S transform proposed in this thesis are better than Short Time Fourier Transform, it is still far from ideal. Heisenberg's uncertainty principle offers a restriction on the achievable time frequency resolution. Much remains to be done to achieve perfect time frequency resolution. The gradient of the local variance can be used to calculate the frequency trend of a time series, but it requires refinement and the development of a mathematical basis. S transform can be applied for the analysis of time series from diverse areas. Even though business cycles were analyzed in detail, more study is necessary on its relationship to various other factors that affect the GDP of a country. The interest rate is one such factor that may be analyzed. The S transform of a time series of length N produces an S matrix of size N × N/2.Each point in the S matrix is a complex number. Thus the S transform requires a good amount of memory for storage of the 2D complex matrix. As the Gaussian window gets wider in the frequency domain, in the higher frequency range, the incremental width of the Gaussian window with an increase in frequency is often less than one sample interval.

The S transform has a frequency step computation procedure. i.e. The N/2 frequency voices are computed separately one at a time. This increases the computation time of the S transform. The S transform computation time can be improved by making use of the properties of a Toeplitz matrix, which is a matrix in which each descending diagonal from left to right is constant. Time Variance (TV) is another possible candidate for analysis of a time series.

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