Reliability Analysis of Various Structures Subjected To Fatigue Loading

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ABSTRACT:
In literature, there exists an effective analytical approach without Monte Carlo simulation to evaluate the fatigue reliability of structural components. This analytical approach required selection of a reference S-N curve for damage evaluation. Overall reliability of a given life is based on the assumption of weighted sums of the equivalent damage rates of all steps in the load spectrum. A sensitivity study indicated that the reference S-N curve chosen might influence the accuracy of the solution. This present work follows the development of the S-N curve approach method. Reliability is evaluated using the S-N curve that produces the same damage sums at its retirement time when load and usage cycles are applied in distributions. This method does not require selection of reference S-N curve or weighted reliability assumptions. It is easier to understand and implement from a computation point of view. Numerical examples presented to demonstrate the applicability of the method for performing reliability analysis. Reliability results of the S-N curve approach method are compared to both analytical and Monte Carlo simulation [1] approaches available in literature.

1. INTRODUCTION:
The fatigue process of mechanical components under service loading is stochastic in nature. The prediction of time dependent fatigue reliability is critical for the design and maintenance planning of many structural components. Despite extensive progress made in the past decades, life prediction and reliability evaluation is still a challenging problem. Fatigue failures are common phenomena in the engineering structures and mechanical components, which will lead to fatal accidents. According to study conducted by the ASCE (American Society of Civil Engineering) committee on Fatigue and Fracture Reliability, 80-90% of failures in steel structures are related to fatigue and fracture. Therefore, it is a great significance to consider fatigue reliability when perform optimal design on a structural components, and it will lead to higher safety operating performance and economic efficiency. This presented work has been provides an S-N curve approach method for fatigue reliability analysis of structural components. This approach requires no Monte Carlo simulation. It allows the evaluation of fatigue reliability on a given spectrum with or without load variability. At present, S-N curve approach has been widely used in the design of offshore structures, vehicle knuckles, ship structures, aerospace structure components and scientific applications.

2. LEVELS OF RELIABILITY METHODS:
Level I methods, the reliability of the design deviate from the target value, and the objective is to minimize such an error. Load and Resistance Factor Design (LRFD) method comes under this category.

Level II methods, which employ two values of each uncertain parameter (i.e., mean and variance), supplemented with a measure of the correlation between parameters, are classified as level III methods.

Level III methods encompass complete analysis of the problem and also involve integration of the multidimensional joint probability density function of the random variables extended over the safety domain.

Level IV methods are appropriate for structures that are of major economic importance, involve the
principles of engineering economic analysis under uncertainty, and consider costs and benefits of construction, maintenance, repair, consequences of failure, and interest on capital, etc. Foundations for sensitive projects like nuclear power projects, transmission towers, highway bridges, are suitable objects of level IV design.

**General definition of the reliability index:**

A version of the reliability index was defined as the inverse of the coefficient of variation. The reliability index is the shortest distance from the origin of reduced variables to the point illustrated in Fig line \( g(Z_R, Z_Q) = 0 \)

\[
\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}
\]

Cornell defined the reliability index as the ratio of the expected value of \( Z \) over its standard deviation. The Cornell reliability index \( (\beta_C) \) is the absolute value of the ordinate of the point corresponding to \( Z = 0 \) on the standardized normal probability plot as given in fig.

**Stages of Fatigue Failure**

a) **Crack initiation:** Areas of localized stress concentrations such as fillets, notches, key ways, bolt holes and even scratches or tool marks are potential zones for crack initiation. As a result of the local stress concentrations at these locations, the induced stress goes above the yield strength and cyclic plastic straining results due to cyclic variations in the stresses. On a macro scale the average value of the induced stress might still be below the yield strength of the material.

b) **Crack propagation:** This further increases the stress levels and the process continues, propagating the cracks across the grains or along the grain boundaries, slowly increasing the crack size. As the size of the crack increases the cross sectional area resisting the applied stress decreases and reaches a threshold level at which it is insufficient to resist the applied stress.

c) **Final fracture:** As the area becomes too insufficient to resist the induced stresses any further a sudden fracture results in the component.

4. **FATIGUE RELIABILITY METHODS:**

Two key steps in the fatigue reliability analysis are to evaluate the reliability for each load step of a given multi-load steps spectrum and to determine the appropriate way to combine reliability from each...
load step. At the beginning of this development, two assumptions were made. The first assumption involved “constant damage rate tracking” for each stress level. The second assumption was made, such that, overall fatigue reliability should be calculated using the weighted average of individual reliability on its damage rate. The following sections illustrate the proposed analytical approach.

4.1 Monte Carlo Simulation
The technique consists of three steps 1. Generating a set of values \( y_{ik} \) for the material properties and geometric parameters \( Y_i \) in accordance with empirically determined or assumed density functions \( f_{yi} \). The suffix \( i \) is used to denote the \( i \)th variable and suffix \( k \) is used to represent the \( k \)th set of values \( (y_{1k}, y_{2k}, ..., y_{ik}, ..., y_{nk}) \) of the corresponding variables \( (Y_1, Y_2, ..., Y_i, ..., Y_n) \).

2. Calculating the value \( r_k \) corresponding to set of values \( y_{ik} \) obtained in step 1, by means of the appropriate response equation for resistance of the section. That is

\[
r_k = g(y_{1k}, y_{2k}, ..., y_{ik}, ..., y_{nk})
\]

3. Repeating step 1 and 2 to obtain a large sample of the values of \( R \) and therefore, estimating \( f_R(r) \).

4.2 Analytical Fatigue Reliability

Fig illustrates the methodology to calculate the reliability if the number of cycles that is changed while the stress or applied load is kept constant.

If the stress or applied load is changed for a given cycle, the change in stress or load will reflect in the endurance as shown in Fig. The endurance change at \( \sigma_i \) can be referred to a known reliability curve at \( \sigma_i \) with the reference endurance \( E_i \) such that

\[
\sigma_f/E_j = \sigma_i/E_i
\]

4.3 Combined Reliability Method

In the analytical approach for a multiple load step spectrum requires combining the reliability calculated from each load step. In the development, the reliability is combined using a weighted average of the damage cumulative rates for each load step. The equation to calculate the reliability for a multiple load steps spectrum is as follows

\[
R(t) = \frac{\sum_{i=1}^{n} \frac{n_i}{N_i} \times R_i}{\sum_{i=1}^{n} \frac{n_i}{N_i}}
\]

Where \( R(t) \) is component reliability for the fatigue life at time \( t \).

4.4 S-N Curve Approach Method

The most commonly used model for fatigue behavior under constant amplitude loading is of the form

\[
NS^n = K
\]
In which \( m \) and \( K \) are empirical constants denoting slope of S-N line and intercept on S axis respectively. \( N \) is number of cycles to failure and \( S \) is the applied stress range. When plotted on log-log scale, the S-N relationship has a linear form (Fig 4.3) as given below

\[
\log N = \log K - m \log S
\]

*Fig. S-N Curve on log-log plot.*

Transformation of resistance

Quantities i.e., either cycles to failure or stress range. That is both load and resistance curves calculation of reliability requires that load and resistance are expressed in terms of the same basic are to be plotted on the same axis. Hence, in the present case, one of the curves is to be transformed. Transforming the resistance, when distribution of resistance is plotted along the vertical line through point b (Fig 4.5), the points with the same survival probability must lie on the same line parallel to the mean resistance. From geometry, it is clear that

\[
\sigma'_R = \frac{\sigma_R}{m} = \frac{\sigma_{\log N}}{m}
\]

\( \sigma'_R \) Indicates the standard deviation of resistance measured along the vertical line.

Reliability index is given by

\[
\beta = \frac{\mu_M}{\sigma_M} = \frac{1}{m} \left( \frac{\mu_R - \mu_Q}{\sigma_Q^2 + \sigma_R^2} \right)^{1/2} = \frac{(\log N - \log N_d)}{\sigma_t}
\]

Where

\[
\sigma_t = \sqrt{ \left( m \times \sigma_{\log S} \right)^2 + \left( \sigma_{\log N} \right)^2 }
\]

5. NUMERICAL EXAMPLES AND DISCUSSIONS

5.1 Analysis with no load variability

The first problem [1] two load step spectrum (Table 5.1) is considered with normal distribution in strength and no statistical variation in loads. The constant Coefficient of Variation (COV) is used for the entire mean S-N curve (Fig 5.1) drawn using the NS\(^6=10^{28}\). The mean S-N curve for the fatigue strength is as follows.
\[
\frac{S}{E} = 0.7 + 0.12 \frac{N^{0.5}}{0.7848528}
\]

Where,

- \(N\) is the number of the cycles in millions
- \(S\) is the fatigue strength

The factor 0.7848528 is a normalized factor for the endurance at 2,000,000 cycles

- \(E\) is fatigue mean strength 5090 N/mm²

Coefficient of Variation (COV) is 10%

Using the data available from fig and table 5.1 drawn on log-log graph, and as explained in the earlier sections, fig is obtained.

The following is from fig 5.2

\[
\log N = \log 2 \times 10^6 = 6.30
\]

\[
\log N_d = \log 1.7 \times 10^5 \times 5.2304
\]

Slope of the curve \(m = 6\)

COV of \(\delta_N = 10.71\%\) and \(\delta_{Se} = 10.71\%\)

Standard deviation of endurance limit is

\[
\sigma_{\log Se} = [0.4343 \log (1 + \delta_{Se}^2)]^{1/2}
\]

\[
= 0.0463
\]

Standard deviation of cycles

\[
\sigma_{\log N} = [0.4343 \log (1 + \delta_N^2)]^{1/2}
\]

\[
= 0.0463
\]

Reliability index \(\beta\)

\[
\beta = \frac{( \log N - \log N_d )}{\sqrt{[ (m \times \sigma_{\log Se})^2 + (\sigma_{\log N})^2]}}
\]

\[
= \frac{(6.3 - 5.2304)}{\sqrt{[(6 \times 0.0463)^2 + (0.0463)^2]}}
\]

\[
= 3.7975
\]

Probability of failure \(P_f\) = \(\Phi(-\beta)\)

\[
= \Phi(-3.7975)
\]

\[
= 7 \times 10^{-5} \quad \text{(from standard normal tables [11])}
\]

Reliability = 1 - \(P_f\)

Table 5.1 No reduction of load and cycles usage

<table>
<thead>
<tr>
<th>S.No</th>
<th>Cycles</th>
<th>Load</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46633</td>
<td>5000</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>6995</td>
<td>7000</td>
<td>Normal</td>
</tr>
</tbody>
</table>
\[
= 1 - 7 \times 10^{-5} \\
= 0.99993
\]

**Case 1**: Reduction of cycles usage and no reduction of load (100% load and 75% cycles)

**Table 5.2 Reduction of cycles and no reduction of load**

<table>
<thead>
<tr>
<th>S.No</th>
<th>Cycles</th>
<th>Load</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34875</td>
<td>5000</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>5246</td>
<td>7000</td>
<td>Normal</td>
</tr>
</tbody>
</table>

**Case 2**: No reduction of cycles usage and reduction of load

**Table 5.3 Reduction of cycles and load**

<table>
<thead>
<tr>
<th>S.No</th>
<th>Cycles</th>
<th>Load</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34875</td>
<td>5000</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>5246</td>
<td>7000</td>
<td>Normal</td>
</tr>
</tbody>
</table>

**RESULTS:**

Reliability for variation of cycles and/or loads

<table>
<thead>
<tr>
<th>Cases</th>
<th>Cycle Spectrum</th>
<th>Load Level</th>
<th>Description</th>
<th>Analytical Reliability</th>
<th>Monte Carlo Simulation</th>
<th>S-N Curve Approach</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34 87 5 52 46</td>
<td>50 00 70 00</td>
<td>100% load 75% cycles</td>
<td>0.99 9668</td>
<td>0.999 647</td>
<td>0.99 989</td>
<td>0.00 243</td>
</tr>
<tr>
<td>2</td>
<td>46 63 3 69 95</td>
<td>45 00 63 00</td>
<td>90% load 100% cycles</td>
<td>0.99 986</td>
<td>0.999 876</td>
<td>0.99 995</td>
<td>0.00 074</td>
</tr>
<tr>
<td>3</td>
<td>34 87 5 52 46</td>
<td>45 00 63 00</td>
<td>90% load 75% cycles</td>
<td>0.99 968</td>
<td>0.999 958</td>
<td>0.99 9999 3</td>
<td>0.00 0007</td>
</tr>
<tr>
<td>4</td>
<td>46 63 3 77</td>
<td>55 00 110% load 100%</td>
<td>0.99 1010</td>
<td>0.999 96</td>
<td>0.99 965</td>
<td>0.00 869</td>
<td></td>
</tr>
</tbody>
</table>
### Effect of coefficient of variation on reliability index

From fig it is observed that the reliability index decreased with increase in the coefficient of variation.

### Effect of load on reliability index

From fig it is observed that the reliability index decreases with the increases in load.

### Effect of cycles usage on reliability index

From fig it is observed that the reliability index decrease with increases in cycle usage.

### Effect of cycles usage on failure rate

From fig it is observed that the failure rate increases with increases in cycle usage.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Description</th>
<th>Analytical Reliability</th>
<th>Monte Carlo Simulation</th>
<th>S-N Curve Approx with COV of 20%</th>
<th>S-N Curve Approx with COV of 30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100% load 75% cycles</td>
<td>0.99 9668</td>
<td>0.999 647</td>
<td>0.999 89</td>
<td>0.9767 0</td>
</tr>
<tr>
<td>2</td>
<td>90% load 100% cycles</td>
<td>0.99 986</td>
<td>0.999 876</td>
<td>0.999 95</td>
<td>0.9816 9</td>
</tr>
<tr>
<td>3</td>
<td>90% load 75% cycles</td>
<td>0.99 9968</td>
<td>0.999 958</td>
<td>0.999 9993</td>
<td>0.9967 4</td>
</tr>
<tr>
<td>4</td>
<td>110% load 100% cycles</td>
<td>0.99 1010</td>
<td>0.990 96</td>
<td>0.999 65</td>
<td>0.9650 5</td>
</tr>
<tr>
<td>5</td>
<td>110% load 75% cycles</td>
<td>0.99 7535</td>
<td>0.997 5</td>
<td>0.999 57</td>
<td>0.9632 7</td>
</tr>
</tbody>
</table>
From fig it is observed that the failure rate increases with the number of cycles.

6. CONCLUSIONS:

The present work has shown an effective method to quantify fatigue reliability of structural components. The S-N curve approach method is used to quickly identify a safe life and associated reliability based on engineering analysis and available data. On application of the method successfully, the following conclusions have been derived.

- A technique to compute the reliability of structural components has been presented.
- Varying the parameters like load, coefficient of variation, and cycles to usage the change in reliability index is observed.
- On condition inspection and part replacement before they reach retirement time.
- Reliability index decreases with the increase in the coefficient of variation.
- Failure rate increases with increase in cycles usage.
- Reliability index decreases with the increase in load.
- Reliability index decreases with increase in cycles usage.

REFERENCES


