

## **Spectrum Sensing of OFDM Signals over Multipath Fading Channels for Cognitive Radios**

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### **Abstract:**

OFDM symbols are used in this research as it is a popular transmission technology in cognitive radio networks and also utilized to improve the reliability of spectrum sensing of secondary users. However spectrum sensing over multipath fading channels remains an important and challenging issue. This paper proposes a new scheme i.e., optimal NP detector for spectrum sensing using cyclic prefix to improve detection performance. This approach involves multipath fading channels and classical NP detector. To detect OFDM signal of Primary Users (PU) the log likelihood ratio(LLR) test is formulated by using the correlation characteristics of the redundancy of cyclic prefix. In this paper according to the analytical results the likelihood ratio of received samples is equivalent to their log likelihood function (LF) plus the LR of an Energy detector. Since many unknown parameters need to be resolute, a practical generalized log likelihood ratio test (GLRT) and Channel Independent GLRT are presented to achieve good performance over multipath fading channels. Simulation results of proposed detectors compared with the state-of-the-art detectors.

### **Keywords:**

Cognitive Radio, Spectrum Sensing, Neyman-Pearson Detector, GLRT, CI-GLRT.

### **I. INTRODUCTION:**

In CR, the PUs is referred to those users who have priority on the usage of a part of the spectrum. Spectrum sensing is a key element in CR communications, as it enables the CR to adapt to its environment by detecting spectrum holes.

The majority effective way to detect the spectrum is to detect the PUs that are receiving data within the range of a CR. The task of spectrum sensing involves the following subtasks are Detection of spectrum holes, Spectral resolution of each spectrum vacancies, Identification of the spatial directions of incoming interferes and Signal classification. Today OFDM (Orthogonal Frequency Division Multiplexing) techniques are adopted by progressing wireless communication standards. Spectrum sensing helps to detect the spectrum holes (unutilized bands of the spectrum) providing high spectral resolution capability. During employing CR, Secondary Users (SUs) unable to interfere with other licensed users using the spectrum; so to guarantee an interference-free communication between rental users, the spectrum sensing information between multiple cognitive radio devices needs to be shared to decrease the probability of interference with licensed users. The signal detection problem is solved by the decision between the two hypotheses

$$\begin{cases} H_0 : \text{primary user not present} \\ H_1 : \text{primary user present} \end{cases} \quad (1)$$

The decision between the two hypothesis made by comparing a test statistics  $T$  with a threshold  $\gamma$ . The detector performance is characterized as probability of detection and probability of false alarm. In order to use the Likelihood Ratio Test (LRT), perfect knowledge of the parameters, such as, the noise and source signal distributions as well as the channels characteristics, is usually required. However, in cognitive radio scenarios, this information is sometimes unavailable.

In such cases, other approaches like the Generalized Likelihood Ratio test (GLRT) and the Channel Independent Generalized Likelihood Ratio test (CI-GLRT) are more adequate. Different spectrum sensing detector based on OFDM signals are introduced and emphasized in this paper. To enhance the detection probability, many signal detection techniques can be used in spectrum sensing. In this paper, the work proposes an optimal NP detector by discovering a relationship between the log-likelihood function (LF) of received signals and the log-likelihood ratio (LR) of the ED.

**II.OFDM SIGNAL MODEL AND CORRELATION CHARACTERISTICS**

In this paper a OFDM with N subcarriers is considered. The complex data are modulated onto the N subcarriers used in the spectrum sensing process through inverse discrete Fourier transform (IDFT). Here a Cyclic Prefix of length  $N_G$ , which is a duplicate of the last  $N_G$  samples of a symbol, is inserted at the beginning of each OFDM symbol to prevent inter symbol interference (ISI) and also preserve mutual orthogonality among all subcarriers. Consider  $h(l)$  denote the impulse response of a multipath channel with  $(L+1)$  uncorrelated taps.

Following parallel-to-serial conversion, the transmitted OFDM data  $x(n)$ , i.e., the elements of  $x = [... x_{m-1}^T x_m^T x_{m+1}^T ...]^T$ , where n denotes the sampling index,  $(.)^T$  denotes a transpose operation,  $x_m = [x_m(0) x_m(1) ... x_m(N + N_G - 1)]^T$  is the mth symbol and  $x_m(n) \equiv x(m(N + N_G) + n), n \in \{0, 1, \dots, N + N_G - 1\}$ , are finally transmitted through a multipath channel  $h(l)$ . The channel length is assumed to be shorter than the CP length. Owing to the CP, for  $n_1 \neq n_2$ , the auto correlation of  $x_m(n)$  is nonzero, i.e.,  $E[x_m(n_1)x_m^*(n_2)] = \sigma_x^2$ , only for  $0 \leq n_1 \leq N_G - 1$  in the CP and  $n_2 = n_1 + N$ , where  $\sigma_x^2$  and  $(.)^*$  represents the signal power on the transmitter side and complex conjugate, respectively; otherwise,  $E[x_m(n_1)x_m^*(n_2)] = 0$ .

On the receiver side, owing to the multipath channel effect, the received sampled data  $\tilde{x}_m(n)$  can be written as

$$\tilde{x}_m(n) = e^{j\frac{2\pi n\epsilon}{N}} \sum_{l=0}^L h(l) x_m(n - l - \theta) + w(n) \tag{2}$$

Where  $\epsilon$  is carrier frequency offset (CFO);  $\theta$  represents symbol timing delay, and  $w(n)$  is the AWGN with zero mean and variance  $\sigma_w^2$ .

As  $x(\cdot)$ ,  $h(\cdot)$ , and  $w(\cdot)$  are mutually uncorrelated, assume that the channel coefficients are wide-sense stationary and uncorrelated scattering (WSSUS), considering ISI introduced from neighboring symbols, the correlation between  $\tilde{x}_m(n)$  and  $\tilde{x}_m(n + N)$  can be expressed as

$$E[\tilde{x}_m(n)\tilde{x}_m^*(n + N)] = \begin{cases} e^{-j2\pi\epsilon} \sigma_x^2 \sum_{l=0}^{n-\theta} \sigma_{h(l)}^2, & n \in I_1 \\ e^{-j2\pi\epsilon} \sigma_x^2 \sum_{l=0}^L \sigma_{h(l)}^2, & n \in I_2 \\ e^{-j2\pi\epsilon} \sigma_x^2 \sum_{l=0}^L \sigma_{h(l)}^2, & n \in I_3 \end{cases} \tag{3}$$

where  $[\tilde{x}_m(n) \equiv \tilde{x}(m(N + N_G) + n)]$  denote the m-th received symbol,  $n \in \{\theta, \theta + 1, \dots, \theta + N + N_G - 1\}$ ,  $\sigma_{h(l)}^2 \equiv E[|h(l)|^2]$ ,  $I_1 \equiv \{\theta, \theta + 1, \dots, \theta + L - 1\}$ ,  $I_2 \equiv \{\theta + L, \theta + L + 1, \dots, \theta + N_G - 1\}$ ,  $I_3 \equiv \{\theta + N_G, \theta + N_G + 1, \dots, \theta + N_G + L - 1\}$ , and  $I_4 \equiv \{\theta + N_G + L, \theta + N_G + L + 1, \dots, \theta + N + N_G - 1\}$ . Moreover the correlation characteristic exists for all symbols. It can be seen that the correlation characteristics (3) under multipath channels are not flat, which makes the subsequent ratio test complicated compared to conventional empirical correlation coefficient –based detector considering only AWGN channels. Additionally,

$$E[\tilde{x}_m(n)\tilde{x}_m^*(n + N)] = 0, \text{ for } n_1 \neq 0 \text{ and } N. \tag{4}$$

**II. NEYMAN PEARSON DETECTION SCHEME**

The Neyman-Pearson (NP) detector is the familiar conventional detector which optimizes the detection probability given a fixed level of false alarm probability. It is based on the observed data only and does not require any prior information about the hypotheses. As a result of this it can be applied in almost all detection problems. The Neyman-Pearson criterion is stated in terms of certain probabilities associated with a particular hypothesis test. The Neyman-Pearson criterion says that we should construct our decision rule to have maximum probability of detection while not allowing the probability of false alarm to exceed a certain value  $\alpha$ . This section presents the classical NP detector over multipath fading channels directly based on the received samples in order to improve the detection performance. According to the received signal model, the received data is determined by using the NP hypothesis test as

$$\begin{aligned} \mathcal{H}_0 : \tilde{x}_m(n) &= w(n) \\ \mathcal{H}_1 : \tilde{x}_m(n) &= e^{j\frac{2\pi n\epsilon}{N}} \sum_{l=0}^L h(l) x_m(n-l-\theta) + w(n) \end{aligned} \quad (5)$$

Where  $\mathcal{H}_1$  and  $\mathcal{H}_0$  denote the hypothesis with and without the presence of PU signal, respectively. Each SU solves the hypothesis testing problem to make a decision about the presence or absence of the PU. When N is large, the sampled data  $\tilde{x}(n)$  can be modeled approximately as complex Gaussian by using the central limit theorem; in addition, the probability density function (pdf) of each sample of the m-th symbol is given by

$$f(\tilde{x}_m(n) | \mathcal{H}_1, \sigma_1^2) = \frac{\exp\left\{-\frac{|\tilde{x}_m(n)|^2}{\sigma_1^2}\right\}}{\pi \sigma_1^2} \quad (6)$$

Where

$$\sigma_1^2 \equiv E[|\tilde{x}_m(n)|^2] = \sigma^2 + \sigma_w^2 \quad \text{and} \quad \sigma^2 \equiv \sigma_x^2 \sum_l \sigma_h^2(l) \quad (7)$$

Where  $\sigma^2$  denotes the received signal power. Based on (6) and (7), owing to CP, the received samples,  $\tilde{x}_m(n)$  and  $\tilde{x}_m(n+N)$ , for  $n \in I \equiv I_1 \cup I_2 \cup I_3$ , are jointly Gaussian with pdf

$$f(\tilde{x}_m(n), \tilde{x}_m(n+N) | \mathcal{H}_1, \epsilon, \rho_n, \sigma_1^2) = \exp\left(\frac{\frac{|\tilde{x}_m(n)|^2 - 2\rho_n \text{Re}\{e^{2\pi n\epsilon} \tilde{x}_m(n)\tilde{x}_m^*(n+N)\} + |\tilde{x}_m(n+N)|^2}{\sigma_1^2(1-\rho_n^2)}}{\pi \sigma_1^4(1-\rho_n^2)}\right) \quad (8)$$

Where  $\text{Re}\{\cdot\}$  denotes the real part, and the correlation coefficient

$$\rho_n \equiv \left| \frac{E[\tilde{x}_m(n)\tilde{x}_m^*(n+N)]}{\sqrt{E[|\tilde{x}_m(n)|^2]E[|\tilde{x}_m(n+N)|^2]}} \right| \quad (9)$$

Similarly,  $f(\tilde{x}_m(n) | \mathcal{H}_0, \sigma_w^2)$  can be obtained as (6) by replacing  $\sigma^2$  with zero. With the pdfs in (6) and (8), the likelihood ratio used for the NP hypothesis test (5) on signal detection is analyzed. To design the optimal NP detector, the following theorem is proven first.

**Theorem1:**

Likelihood ratio of the received samples equals the product of likelihood function under  $\mathcal{H}_1$  hypothesis and the likelihood ratio of ED.

**Proof:** For an observation window of length  $(2N+N_G)$ , the window length covers one complete interval of nonzero correlation (3), which is assumed to belong to the m-th symbol. The likelihood ratio of the received m-th symbol for the NP detector can be expressed as

$$\begin{aligned} \Lambda_m^{NP}(\theta, L, \epsilon, \rho_n, \sigma^2, \sigma_w^2) &\equiv \frac{f(\tilde{x}_m | \mathcal{H}_1)}{f(\tilde{x}_m | \mathcal{H}_0)} \\ &= \frac{\prod_{n \in I} f(\tilde{x}_m(n), \tilde{x}_m(n+N) | \mathcal{H}_1) \prod_{n \in I \cup I'} f(\tilde{x}_m(n) | \mathcal{H}_1)}{\prod_n f(\tilde{x}_m(n) | \mathcal{H}_0)} \\ &= \frac{\prod_{n \in I} \frac{f(\tilde{x}_m(n), \tilde{x}_m(n+N) | \mathcal{H}_1)}{f(\tilde{x}_m(n) | \mathcal{H}_1)} \prod_n f(\tilde{x}_m(n) | \mathcal{H}_1)}{\prod_n f(\tilde{x}_m(n) | \mathcal{H}_0)} \end{aligned}$$

$$= \frac{\prod_{n \in I} \frac{f(\tilde{x}_m(n), \tilde{x}_m(n+N) | \mathcal{H}_1)}{f(\tilde{x}_m(n) | \mathcal{H}_1) f(\tilde{x}_m(n+N) | \mathcal{H}_1)}}{\prod_{n \in I} \frac{f(\tilde{x}_m(n) | \mathcal{H}_1)}{f(\tilde{x}_m(n) | \mathcal{H}_0)}}}$$

Likelihood Function                  Likelihood ratio of ED

$$= \Lambda_m^{LF} |(\theta, L, \epsilon, \rho_n, \sigma^2, \sigma_w^2) \times \Lambda_m^{ED} |(\sigma_1^2, \sigma_w^2) \quad (10)$$

Where  $(.) | (\theta, L, \epsilon, \rho_n, \sigma^2, \sigma_w^2)$  denotes the quantity conditioned on  $(\theta, L, \epsilon, \rho_n, \sigma^2, \sigma_w^2)$ ,  $\tilde{x}_m$  denote the received samples in the observation window,  $I \equiv I_1 \cup I_2 \cup I_3$  and  $I'$   $\equiv$  the interval that is obtained by shifting  $I$  to the right by  $N$  samples,  $\times$  denotes multiplication, and  $\Lambda_m^{LF}$  and  $\Lambda_m^{ED}$  represent the likelihood function, which is related to unknown parameter estimation, and likelihood ratio of ED, respectively

### III. PROPOSED NP DETECTOR:

The previous section is the formulation of NP detector and its relationship with the LLF and the ED. This section further develops the algorithm for optimizing the NP detector. To derive an ideal case for subsequent study, all involved parameters are assumed to be known. The LLF and LLR of ED are discussed below

#### A. Log-Likelihood Function of Received Signals

This approach yields optimal detector but in general it does not lead to closed-form expression derivation of the detector. After  $M$  symbols have been received, the LF of received signals  $\Lambda^{LF}$ , can be expressed as

$$\Lambda^{LF} = \log(\prod_{m=0}^{M-1} \Lambda_m^{LF}) = M \sum_{n \in I} \Lambda_n^{LF} \quad (11)$$

Where

$$\Lambda_n^{LF} = \frac{2(\rho_n \Gamma(n) - \rho_n^2 \Phi(n))}{\sigma_1^2 (1 - \rho_n^2)} - \log(1 - \rho_n^2) \quad (12)$$

Denotes the LF at each sampling point, and

$$\Gamma(n) \equiv \frac{1}{M} \sum_{m=0}^{M-1} \gamma_m(n), \quad \Phi(n) \equiv \sum_{m=0}^{M-1} \phi_m(n),$$

$$\gamma_m(n) \equiv \text{Re}\{e^{j2\pi\epsilon} \tilde{x}_m(n) \tilde{x}_m^*(n+N)\},$$

$$\phi_m(n) \equiv \frac{1}{2} (|\tilde{x}_m(n)|^2 + |\tilde{x}_m(n+N)|^2) \quad (13)$$

It is clearly shown that, besides received samples, the likelihood function requires knowledge of  $(\theta, L, \epsilon, \rho_n, \sigma_1^2)$ . Besides, owing to the non-uniform profile of  $\rho_n$ , the distribution of the detector based on the pure likelihood function does not have a simple formulation. Therefore, its threshold has to be determined empirically, and it can be regarded as the benchmark for all practically CP-based detectors.

#### B. Log-likelihood Ratio of ED

Energy detection is most selective for detecting independent and identically distributed (iid) signals in high SNR conditions, but not optimal for detecting correlated signals. The idea is to measure the received energy on the specific portion of the spectrum. If the measured energy is below a threshold value, the channel is considered available. Its simplicity and low signal processing requirement make this method very attractive for spectrum sensing. Furthermore, the threshold used in energy selection depends on the noise variance, and small noise power estimation errors can result in significant performance loss.

After receiving  $M$  symbols, there are  $N_1 = M(N + N_G)$  samples in the observation window. Based on (5) and (9), the LR of the ED,  $\Lambda^{ED}$ , can be obtained as

$$\Lambda^{ED} = \log(\prod_{m=0}^{M-1} \Lambda_m^{ED})$$

$$= \log\left(\prod_{n=0}^{N_1-1} \frac{f(\tilde{x}_m(n) | \mathcal{H}_1)}{f(\tilde{x}_m(n) | \mathcal{H}_0)}\right)$$

$$= C \sum_{n=0}^{N_1-1} \frac{|\tilde{x}_m(n)|^2}{\sigma_w^2} + C_1 \quad (14)$$

where the signal-to-noise ratio (SNR)  $\zeta \equiv \sigma^2 / \sigma_w^2$ ,  $C = \frac{\sigma^2}{\sigma_1^2} = \frac{\sigma^2}{\sigma^2 + \sigma_w^2} = \frac{\zeta}{\zeta + 1}$ , and  $C_1 = N_1 \log(\frac{1}{\zeta + 1})$  are constants.

It is clearly shown that, besides received samples, the ED only requires knowledge of  $(\sigma_w^2, \sigma^2)$ .



If  $\sigma_w^2$  and  $\sigma^2$  are known and regarded as constants, they can be removed from (14), and the decision metric of the conventional energy detection (CED) can be obtained as

$$\Lambda^{CED} = \sum_{n=0}^{N_1-1} |\tilde{x}_m(n)|^2 \geq \eta^{CED} \quad (15)$$

Where  $\eta^{CED}$  denotes the decision threshold of the CED.

### C. Proposed Optimal Neyman-Pearson Detector

According to theorem 1, the LR of the proposed optimal NP detector is the summation of (11) and (14) i.e.,

$$\Lambda^{NP} = \log\left(\prod_{m=0}^{M-1} \Lambda_m^{NP}\right) = \Lambda^{LF} + \Lambda^{ED} \quad (16)$$

Unfortunately, obtaining the exact distribution of  $\Lambda^{NP}$  is quite complex. We will determine its decision threshold empirically.

### IV. PROPOSED GLRT

Here both the hypotheses contain unknown parameters, finding the neyman Pearson solution becomes very tedious and often the involved integrals do not yield closed-form solution. On account of the above limitation, this paper presents the use of an alternative hypothesis testing approach referred as generalized likelihood ratio test (GLRT). In this approach, the unknown parameters are first estimated from the observed data from both the hypothesis. In the GLRT method, the utmost likelihood estimation (ML) is used to estimate the value of the unknown parameters which are, in turn, used in a normal LRT test. In the GLRT, the unknown parameters are replaced by their maximum likelihood estimate (MLE) in the likelihood ratio. There is no optimality associated with the GLRT, in practice, it work quite well.

#### A. All parameters are unknown

First, the estimation of  $\sigma_w^2$  under  $\mathcal{H}_0$  hypothesis is determined. The ML estimate of  $\sigma_w^2$

$$\hat{\sigma}_w^2 | \mathcal{H}_0 = \frac{1}{N_1} \sum_{n=0}^{N_1-1} |\tilde{x}_m(n)|^2 \quad (17)$$

Where  $(.)|_{\mathcal{H}_0}$  denotes the quantity conditioned on  $\mathcal{H}_0$ .

Next, the estimation of  $\rho_n$  and  $\sigma_1^2$  is jointly considered. It can be observed that

$$\begin{aligned} \Gamma(n) &= \frac{1}{M} \sum_{m=0}^{M-1} \gamma_m(n) = \text{Re}\{e^{j2\pi\epsilon} \Psi(n)\} \\ &= |\Psi(n)| \cos(2\pi\epsilon + \angle\Psi(n)) \end{aligned} \quad (18)$$

where  $\psi(n) \equiv \frac{1}{M} \sum_{m=0}^{M-1} \tilde{x}_m(n) \tilde{x}_m^*(n+N)$ . Obviously, the estimation  $\angle\Psi(n)$  is an estimate of CFO,  $-2\pi\epsilon$ . Hence, as  $M \rightarrow \infty$ ,  $\cos(2\pi\epsilon + \angle\Psi(n)) \approx 1$ , i.e.,  $\Gamma(n) \approx |\Psi(n)|$ . The ML estimate of  $\rho_n$  can be reduced to

$$\rho_n = \frac{|\Psi(n)|}{\Phi(n)} \quad (19)$$

Notably,  $\Gamma(n)$  is related to unknown  $\epsilon$ . In contrast,  $\Psi(n)$  is independent of  $\epsilon$ . Doing so can get rid of the estimation of  $\epsilon$ , which is important, because estimation accuracy is limited at low SNRs. The LF only related to  $\sigma_1^2$  is found to be

$$\hat{\sigma}_1^2 | \mathcal{H}_1 = \frac{1}{N_1} \sum_{n=0}^{N_1-1} |\tilde{x}_m(n)|^2 \quad (20)$$

The GLRT can thus be given by

$$\begin{aligned} \Lambda^{GLRT} &\equiv \log\left(\frac{\max_{(\theta, L, \epsilon, \rho_n, \sigma_1^2)} f(\tilde{x}_m | \mathcal{H}_1, \theta, L, \rho_n, \sigma_1^2)}{\max_{(\sigma_w^2)} f(\tilde{x}_m | \mathcal{H}_0, \sigma_w^2)}\right) \\ &= \max_{(\theta, L)} (\Lambda^{LF} + \Lambda^{ED}) \approx \max_{(\theta, L)} (M \sum_{n \in I} \hat{\rho}_n^2) \end{aligned} \quad (21)$$

Where  $\Lambda^{LF}$  is given in (10) with those known parameters replaced by their estimates  $\Lambda^{ED} = 0$ , i.e., the GLRT is irrespective of the robust ED because the GLRT for the likelihood ratio of the ED is a constant of 1.

**B. Channel – Independent GLRT (CI-GLRT)**

The GLRT depends on the  $\rho_n$ , which depends highly on the power delay profile(PDP) of multipath channels and SNR conditions. In contrast, this section intends to derive a modified GLRT independent of multipath channel profiles. Moreover, CI-GLRT can reduce the computational complexity and get rid of estimating too many unknown parameters. The correlation (3), i.e.  $E[\tilde{x}_m(n)\tilde{x}_m^*(n+N)] \approx \frac{1}{M} \sum_{m=0}^{M-1} \tilde{x}_m(n)\tilde{x}_m^*(n+N) = \Psi(n)$ , has a complementary property for the correlations in  $I_1$  and  $I_2$ , which are complementary to separated-by- $N_G$  correlations in  $I_3$  and  $I_4$ , respectively. Namely, for  $n \in I_1$  and  $n \in I_2$ ,  $|\Psi(n) + \Psi(n+N_G)| \approx \sigma^2$  (22)

Notably, when  $n \in I_1$  and  $n \in I_2$ ,  $n+N_G \in I_3$  and  $n+N_G \in I_4$  respectively. Additionally,  $\sigma^2 \approx |\Psi(n)|$  for  $n \in I_2$ . Based on the complement property, given  $(\theta, L)$ , one can determine the aliased version of  $\Psi(n)$  for  $n \in I_1$ , i.e.  $|\Psi^{alias}(n)| = |\Psi(n) + \Psi(n+N_G)| \approx \sigma^2$ , which equals  $|\Psi(n)|$  for  $n \in I_2$ . As such,  $|\Psi^{alias}(n)|$  for  $n \in I_1$  and  $|\Psi(n)|$  for  $n \in I_2$  form a flat correlation characteristic for  $n \in I_1 \cup I_2$ , which is the same as that of the flat fading or AWGN channels. Therefore, the proposed CI-GLRT entails estimating only one  $\rho$  as

$$\hat{\rho} = \frac{\frac{1}{N_G} (|\sum_{n \in I_1} \Psi^{alias}(n) + \sum_{n \in I_2} \Psi(n)|)}{\frac{1}{M(N+N_G)} \sum_{m=0}^{M-1} \sum_{n=0}^{N+N_G-1} |\tilde{x}_m(n)|^2}$$

$$= \frac{M(N+N_G)}{N_G} \frac{|\sum_{n \in I} \Psi(n)|}{\sum_{m=0}^{M-1} \sum_{n=0}^{N+N_G-1} |\tilde{x}_m(n)|^2} \quad (23)$$

Since there are more samples to estimate one  $\rho$  than to estimate each  $\rho_n$  for  $\forall n \in I$ , the estimation accuracy for  $\rho$  is much higher than that for  $\rho_n$ . Another advantage is that the proposed CI-GLRT is independent of multipath channel PDPs.

Hence, according to the original GLRT, the decision metric of the CI-GLRT becomes

$$\Lambda^{CI-GLRT} \approx \max_{(\theta, L)} (MN_G \hat{\rho}^2) \geq \eta^{CI-GLRT} \quad (24)$$

Where  $\eta^{CI-GLRT}$  is the decision threshold of the CI-GLRT. Under  $\mathcal{H}_0$  hypothesis, the distribution of the decision metric can be obtained using distribution fitting as weibull distribution.  $\eta^{CI-GLRT}$  Can be determined using the CDF of weibull distribution for given probability of false alarm  $P_{fa}$ .

**VI. COMPARISON WITH STATE OF ART DETECTORS**

**AC Detector:**

The detector directly employs the second-order statistic,  $\rho_n$ , for the decision metric. By averaging  $\text{Re}\{\Psi(n)\}$  over all samples, the detector can blindly make a decision independent of  $\theta$ . The decision metric was designed as

$$\frac{\frac{1}{N+N_G} \sum_{n \in I_1 \cup I_2} \text{Re}\{\Psi(n)\}}{\frac{1}{N_1} \sum_{n=0}^{N_1-1} |\tilde{x}(n)|^2} \geq \eta^{AC}, \quad (25)$$

Where  $\eta^{AC} = \frac{1}{\sqrt{2N_1}} Q^{-1}(P_{fa})$  has a CF expression. The AC detector does not require information on  $\sigma_W^2$  and the SNR. Another detector, namely an AC detector featuring perfect synchronization (syncAC), over AWGN channels was expressed as

$$\frac{\frac{1}{N_G} \sum_{n \in I_1 \cup I_2} \text{Re}\{\Psi(n)\}}{\frac{1}{N_1} \sum_{n=0}^{N_1-1} |\tilde{x}(n)|^2} \geq \eta^{syncAC} \quad (26)$$

$$\text{Where } \eta^{syncAC} = \frac{1}{\sqrt{2MN_G}} Q^{-1}(P_{fa})$$

**Sliding Window:**

The sliding-window (SW) detector over AWGN channels is expressed as

$$\max_{\theta} |\sum_{n \in I_1 \cup I_2} \Psi(n)| \geq \eta^{SW} \quad (27)$$

The distribution of the decision metric has no CF expression; therefore, the threshold must be determined empirically

**VII.PERFORMANCE EVALUATION**

In this section some simulation results of the proposed schemes and the conventional detectors are given for sensing the OFDM signals over multipath fading channels. During the simulation process we fix the probability of false alarm  $P_{fa} = 0.05$  to get the thresholds of the detectors proposed. The performance of these presented detectors with estimated parameters and simulation results are shown in this paper. Monte Carlo simulations were conducted to contact the performance of the proposed detector. An OFDM system with total subcarriers  $N = 64$  and  $N_G = 16$  was considered. The OFDM symbol duration including the CP was  $16\mu s$ . In addition, the simulation results were assessing in the presence of CFO of 30% subcarrier spacing. The channel taps were randomly generated using independent zero- mean unit- variance complex Gaussian random variables. During each trial, unless otherwise stated,  $M = 20$  OFDM symbols were tested and  $P_{fa} = 0.05$ .

The detection algorithm involves two probabilities: probability of detection  $P_d$  and probability of false alarm  $P_{fa}$ .  $P_d$  is the probability of detecting a signal on the measured frequency when it is really present. Therefore a large detection probability is desired. The threshold value ( $\lambda$ ) for the detector is fixed either from the fixed probability of detection  $P_d$  or from the fixed probability of false alarm  $P_{fa}$ . Fig. 1. plots the probability of detection versus the SNR for the proposed NP detector, the LLF and the ED over multipath fading channels. It compares the performance of the proposed NP detector with the LLF and ED detectors. According to the paper the NP detector is a combination of LLF and the ED, when compared to ED detector and LLF detector, the NP detector exhibits the best performance. It is apparent that the NP detector achieves significant performance that it outperforms the other existed detectors.

Fig.2. plots the probability of detection versus the SNR for the NP detector , the LLF detector , ED, GLRT detector, and CI-GLRT detector. Based on Fig. 2 we can make the conclusion that although the proposed detectors do not have a better performance compared with the existed ones. The GLRT detector performance is worse when compared to the other detectors, as the estimation of unknown parameters is not sufficiently accuracy . the estimation of all the parameters will not improve the system performance. In other words it degrades the performance of the detector. Fig 3. Compares all CP- based detectors. Fig 4. Plots are probability of false alarm versus probability of detection. Actually the fig displays the receiver operating characteristics at  $SNR = -9dB$  in the scenario considered in fig 3.

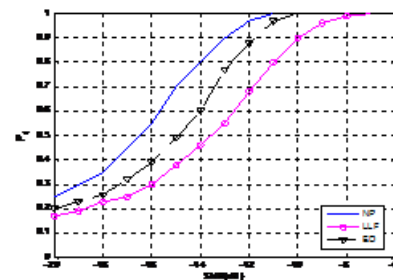
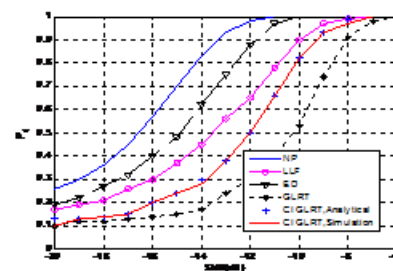
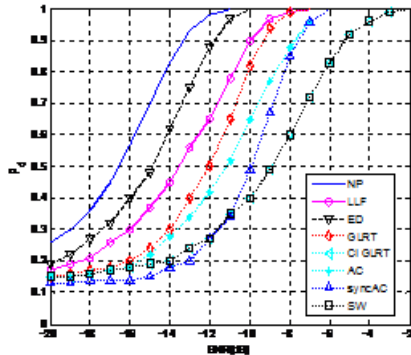


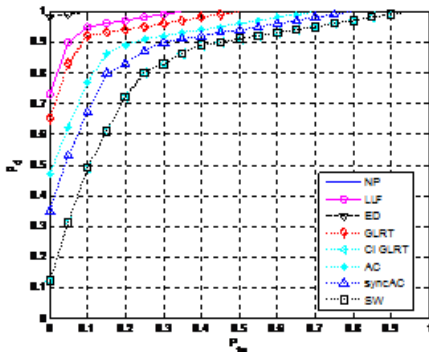
Fig: 1. Probability of detection plotted as a function of SNR for the NP detector , the LLF detector, and the ED.



**Fig. 2. Probability of detection plotted as a function of SNR for the NP detector, LLF detector, ED, GLRT detector, and CI-GLRT detector**



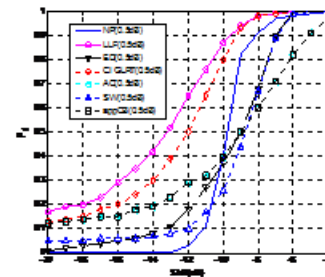
**Fig.3. Comparison of all CP- based detectors.**



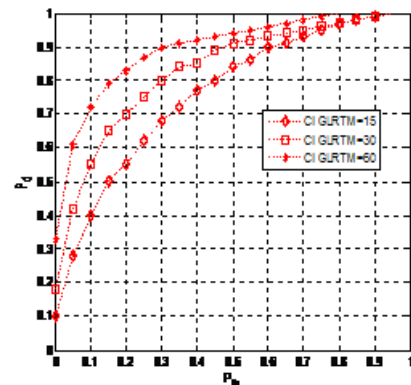
**Fig. 4. Comparison of all CP-based detectors using the ROC. SNR= -9dB**

Noise uncertainty is an important factor which would affect the performance of the detector. In this paper we assume that the accurate noise power  $\sigma_n^2$  is obtained, while it is hard to be achieved in the practical applications. Therefore it is necessary to examine the influence of noise uncertainty for our proposed detectors. Fig.5. Shows the performance of all the detectors considered in this paper with the noise uncertainty is equal to 0.5dB. We can see that at a low uncertainty the performance of the ED based detectors, including ED and NP detector, degrade correspondingly. Noise uncertainty can cause some performance degradations. Here the performance of the proposed NP detector is also presented, which exhibits performance degradation in response to noise uncertainty.

Therefore based on Fig. 5 we can make the conclusion that even though the proposed detectors do not have very strong ability to resist noise uncertainty, the detectors can get the novel performance when the accurate or relatively accurate noise power might be obtained.



**Fig. 5. Comparison between all CP-based GLRT detectors and the ED, The influence of unknown parameters,  $\sigma_w^2$ , with 0.5dB uncertainty is demonstrated.**



**Fig. 6. ROC is proposed CI-GLRT detector under the effects of M with noise uncertainty of 0.5dB. SNR= -12dB**

Fig. 6 shows the ROC curve for CI-GLRT at SNR = -12 dB under the effects of M (the number symbols) with noise uncertainty 0.5dB. It is apparent that the noise uncertainty might impact the probability of detection of all detectors.



## VIII. CONCLUSION

In this paper two new spectrum sensing algorithms for OFDM signals are investigated under low SNR environment with the presence of a timing delay and also GLRT and CI-GLRT detection algorithm based on the differential characteristics for the sensing the OFDM signals are proposed. The simulation results show that the proposed NP detector can achieve the best performance among all the detectors considered in this paper. In addition it proposes a new way to estimate the parameters of the NP detector. In this paper we just analyze two algorithms, while there are many detectors of sensing the OFDM signals that can employ the differential operation to improve the detection performance. This is a topic for the future research.

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