

Linear Precoder Design To Minimize Peak To Average Power Ratio (PAPR) For MIMO-OFDM System

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Abstract: We propose a direct precoding plan for a solitary client numerous input-multiple-output Orthogonal Frequency Division Multiplexing (OFDM) framework to minimize top to normal force degree (PAPR) by utilizing repetitive spatial assets at the transmitter through a particular worth decay based summed up converse. The proposed precoder focused around the summed up converse is made out of two sections. One is for minimizing Peak to Average Power Ratio (PAPR), and the other is for getting the multiplexing addition. Additionally, the proposed precoder contains a scalar parameter B that measures the got sign to-clamor power proportion (SNR) misfortune at the expense of PAPR diminishment. Indeed in instances of little SNR misfortune, the proposed plan drastically lessens PAPR. Moreover, reenactment results demonstrate that we can acquire a PAPR near to 0.5 by utilizing many transmission radio wires with little SNR misfortune.

Index terms: Orthogonal Frequency Division Multiplexing, Multi Input Multi Output, Peak to Average Power Ratio, Precoding, Convex Optimization, PAR.

1. Introduction

Wireless systems employing multi element antenna arrays at both the transmitter and receiver, known as multiple-input multiple-output (MIMO) systems, promise large gains in capacity and quality compared with single antenna links [1]. Spatial multiplexing is a simple MIMO signaling approach that achieves large spectral efficiencies with only moderate transmitter

complexity. Receivers for spatial multiplexing range from the high-complexity, low-error-rate maximum likelihood decoding to the low-complexity, moderate-error-rate linear receivers [2], [3]. Linear transmit precoding, where the transmitted data vector is pre-multiplied by a precoding matrix that is adapted to some form of channel information, adds resiliency against channel conditioning. Linear precoded spatial multiplexing has been proposed for transmitters with full channel state information.

Multi Input Multi Output and Orthogonal Frequency Division Multiplexing (MIMO OFDM) systems are widely used in wireless broadband communication, this system provide high spectral efficiency in communication. OFDM system has a major drawback that is high Peak to Average Power Ratio (PAPR) because in efficient use of Power Amps [4]. Different authors have been developed to reduce the PAPR of OFDM signals [5]-[9]. Among all these existing methods, the Iterative Clipping and Filtering (ICF) method [5] and optimized ICF method [6] procedure maybe the simplest to approach a specified PAPR threshold in the processed OFDM signals, this methods reduce PAPR for SISO-OFDM but these methods cannot be extended to MIMO-OFDM because clipping time domain signals causes out-of-band spectral regrowth and in-band distortion.

The convex optimization based conventional method [7] shows good performance to reduce PAPR for MIMO-OFDM systems, this method can extend to MIMO pulse shaping (precoding) [8] system. In this method large numbers of antennas are used at the

Base-Station (BS) would serve a large number of users concurrently in the same frequency band, but with number of BS antennas being much larger than the number of users it says that hundred antennas serving ten users and it reduce the operational power consumption at the transmitter. The author [9] also improved to reduce PAPR performance using null spaces, it shows that the performance is unitary PAPR (i.e. ≈ 1) is achieved by using the number of antennas is infinite. The current method also reduces PAPR performance better than the method [10] and provides PAPR close to 0.5 ($\text{PAPR} \approx 0.5$) for the transmission antennas large enough.

In this paper, the pulse shaping or pre-coding technique is an efficient and flexible way for reducing the PAPR of OFDM signals. The precoder contains two matrices one is for minimizing PAPR and other one is for obtaining the multiplexing gain. In this method, each data block is multiplied by a pre-coding matrix prior to OFDM modulation and transmission. This method is data-independent and, thus, avoids block based optimization. It also works with an arbitrary number of subcarriers and any type of baseband modulation used. We introduce a scalar parameter B that quantifying the Signal to Noise Ratio (SNR) at the cost of PAPR reduction. The current method improves PAPR performance since the maximum amplitude of time domain signals is minimized while keeping the average transmission power at a certain level.

Notations

In this paper indicating the following notations are, $(\bullet)^T$ – transpose, $(\bullet)^H$ – conjugate transpose, $(\bullet)^{-1}$ – inverse, $(\bullet)^+$ – pseudo inverse, $(\bullet)^-$ – generalized inverse, $(\bullet)^\perp$ – orthogonal projection, $\|\bullet\|_\infty$ – infinite norm, $\|\bullet\|_F$ – frobenius norm, $\text{blkdiag}(\bullet)$ – block digitalization, $\text{tr}(\bullet)$ – trace and $(\bullet)^{r_{i,k}}$ – trace of k^{th} subcarrier i th row vector of a matrix.

2. Block Diagram Description

A) MIMO OFDM System description

The block diagram of MIMO OFDM (down link) system is shown in the following figure.1, this system contains number of transmission antennas M_T and number of receiver antennas M_R , in this system transmission antennas are greater than the receiver antennas i.e. $M_T > M_R > d_k$. Where k representing as k^{th} subcarrier $\forall k \in \{1 \dots N_C\}$.

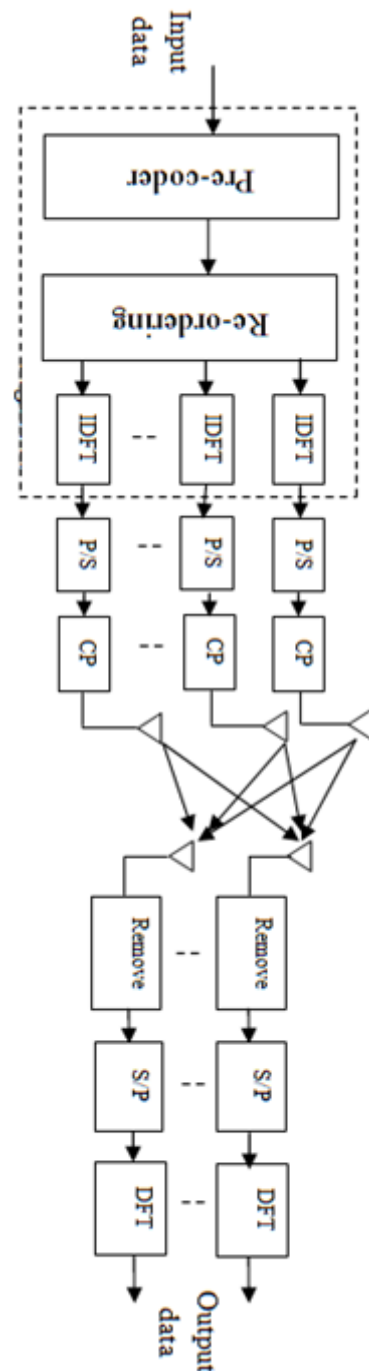


Figure.1: Block diagram of MIMO OFDM system

The transmitter transfer $d_k \times 1$ symbol vectors $s_k = [s_{k,1} \dots s_{k,d_k}]^T$ satisfy the following equation $E_{s_k} [s_k s_k^H] = (p_s/d_k I)$, the received signal y_k is defined as $y_k = R_k H_k F_k s_k + R_k n_k$ (i)

Here F_k denotes the transmission precoder for k^{th} subcarrier and is satisfying $E_{s_k} [F_k s_k s_k^H F_k^H] \leq p$. R_k denotes the receiver filter of k^{th} subcarrier, n_k is Gaussian noise vector and it satisfy $E[n_k n_k^H] = \sigma_n^2 I$. H_k is the $M_T \times M_R$ MIMO fading channel. The entire frequency selective fading channel is subdivided into number of series narrowband fading channels. Rewrite the equation (i) for all N_C subcarriers is defined as

$$y = R H F s + R n \quad (ii)$$

Here $n = [n_1^T \dots n_{N_C}^T]^T$, $s = [s_1^T \dots s_{N_C}^T]^T$, $F = \text{blkdiag}(F_1 \dots F_{N_C})$, $R = (R_1 \dots R_{N_C})$ and $H = \text{blkdiag}(H_1 \dots H_{N_C})$.

B) OFDM Signal

In OFDM system, the message bits are grouped in blocks $\{X_n, n = 0, 1 \dots N - 1\}$ and modulate in amplitude a set of N subcarriers $\{f_n, n = 0, \dots N - 1\}$. These sub-carriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where $\Delta f = 1/T$, and T is the OFDM symbol period. The resulting signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t} \quad (iii)$$

C) PAPR of OFDM Signals and Generalized Inverse

The PAPR is a measure commonly used to quantify the envelope fluctuations of multicarrier signals. For a discrete time signal $x(n)$, the PAPR is defined as the ratio of the maximum power to the average power. For MIMO OFDM system (for IDFT consideration at the transmitter) PAPR is defined as

$$PAPR = \frac{\max_{j \in [1, L, N_C], i \in [1, M_T]} |\tilde{x}_j^{(i)}(n)|^2}{\frac{1}{L N_C M_T} \sum_{i=1, j=1}^{M_T, L, N_C} |\tilde{x}_j^{(i)}(n)|^2} \quad (iv)$$

Where denominator is the average power of the signal $\tilde{x}_j^{(i)}(n)$, numerator is the maximum power of the

signal $\tilde{x}_j^{(i)}(n)$ and l is the oversampling factor. The different PAPR reduction techniques are shown in below table.1

Methods	Average Power	Computational Complexity	Bandwidth Expansion	BER Degradation	Side Information
Clipping	No	Low	No	Yes	No
PTS/SLM	No	High	Yes	No	Yes
TR/TI	Yes	High	Yes	No	No
Precoding	No	Low	Yes	No	No

Table.1: Precoding technique compared with different PAPR reduction techniques

H_k is divided into three parts based on singular value decomposition (SVD) i.e. $H_k = U_k \Sigma_k V_k^H$, where V_k^H , U_k are denotes the $M_T \times M_T, M_R \times M_R$ unitary matrix and Σ_k is denotes $M_R \times M_T$ diagonal matrix with singular values on the diagonal. Vector Symbol Error Rate (VSER) is minimizing by using precoding matrix for MIMO OFDM data transmission. VSER is defined as $F_k = \bar{V}_k$ [11], here \bar{V}_k is composed of first d_k columns of V_k , $\text{tr}(\bar{V}_k^H \bar{V}_k) = d_k$. The generalized inverse \bar{V}_k^H is denoted as \bar{V}_k^{H-} is satisfying $\bar{V}_k^H \bar{V}_k^{H-} \bar{V}_k^H = \bar{V}_k^H$ [12]. Then the matrix \bar{V}_k^{H-} is defined as

$$\bar{V}_k^{H-} = \bar{V}_k^{H+} + P_k^\perp T_k \quad (v)$$

Here $P_k^\perp = \bar{V}_k^{H+} \bar{V}_k^{H-}$ is orthogonal projection matrix and T_k is $M_T \times d_k$ random matrix.

3. Precoder Design For MIMO

The main objective is to design MIMO precoder, it minimize the PAPR while providing a subsequence data rate performance based on MIMO precoding gain. In this, first we discuss on PAPR minimization.

Direct method: In this method, the maximum amplitude is minimized while keeping the average power consumption at certain level and it is defined as

$$\text{minimize } \max_{F_k, \forall k} \forall j, \forall i \left[\left| \tilde{x}_j^{(i)}(n) \right|^2 \right] \quad (\text{vi})$$

$$\text{Subject to } P_{avg}^- \leq E_{i,j} \left[\left| \tilde{x}_j^{(i)}(n) \right|^2 \right] \leq P_{avg}^+ \quad (\text{vii})$$

Here P_{avg}^- and P_{avg}^+ are denotes the boundaries of the average power and is satisfying $P_{avg}^- < P_{avg}^+$ and $P_{avg}^- < P_{avg}^+ \approx 0$. The above problem is nonconvex due to lower bound in equation (vii) PAPR is minimize by considering the convex problem based on this direct approach.

Precoding matrix selection: The precoder matrix will determine the performance of the entire system. Because we are interested in constructing a high-rate signaling scheme with low error rates, we will present bounds on the probability of vector symbol error (i.e., the probability that the receiver returns at least one symbol in error). Here the design factor is used the null space of the right singular matrix H_k and it exists in generalized inverse of \bar{V}_k^H . From equation (i) the proposed precoding matrix is defined as

$$F_k = \sqrt{B} \cdot \bar{V}_k^{H+} + P_k^\perp T_k \quad (\text{viii})$$

Here B is the received SNR loss factor and it satisfying the condition $0 < B < 10$. The parameter B is allowed to control the power consumption between the control signal $P_k^\perp T_k$ and the effective data transmission based on the SVD generalized inverse. It is given in below equation.

$$\text{tr}(F_k^H F_k) = \sqrt{B} \cdot \text{tr}(T_k^H P_k^\perp \bar{V}_k + \bar{V}_k^H P_k^\perp T_k) + \text{tr}(B \cdot \bar{V}_k^H \bar{V}_k + T_k^H P_k^\perp T_k) \quad (\text{ix})$$

Here $P_k^{+H} = P_k^\perp$, $P_k^\perp P_k^\perp = P_k^\perp$ and the first term is zero because of $P_k^\perp \bar{V}_k = 0_{M_T \times d_k}$ and $\bar{V}_k^H P_k^\perp = 0_{d \times M_T} \cdot \sqrt{B} \bar{V}_k$ is contributes the data transmission, $P_k^\perp T_k$ is used in PAPR reduction by minimizing the peak amplitude of transmission signals, it is removed by the wireless channel. $\sqrt{B} \bar{V}_k$, is also used in PAPR reduction at the lower bound P_{avg}^- in equation (vii). From equation (viii) precoder and receiver $R_k = P_k^H$, then the received signal is given as

$$y_k = \sqrt{B} \sum_k s_k + U_k^H n_k \quad (\text{x})$$

Where $\bar{\Sigma}_k$ is denotes the first d_k column vectors of Σ_k and $U_k^H H_k F_k = \sqrt{B} \bar{\Sigma}_k I$. The second term in equation (viii) is not effect on the received signal.

PAPR minimization:

From the design parameter T_k and is given in equation (viii) then reformulate the problem defined in equations (vi), (vii) and its are shown in below equations that are

$$\text{minimize } \max_{T_k, \forall k} \forall j, \forall i \left[\left| \tilde{x}_j^{(i)}(n) \right|^2 \right] \quad (\text{xi})$$

$$\text{subject to } P_{avg}^- \leq E[s_k^H F_k^H F_k s_k] \leq P_{avg}^+ \quad (\text{xii})$$

$$\text{Where } \tilde{X}^{(i)} = Q^{IDFT} [f_{1,i}^r s_{1,1} \dots f_{N_c,i}^r s_{N_c,1}]^T \quad (\text{xiii})$$

Here Q^{IDFT} denotes the $LN_C \times LN_C$ IDFT matrix. From equation (viii), F_k is not in function of the $M_T \times 1$ target vector variable $t_{k,l}$, but these are contained in row vector for the precoding matrix F_k , it can be defined as

$$f_{k,i}^r \sqrt{B} \cdot \bar{v}_{k,i}^r + [P_{k,i}^{\perp,r} t_{k,1} \dots P_{k,i}^{\perp,r} t_{k,d_k}] \quad (\text{xiv})$$

Substitute equation (viii) in equation (xii) then it is given as

$$P_{avg}^- \leq B \frac{p_s}{d_k} \cdot \text{tr}(\bar{V}_k^H \bar{V}_k) + \frac{p_s}{d_k} \sum_{l=1}^{d_k} t_{k,l}^H P_k^\perp t_{k,l} \leq P_{avg}^+ \quad (\text{xv})$$

The average power consumption is larger than Bp_s because the average power is minimized when the design parameters are $t_{k,1} \dots t_{k,d_k} = 0$. We assume that the $P_{avg}^- = p_s$, and $P_{avg}^- = Bp_s$. Here Bp_s is the transmission power for carrying the data signals. In equation (xv) second boundary is neglected then the average power consumption is maintained in between Bp_s and p_s . So we can have an opportunity to minimize PAPR is close to 0.5.

Let us consider scalar parameter B . We can intuitively predict that the constant B would be selected close to 0.5 when PAPR negligibly affects the error rate performance so that MIMO precoding gain is more critical than PAPR performance. However, it is noted that minimizing the maximum amplitude at the

expense of the received SNR loss does not always guarantee the *peak-to-average power ratio* reduction due to the variation of the average transmission power. It implies that there is B which provides the best tradeoff between the PAPR performance and SNR loss given system parameters such as d_k , modulation order, dynamic range and M_T .

4. Results And Analysis

The PAPR dB scale performance of the proposed scheme is compared with conventional scheme [8] and PMP [9] for $4 \times 2, 8 \times 2$ MIMO is shown in below figure.2.

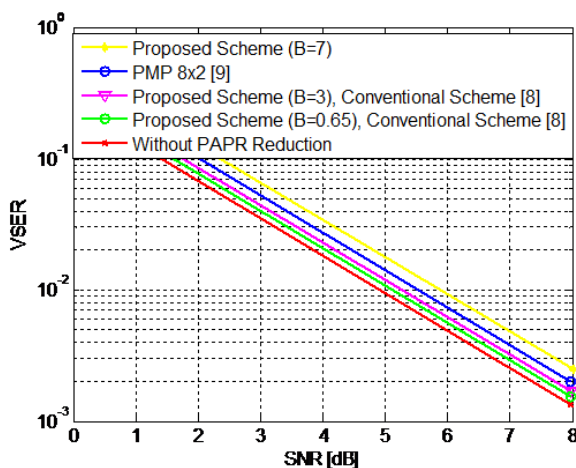


Figure.2: PAPR dB scale performance of the proposed scheme, conventional scheme [8] and PMP [9].

Let us consider simulation parameters ($M_T, M_R, d_k = M_R, N_C$) is assumed like as (4, 2, 2, 64), (8, 2, 2, 64) with 16-QAM. As the baseline approaches for PAPR performance evaluation, we focus on the convex optimization approaches of an Error Vector Magnitude (EVM) based optimization scheme [8], proposed symbol optimization based PMP [9] and a MIMO-OFDM signal without PAPR reduction, where the maximum EVM is -30 dB and -20 dB from [8]. Here the EVM is defined as $\sqrt{d_k/p_s \cdot \sum_{k=1}^{N_c} \|e_k\|^2}$, where e_k is difference between the original and distorted signal. -30 dB and -20 dB EVM are equivalent to -15 dB and -10 dB SNR loss [7], and their corresponding B in the proposed scheme are 0.65, 3. Then compare algorithm

performance with [8] and [9]. We evaluate PAPR performance for a system parameter 4×2 MIMO of 16-QAM and its corresponding VSER performance. As a reference for SU-MIMO transmission scheme, we consider a transmit Zero Forcing Beam Forming (ZF-BF) without the PAPR reduction scheme with $R_k = 1$. Also the PAPR reduction scheme [8] and [9] are applied to SU-MIMO based on ZF-BF with $R_k = 1$. The VSER performance of our proposed scheme is degraded only depending on B compared to the reference case.

From figure.2, it can be observed that the PAPR value ranges from 0.25 dB to 2.9 dB according to B and M_T . In consideration of 10^{-3} percentage of PAPR performance with $B = 0.65$, the proposed scheme outperforms EVM-based conventional scheme [8] and [9] by more than 4.3 dB and 1 dB respectively.

The vector symbol error rate performance of the proposed scheme is compared with the conventional scheme [8] and PMP [9] for 4×2 is shown in below figure.3.

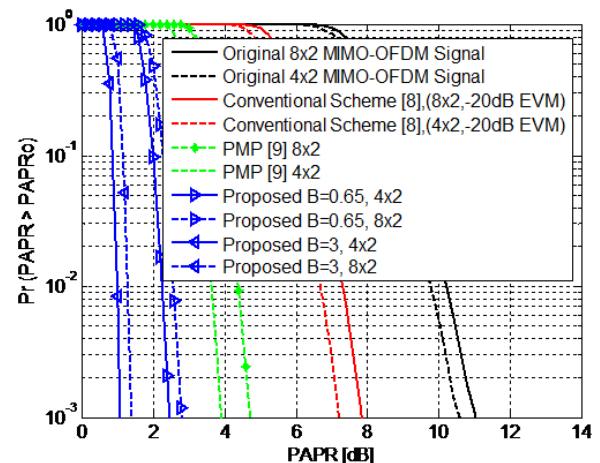


Figure.3: VSER performance of the proposed scheme, conventional scheme [8] and PMP [9].

From figure.3, it can be observed that the proposed method also show the good VSER performance over PMP [9] for $B = 0.65$ and $B = 3$. When $B = 0.65$, the proposed scheme shows PAPR gain is 3.2 dB and $B = 3$, the proposed scheme shows VSER gain is 0.9 dB

over [9], it can be seen in figures.2 and 3. Recall that the transmission power for data transmission is maintained as Bp_s , so that the received SNR loss of the proposed scheme will be marginal close to 0.5. Since the PMP scheme [9] also uses the null space of the MIMO channel, but without the constraint of the desired signal space power, relatively large power consumption is allowed to minimize peak power, which may degrade VSER performance compared to the proposed scheme. The proposed scheme and conventional scheme [5] show the same VSER performance due to the same cost for both schemes.

The PAPR performances of the proposed scheme for different MIMO configurations are shown in below figure.4.

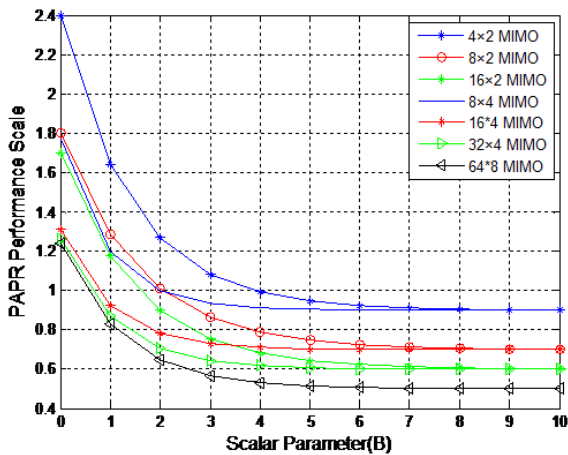


Figure.4: PAPR performance of the proposed scheme with different MIMO configurations.

The PAPR performance is significantly improved with small reduction of B when PAPR close to 0.5. In contrast, B decreases. From figure.4, it can be also observed that the PAPR performance is marginally improved even if M_T is doubled. However, the PAPR is significantly reduced as scalar parameter B increases then PAPR close to 0.5 and PAPR converge to a certain level. The proposed method reducing PAPR since the maximum amplitude of the time domain signal is minimized while keeping the average transmission power at a certain level. From figure.4, the performance curves of 13×2 , 32×4 and 64×8

MIMO, it can be observed that PAPR performance approaches the same level as B increase in case that M_T/M_R (M_T/d_k) is the same, where $M_R = d_k$. When M_T and M_T/M_R are simultaneously increased, the limiting PAPR can be achieved with relatively large B . Thus, it is observed that the proposed scheme can be achieve a PAPR close to 0.5 with increasing number of transmitters (M_T) and number of transmitters or number of receivers (M_T/M_R). The amplitude levels of different MIMO configuration performance of the proposed scheme is shown in below table.2.

	0	1	2	3	4	5	6	7	8	9	10
4x2 MIMO	2.4	1.62	1.28	1.1	1	0.97	0.92	0.9	0.9	0.9	0.9
8x4 MIMO	1.79	1.2	1	0.93	0.92	0.91	0.9	0.9	0.9	0.9	0.9
8x2 MIMO	1.8	1.3	1	0.88	0.8	0.76	0.74	0.71	0.7	0.7	0.7
16x4 MIMO	1.3	0.96	0.8	0.74	0.72	0.7	0.7	0.7	0.7	0.7	0.7
16x2 MIMO	1.7	1.19	0.9	0.77	0.68	0.64	0.62	0.61	0.6	0.6	0.6
32x4 MIMO	1.26	0.89	0.7	0.64	0.62	0.6	0.6	0.6	0.6	0.6	0.6
64x8 MIMO	1.25	0.84	0.64	0.58	0.54	0.52	0.51	0.5	0.5	0.5	0.5

Table.2: PAPR performance of the proposed scheme

5. Conclusion

The proposed MleMO precoding scheme consists of two parts, one is minimizing PAPR and other one is obtaining the multiplexing gain. The PAPR is minimized when the maximum amplitude of time domain signals are minimized while keeping the average power at certain level. The PAPR performance is close to 0.5, the proposed method minimizes the errors and it gives effective data rate transmission.

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