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Reliability Analysis of Various Structures Subjected To Fatigue Loading

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ABSTRACT:

In literature, there exists an effective analytical approach without Monte Carlo simulation to evaluate the fatigue reliability of structural components. This analytical approach required selection of a reference S-N curve for damage evaluation. Overall reliability of a given life is based on the assumption of weighted sums of the equivalent damage rates of all steps in the load spectrum. A sensitivity study indicated that the reference S-N curve chosen might influence the accuracy of the solution. This present work follows the development of the S-N curve approach method. Reliability is evaluated using the S-N curve that produces the same damage sums at its retirement time when load and usage cycles are applied in distributions. This method does not require selection of reference S-N curve or weighted reliability assumptions. It is easier to understand and implement from a computation point of view. Numerical examples presented to demonstrate the applicability of the method for performing reliability analysis. Reliability results of the S-N curve approach method are compared to both analytical and Monte Carlo simulation [1] approaches available in literature.

1. INTRODUCTION:

The fatigue process of mechanical components under service loading is stochastic in nature. The prediction of time dependent fatigue reliability is critical for the design and maintenance planning of many structural components. Despite extensive progress made in the past decades, life prediction and reliability evaluation is still a challenging problem. Fatigue failures are common phenomena in the engineering structures and mechanical components, which will lead to fatal accidents. According to study conducted by the ASCE (American Society of Civil Engineering) committee on Fatigue and Fracture Reliability, 80-90% of failures in steel structures are related to fatigue and fracture. Therefore, it is a great significance to consider fatigue reliability when perform optimal design on a structural components, and it will lead to higher safety operating performance and economic efficiency.This presented work has been provides an S-N curve approach method for fatigue reliability analysis of structural components. This approach requires no Monte Carlo simulation. It allows the evaluation of fatigue reliability on a given spectrum with or without load variability. At present, S-N curve approach has been widely used in the design of offshore structures, vehicle knuckles, ship structures, aerospace structure components and scientific applications.

2. LEVELS OF RELIABILITY METHODS:

Level I methods, the reliability of the design deviate from the target value, and the objective is to minimize such an error. Load and Resistance Factor Design (LRFD) method comes under this category.

Level II methods, which employ two values of each uncertain parameter (i.e., mean and variance), supplemented with a measure of the correlation between parameters, are classified as levelIImethods.

Level III methods encompass complete analysis of the problem and also involve integration of the multidimensional joint probability density function of the random variables extended over the safety domain.

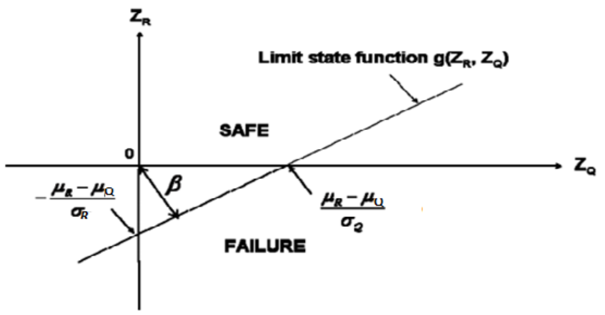
Level IV methods are appropriate for structures that are of major economic importance, involve the principles of engineering economic analysis under uncertainty, and consider costs and benefits of construction, maintenance, repair, consequences of failure, and interest on capital, etc. Foundations for sensitive projects like nuclear power projects, transmission towers, highway bridges, are suitable objects of level IV design.

General definition of the reliability index:

A version of the reliability index was defined as the inverse of the coefficient of variation The reliability index is the shortest distance from the origin of reduced variables to the is illustrated in Fig line $g(Z_R, Z_Q) = 0$

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}.$$

fig.



Reliability index in the space of reduced variables

3. First Order Second Moment (FOSM) Method

This method is also referred to as mean value first-order second moment (MVFOSM) method, and it is based on the first order Taylor series approximation of the performance function linearized at the mean values of the random variables. It uses only second moment statistics (mean and variance) of the random variables.

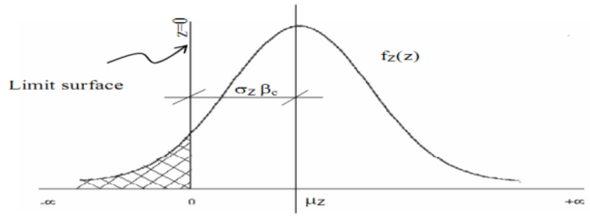


Fig. Definition of limit state and reliability index

Cornell defined the reliability index as the ratio of the expected value of Z over its standard deviation. The Cornell reliability index (β_C) is the absolute value of the ordinate of the point corresponding to Z = 0 on the standardized normal probability plot as given in fig.

Stages of Fatigue Failure

a) Crack initiation: Areas of localized stress concentrations such as fillets, notches, key ways, bolt holes and even scratches or tool marks are potential zones for crack initiation. As a result of the local stress concentrations at these locations, the induced stress goes above the yield strength and cyclic plastic straining results due to cyclic variations in the stresses. On a macro scale the average value of the induced stress might still be below the yield strength of the material.

b)Crack propagation: This further increases the stress levels and the process continues, propagating the cracks across the grains or along the grain boundaries, slowly increasing the crack size. As the size of the crack increases the cross sectional area resisting the applied stress decreases and reaches a threshold level at which it is insufficient to resist the applied stress.

c) Final fracture: As the area becomes too insufficient to resist the induced stresses any further a sudden fracture results in the component.

4. FATIGUE RELIABILITY METHODS:

Two key steps in the fatigue reliability analysis are to evaluate the reliability for each load step of a given multi-load steps spectrum and to determine the appropriate way to combine reliability from each

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load step. At the beginning of this development, two assumptions were made. The first assumption involved "constant damage rate tracking" for each stress level. The second assumption was made, such that, overall fatigue reliability should be calculated using the weighted average of individual reliability on its damage rate. The following sections illustrate the proposed analytical approach.

4.1 Monte Carlo Simulation

The technique consists of three steps 1.Generating a set of values y_{ik} for the material properties and geometric parameters Y_i in accordance with empirically determined or assumed density functions f_{yi}. The suffix i is used to denote the ith variable and suffix k is used to represent the kth set of values (y_{1k} , y_{2k} , ….,y_{ik} ,……..y_{nk}) of the corresponding variables (Y_1, Y_2, …, Y_i ,…., Y_n).

2.Calculating the value r_k corresponding to set of values y_{ik} obtained in step 1, by means of the appropriate response equation for resistance of t he section. That is

$$r_k = g(y_{1k} , y_{2k} , ….,y_{ik} ,……..y_{nk})$$

3. Repeating step 1 and 2 to obtain a large sample of the values of R and therefore, estimating $f_R(r)$.

4.2 Analytical Fatigue Reliability

Fig illustrates the methodology to calculate the reliability if the number of cycles that is changed while the stress or applied load is kept constant.

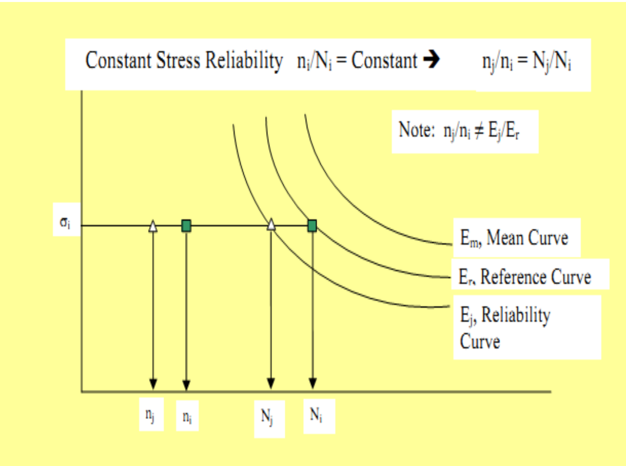

Fig. Reliability for changing the cycles.

If the stress or applied load is changed for a given cycle, the change in stress or load will reflect in the endurance as shown in Fig. The endurance change at □σ_i can be referred to a known reliability curve at σ_i with the reference endurance E_r such that

$$\sigma_j /E_j = \sigma_i/E_r$$

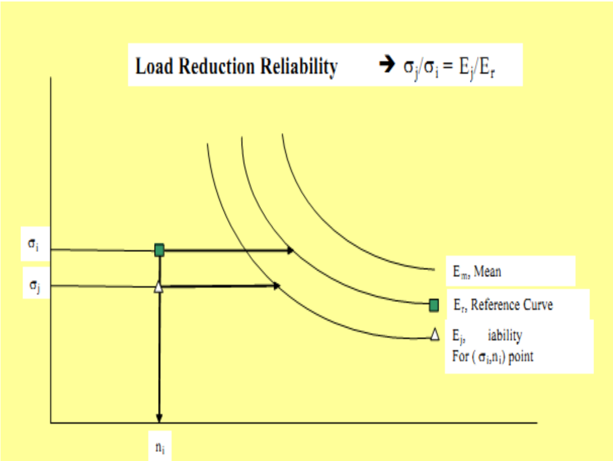

Fig. Reliability for changing the stress or applied load.

4.3 Combined Reliability Method

In the analytical approach for a multiple load step spectrum requires combining the reliability calculated from each load step. In the development, the reliability is combined using a weighted average of the damage cumulative rates for each load step. The equation to calculate the reliability for a multiple load steps spectrum is as follows

$$R(t) = \frac{\sum \frac{n_i}{N_i} \times R_i}{\sum \frac{n_i}{N_i}}$$

Where R (t) is component reliability for the fatigue life at time t.

4.4 S-N Curve Approach Method

The most commonly used model for fatigue behavior under constant amplitude loading is of the form

$$NS^m=K$$

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In which m and K are empirical constants denoting slope of S-N line and intercept on S axis respectively. N is number of cycles to failure and S is the applied stress range.When plotted on log-log scale, the S-N relationship has a liner form (Fig 4.3) as given below

log N = log K- m logS

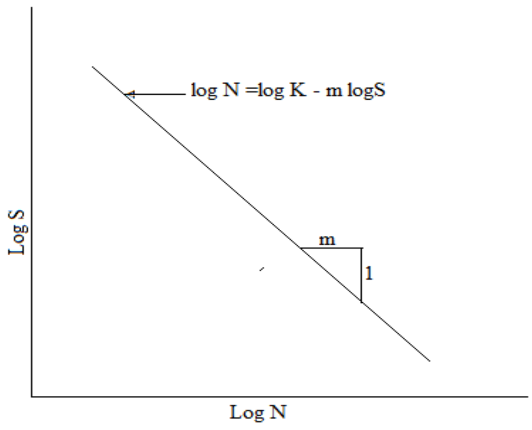

Fig. S-N Curve on log-log plot.

***Transformation of resistance**

Quantities i.e., either cycles to failure or stress range. That is both load and resistance curves calculation of reliability requires that load and resistance are expressed in terms of the same basic are to be plotted on the same axis. Hence, in the present case, one of the curves is to be transformed. Transforming the resistance, when distribution of resistance is plotted along the vertical line through point b (Fig.4.5), the points with the same survival probability must lie on the same line parallel to the mean resistance. From geometry, it is clear that

$$\sigma_R' = \frac{\sigma_R}{m} = \frac{\sigma_{\log\ N}}{m}$$

σ_R' Indicates the standard deviation of resistance measured along the vertical line.

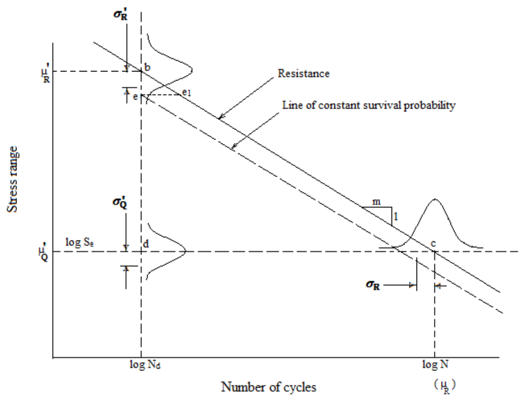

Fig. Transformation of resistance.

The distance between the mean resistance and mean load, measured along the vertical line d-b, in Fig is given by

$$\mu_R'\text{-}\mu_Q' = \frac{1}{m}(\mu_R - \mu_Q)$$

Reliability index is given by

$$\beta = \frac{\mu_M}{\sigma_M}$$

$$\beta = \frac{\frac{1}{m}(\mu_R - \mu_Q)}{(\sigma_Q'^2 + \sigma_R'^2)1/2}$$

$$= \frac{(\log\ N - \log\ N_d)}{\sigma_t}$$

Where

$$\sigma_t = [(m \times \sigma_{\log\ Se})2 + (\sigma_{\log\ N})2]^{1/2}$$

5. NUMERICAL EXAMPLES AND DISCUSSIONS

5.1 Analysis with no load variability

The first problem [1] two load step spectrum (Table 5.1) is considered with normal distribution in strength and no statistical variation in loads. The constant Coefficient of Variation (COV) is used for the entire mean S-N curve (Fig 5.1) drawn using the $NS^6=10^{28}$. The mean S-N curve for the fatigue strength is as follows.

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$$\frac{S}{E} = \frac{0.7+\frac{0.12}{N^{0.5}}}{0.7848528}$$

Where,

N is the number of the cycles in millions

S is the fatigue strength

The factor 0.7848528 is a normalized factor for the endurance at 2,000,000 cycles

E is fatigue mean strength 5090 N/mm^2

Coefficient of Variation (COV) is 10%

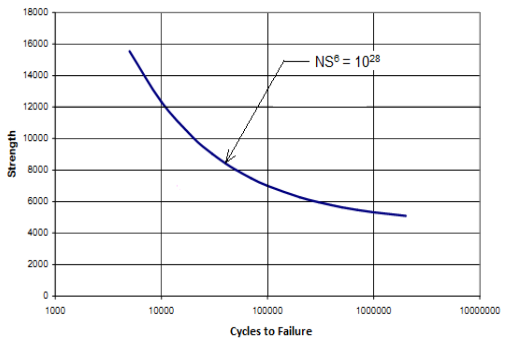

Fig 5.1 Mean S-N curve

Table 5.1 No reduction of load and cycles usage

S.No	Cycles	Load	Distribution
1	46633	5000	Normal
2	6995	7000	Normal

Using the data available from fig and table 5.1 drawn on log-log graph, and as explained in the earlier sections, fig is obtained.

The following is from fig 5.2

logN = log 2×10^6 = 6.30

logN$_d$ = log 1.7×10^5 5.2304

slope of the curve m=6

COV of δ_N= 10.71% and δ_{Se}= 10.71%

Standard deviation of endurance limit is

$\sigma_{\log\ Se}$ = [0.4343 log (1+δ^2_{Se})]$^{1/2}$

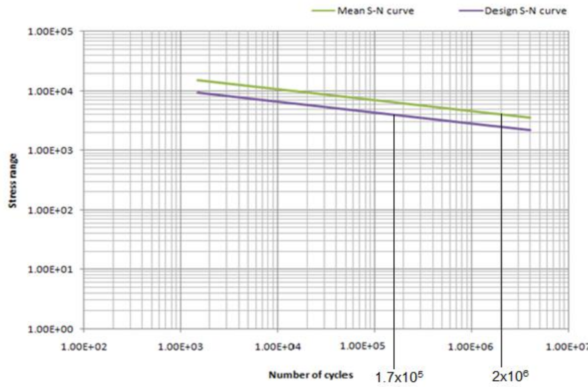

Fig. Mean and design S-N curve with no reduction of load and cycles

$\sigma_{\log\ Se}$ = [0.4343 log (1+0.1071^2)]$^{1/2}$

= 0.0463

Standard deviation of cycles

$\sigma_{\log\ N}$ = [0.4343 log (1+δ^2_{N})]$^{1/2}$

$\sigma_{\log\ N}$ = [0.4343 log (1+0.1071^2)]$^{1/2}$

= 0.0463

Reliabilityindexβ

$$= \frac{(\log N - \log N_d)}{\sqrt{[(m\times\sigma_{\log\ Se})2 + (\sigma_{\log\ N})2]}}$$

$$= \frac{(6.3 - 5.2304)}{\sqrt{[(6\times0.0463)^2 + (0.0463)^2]}}$$

= 3.7975

Probability of failure P_f = $\emptyset$(-β)

= $\emptyset$(-3.7975)

=7×10^{-5} (from standard normal tables [11])

Reliability= 1- P$_f$

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$= 1 - 7\times10^{-5}$

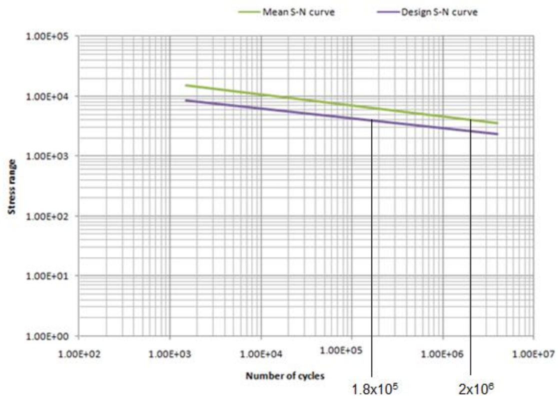
$= 0.99993$

Case 1: Reduction of cycles usage and no reduction of load (100% load and75% cycles)

Table 5.2 Reduction of cycles and no reduction of load

S.No	Cycles	Load	Distribution
1	34875	5000	Normal
2	5246	7000	Normal

Fig for case1



Case 2: No reduction of cycles usage and reduction of load

S.No	Cycles	Load	Distribution
1	34875	5000	Normal
2	5246	7000	Normal

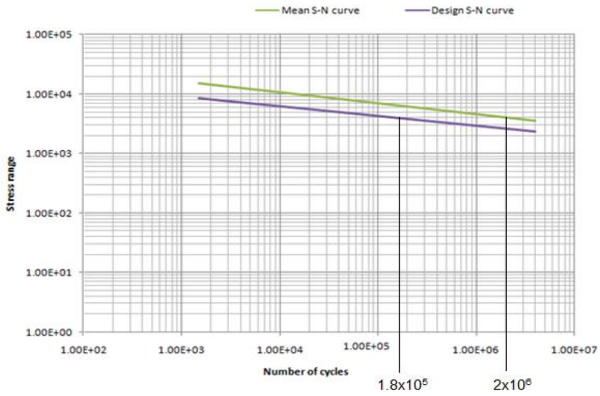


Fig for case 2

RESULTS:

Reliability for variation of cycles and/or loads

Cases	Cycle Spectrum	Load Level	Description	Analytical Reliability	Monte Carlo Simulation	S-N Curve Approach	Error
1	34875 5246	5000 7000	100% load 75% cycles	0.999668	0.999647	0.99989	0.000243
2	46633 6995	4500 6300	90% load 100% cycles	0.99986	0.999876	0.99995	0.000074
3	34875 5246	4500 6300	90% load 75% cycles	0.999968	0.999958	0.9999993	0.0000007
4	46633	5500 77	110% load 100%	0.991010	0.99096	0.99965	0.00869

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	69 95	00	cycles				
5	34 87 5 52 46	55 00 77 00	110% load 75% cycles	0.99 7535	0.997 5	0.99 957	0.00 207

Reliability for changing COV

Cases	Description	Analytical Reliability	Monte Carlo Simulation	S-N Curve Approach	S-N Curve Approach with COV of 20%	S-N Curve Approach with COV of 30%
1	100% load 75% cycles	0.99 9668	0.999 647	0.999 89	0.9767 0	0.9098 8
2	90% load 100% cycles	0.99 986	0.999 876	0.999 95	0.9816 9	0.9222 0
3	90% load 75% cycles	0.99 9968	0.999 958	0.999 9993	0.9967 4	0.9663 8
4	110% load 100% cycles	0.99 1010	0.990 96	0.999 65	0.9650	0.8906 5
5	110% load 75% cycles	0.99 7535	0.997 5	0.999 57	0.9632 7	0.8849 3

Effect of coefficient of variation on reliability index

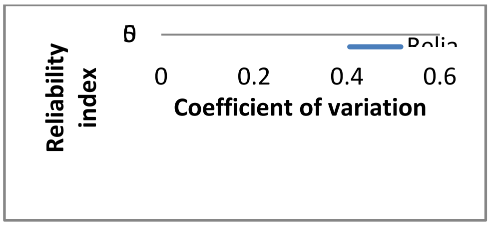


Fig Effect coefficient of variation on reliability index

From fig it is observed that the reliability index decreased with increase in the coefficient of variation.

Effect of load on reliability index

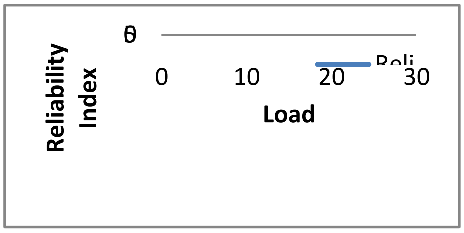
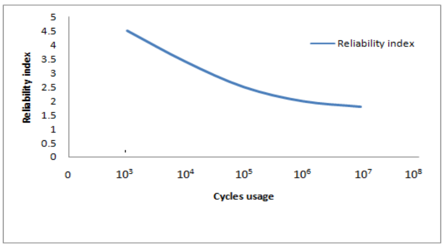


Fig Effect of load on reliability index

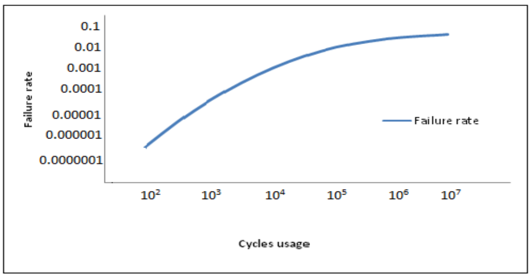
From fig it is observed that the reliability index decreases with the increases in load.

Effect of cycles usage on reliability index



From fig it is observed that the reliability index decrease with increases in cycle usage.

Effect of cycles usage on failure rate



From fig it is observed that the failure rate increase with increases the number of cycles.

6. CONCLUSIONS:

The present work has shown an effective method to quantify fatigue reliability of structural components. The **S-N curve approach method** is used to quickly identify a safe life and associated reliability based on engineering analysis and available data.On application of method successfully, the following conclusions have been derived.

- A technique to compute the reliability of structural components has been presented.
- Varying the parameters like load, coefficient of variation, and cycles to usage the change in reliability index is observed.
- On condition inspection and part replacement before they reach retirement time.
- Reliability index decreases with the increase in the coefficient of variation.
- Failure rate increases with increase in cycles usage.
- Reliability index decreases with the increase in load.
- Reliability index decreases with increase in cycles usage.

REFERENCES

1. Carlton L.Smith, Jung Hua Chang and Martin H.Rogers, " Fatigue Reliability Analysis of Dynamic Components with Variable Loading without Monte Carlo simulation", presented at the American Helicopter Society 63rd Annual Forum, May 2007.
2. Yongming Liu and Sankaran Madhavan, "Probabilistic Fatigue Life Prediction Using an Equivalent Initial Flaw Size Distribution", International Journal of Fatigue Vol. 31, 2009, pp. 476-487.
3. Yu Chee Tong, Ross A.Antoniou and Chun H.Wang, "Probabilistic Fatigue Life Assessment for Helicopter Dynamic Components", Journal of Structural Integrity and Fracture, Vol.7, 2004, pp. 284-298.
4. Sankaran Madhavan and Yongming Liu, "Stochastic Fatigue Damage Modeling under Variable Amplitude Loading", International journal of Fatigue, Vol. 29, 2007, pp. 1149-1161. Variables", International Journal of Computer Issues, Vol. 10, Jan 2012, pp. 210-223.
5. Andre T.Beck and Edison Rosa, "Structural Reliability Analysis Using Deterministic Fatigue Element Programs", Latin American Journal of Solids and Structures, Vol. 3, 2006, pp. 197-222.
6. F.Klubberg, I.Klopfer and C.Broeckman, "Fatigue Testing of Materials and Component under Mean and Load Condition", Journal of Fracture Mechanics, Vol. 11, 2011, pp. 419-426,
7. Young Ho Park and Jun Tang, "An Efficient Methodology for Fatigue Reliability Analysis for Mechanical Components", Journal of Pressure Vessel Technology, Vol. 128,Aug 2006, pp. 116-129.
8. J.Zhao and D.O Adams, "Achieving Six-Nines Reliability Using an Advanced Fatigue Reliability Assessment Model", presented at American Helicopter Society 66th Annual Forum, 2010.
9. John M.McFarland and David S.Riha, "Uncertainty Quantification Methods for Fatigue Reliability Analysis", presented at The American Helicopter Society 65th Annual Forum, 2009.
10. Ranganathan R, "Structural Reliability Analysis and Design", Jacio Publications, 2000.
11. J.D. Baldmin and J.G Thacker, "A Strain-Based Fatigue Reliability Analysis Method", Journal of Mechanical Design, Vol. 177, 2001, pp. 229-236.
12. Saharapeyma Ali, AbdollahHosseini and Mohmad S, "Life-Cycle Prediction of Steel Bridges using Reliability based Fatigue Deterioration", International Journal of Steel Structures, Vol. 13, June 2012, pp. 229-242.
13. Wirsching H, "Fatigue Reliability for Offshore Structures", Journal of Structural Engineering, Vol. 110, 1984, pp. 2340-2356.
14. Hasofer A.M and N.C.Lind, "An Exact and Invariant First Order Reliability Format", Journal

of Engineering Mechanics, ASCE, Vol. 100, Feb 1974, pp 111-121.
15. Shinozuka M, “Basic Analysis of Structural Safety”, Journal of Structural Division, ASCE, Vol. 109, March 1983, pp. 721-740.
16. Nowak A.S and K.V Collins, “Reliability of Structures” McGrraw-Hill Publications, 2000.