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Comparative Analysis of Design of Controllers For Twin Rotor MIMO System

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***Abstract:* Twin Rotor MIMO system is an experimental model of Helicopter. It is a multi-input multi output system which is confined to two degree of freedom. It is utilized for the verification of controlling techniques and observers for helicopter manoeuvre. In this work Linear Quadratic Gaussian (LQG) controller and Linear Quadratic Gaussian controller with integral action (LQGI) are designed for the Twin Rotor MIMO system. Both control techniques are implemented for the control of Twin Rotor MIMO system, in MATLAB Simulink environment to check the endurance of each controller to meet the desired specifications.**

***Keywords:* MIMO (Multiple Input Multiple Output) system, Manoeuvre, Linearization, Kalman Filter, LQG controller, LQGI controller.**

I. INTRODUCTION

Twin rotor MIMO system is a boon to the Control Engineering Community who work on effectiveness of different control techniques for helicopter manoeuvre. It is cross coupled multiple input multiple output system. The aero dynamic model contains two rotors on each side of the horizontal beam. Both rotors are driven by individual DC motors. One rotor is known as main rotor and the other is tail rotor. The horizontal beam is counter balanced by pivoted beam. The horizontal beam can rotate in horizontal and vertical directions. Main rotor is responsible for up and down motion, i.e it generates lifting force so that the horizontal beam is elevated about pitch axis. Tail rotor is responsible for the rotation of horizontal beam about yaw axis (vertical axis). Various control approaches were implemented for the control of Twin Rotor Multiple Input Multiple Output system. In [2] PID control technique has been proposed for Twin Rotor System.

In [3], authors defined the coupling effect and dynamic modelling of TRMS and cross coupled PID control was achieved using four PID controllers. In [4] Optimal control technique for TRMS was introduced and in [5] advanced adaptive control technique for twin rotor MIMO system was developed. In [6], the author compared the response of TRMS with PID and LQR controllers. In the present work, dynamic state space model of TRMS has been derived from differential equations. A Linear Quadratic Gaussian (LQG)controller and Linear Quadratic Gaussian controller with integral action (LQGI) have been designed separately. The response (steady state and transient) of the system is analysed for step input.

II. MATHEMATICAL MODELLING

As TRMS is a non-linear model direct application of linear quadratic regulator is not valid. So the state space model of TRMS has to be modelled from the dynamic equations. From state space matrices the controller is designed for approximate state space model.

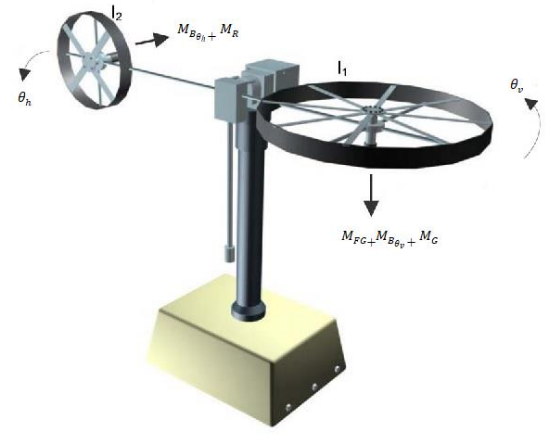

Fig. 1 TRMS Phenomenological Model

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The mathematical model is derived from the phenomenological model shown in Fig.1.
Mathematical equation in vertical plane is given as,

$$I_1 \frac{d^2\theta_v}{dt^2} = M_1 - M_{FG} - M_{B_{\theta_v}} - M_G$$

(1)

Table 2.1 TRMS Physical Parameters

Where $M_1 = c_1 \tau_1^2 + d_1 \tau_1$ - Nonlinear static characteristic (2)

$M_{FG} = M_G \sin\theta_v$ - Gravity momentum (3)

$M_{B_{\theta_v}} = B_{1_{\theta_v}} \left(\frac{d\theta_v}{dt}\right) + B_{2_{\theta_v}} \operatorname{sign}\left(\frac{d\theta_v}{dt}\right)$ - Friction forces momentum (4)

$M_G = K_{gy} M_1 \left(\frac{d\theta_h}{dt}\right) \cos(\theta_v)$ - Gyroscopic momentum (5)

The motor and the electrical control circuit is approximated as a first order transfer function, thus the rotor momentum in Laplace domain is described as-

$$\tau_1 = \left(\frac{k_1}{T_{11}s+T_{10}}\right) u_1$$

(6)

$$I_2 \frac{d^2\theta_h}{dt^2} = M_2 - M_{B_{\theta_h}} - M_R$$

(7)

Where $M_2 = c_2 \tau_2^2 + d_2 \tau_2$

(8)

$$M_{B_{\theta_h}} = B_{1_{\theta_h}} \left(\frac{d\theta_h}{dt}\right) + B_{2_{\theta_h}} \operatorname{sign}\left(\frac{d\theta_h}{dt}\right)$$

(9)

$$M_R = \frac{k_c(T_0 s+1)}{(T_p s+1)} \tau_1$$

(10)

$$\tau_2 = \left(\frac{k_2}{T_{21}s+T_{20}}\right) u_2$$

(11)

The model parameters used in above (2.1)-(2.11) equations are chosen experimentally obtained from reference [1] which makes the TRMS nonlinear model a semi-phenomenological model. The mathematical model given in equation(2.1)- (2.11) are non-linear and in order to design controller for system, the model should be linearized. The first step in linearization technique [14-15] is to find equilibrium point. Equations (2.1) - (2.11) are combined to represent alternate model of TRMS. The alternate model is given as

$$\frac{d^2\theta_v}{dt^2} =$$

(12)

$$\frac{\left(c_1 \tau_1^2 + d_1 \tau_1 - M_G \sin\theta_v - B_{1_{\theta_v}} \left(\frac{d\theta_v}{dt}\right) - B_{2_{\theta_v}} \operatorname{sign}\left(\frac{d\theta_v}{dt}\right) - K_{gy} (c_1 \tau_1^2 + d_1 \tau_1) \left(\frac{d\theta_h}{dt}\right) \cos(\theta_v)\right)}{I_1}$$

$$\frac{d\tau_1}{dt} = \left(\frac{k_1 u_1 - \tau_1 T_{10}}{T_{11}}\right) \quad (13)$$

$$\frac{d^2\theta_h}{dt^2} = \frac{c_2 \tau_2^2 + d_2 \tau_2 - B_{1_{\theta_h}} \left(\frac{d\theta_h}{dt}\right) - B_{2_{\theta_h}} \operatorname{sign}\left(\frac{d\theta_h}{dt}\right)}{I_2}$$

(14)

$$\frac{d\tau_2}{dt} = \left(\frac{k_2 u_2 - \tau_2 T_{20}}{T_{21}}\right)$$

(15)

$$\frac{dM_R}{dt} = \frac{\left(\left(k_c - \frac{k_c T_0 T_{10}}{T_{11}}\right)\tau_1 + \frac{k_c T_0 K_1}{T_{11}} U_1 - M_R\right)}{T_p}$$

(16)

Now let us assume $\theta_v = x_1$

$\theta_h = x_2$

$\tau_1 = x_3$

$\tau_2 = x_4$

$M_R = x_5$

$\frac{d\theta_v}{dt} = x_6$

$\frac{d\theta_h}{dt} = x_7$

Equations (12)- (16) can be represented with state space variable as -

$$\frac{dx_1}{dt} = x_6$$

(17)

$$\frac{dx_2}{dt} = x_7$$

(18)

$$\frac{dx_3}{dt} = -\frac{T_{10}}{T_{11}} x_3 + \frac{k_1}{T_{11}} u_1$$

(19)

$$\frac{dx_4}{dt} = -\frac{T_{20}}{T_{21}} x_4 + \frac{k_2}{T_{21}} u_2$$

(20)

$$\frac{dx_5}{dt} = \frac{\left(k_c - \frac{k_c T_0 T_{10}}{T_{11}}\right) x_3}{T_p} - \frac{1}{T_p} x_5 + \frac{k_c T_0 K_1}{T_{11} T_p} u_1$$

(21)

$$\frac{dx_5}{dt} =$$

(22)

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$$\frac{\left(c_1 x_3^2 + d_1 x_3 - M_G \sin(x_1) - B_{1_{x_1}} x_6 - B_{2_{x_1}} \operatorname{sign}(x_7) - K_{gy}(c_1 x_3^2 + d_1 x_3)\, x_7 \cos(x_1)\right)}{I_1}$$

$$\frac{dx_6}{dt} = \frac{c_2 x_4^2 + d_2 x_4 - B_{1_{x_2}} x_6 - B_{2_{x_2}} \operatorname{sign}(x_7) - x_5}{I_2} \qquad (23)$$

Proposed Controllers:

Advanced control strategies such as optimal controllers make the system to track the reference signal, but in order to make the design reliable in practical environment a Linear Quadratic Gaussian Controller was designed. The integral action makes the system to track reference with zero steady state error and also the advantage of fast tracking LQG controller is also added, this makes controller robust and reliable in practical environment. Following section deals with design of LQG and LQGI controllers for the TRMS.

Linear Quadratic Gaussian Controller:

Linear Quadratic Gaussian (LQG) controller is an optimal controller. It deals with linear system with additive white Gaussian noise and having incomplete state information and undergoing control to quadratic cost. The solution of LQG control problem is unique and consists of Linear dynamic feedback control law that can be easily implemented. Linear Quadratic Gaussian controller is combination of Kalman Filter and Linear Quadratic Regulator. LQG works on separation principle, it means that Kalman Filter and Linear Quadratic Regulator can be designed and computed independently.

LQG controller application can be applied to Linear time invariant system along with Linear time varying system. Here in this work Linear time invariant system is being considered. Designing of system with LQG controller does not guarantee robustness of system. The robustness of system should be checked once the LQG controller has been designed. Figure-4.3 shows block diagram of LQG controller.

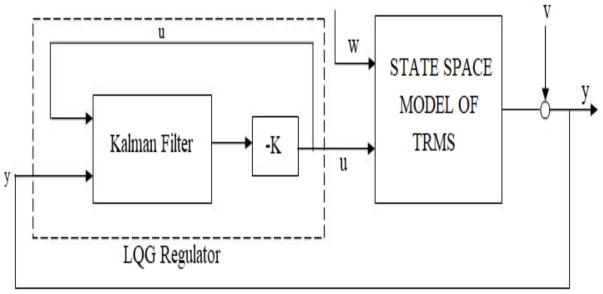

Fig. 2 Block diagram of TRMS with LQG controller

Here in Figure-4.3 it can be seen that Linear Quadratic Gaussian (LQG) controller composed of Kalman Filter (which will estimate all the state of system), followed by Linear Quadratic Regulator (LQR) (which is responsible for controlling the response of system). Along with control input ‘u’ process noise ‘w’ is also applied to system. External white Gaussian noise is added to plant because plant is stochastic with some unknown noise. Measurement noise ‘v’ is also added to system and finally we get response as ‘y’.

Linear Quadratic Gaussian Controller with Integral Action:

In many situations it is desired that the control loop has integral action to guarantee no steady state error to a step input. To impart integral action on the loop, the system is restated in a form that creates a number of additional states equal to the number of outputs that are the output error of the system. In addition to integral action another characteristic that is desired of a controller is robustness. Robustness allows a system to continue to function properly in the face of changes of system parameters or dynamics. There are four types of control theory that can guarantee robustness: deadbeat, robust control theory, sliding mode control and LG based controllers. LQG controller is used in the present work.

Mathematical Modelling:

A Linear Quadratic Gaussian Integral controller involves an addition of integral action to the LQG controlled TRMS. The system is subjected to disturbances w and v and is driven by control u. The integral LQG controller relies on noisy measurements y to generate the control

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signal u. The plant can be represented by state equations,

$$\dot{x} = A\,x(t) + B\,u(t) + F(t)w(t)$$
(4.28)

$$y = Cx(t) + D\,u(t) + v(t)$$
(4.29)

Where $v(t)$=Measuring Noise and $w(t)$ =Process Noise. Both v and w are called as White Noise.

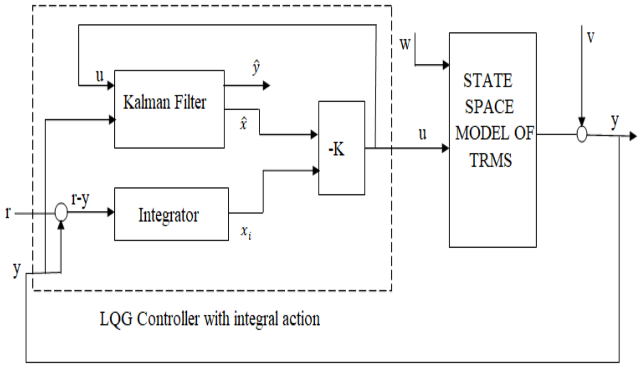

Fig.3 Block diagram of TRMS with LQGI controller

The LQG controller with integral action can be represented as,

$$\begin{bmatrix}\dot{\hat{x}} \\ \dot{x}_i\end{bmatrix} = \begin{bmatrix} A - BK_x - LC + LDK_x & -BK_i + LDK_i \\ 0 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} -K_x & -K_i \end{bmatrix}\begin{bmatrix}\hat{x} \\ x_i\end{bmatrix}$$

$\hat{x}$ represents states estimated by Kalman Filter and x_i is Integrator output

IV. CASE STUDY:
Following graph shows the Pitch response of TRMS for Step reference signal of U1=0.3.The settling time of the response with LQG controller is 9 Seconds and with LQGI controller is 3.5 seconds. The TRMS system shows better response with LQGI controller when compared to LQR and LQG controllers.

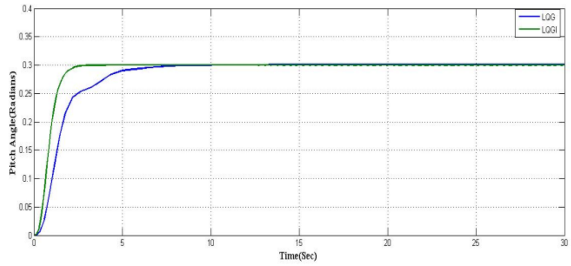

Fig. 4 Comparison of Pitch Angle Response Of TRMS with LQG And LQGI Controllers

Yaw Response Comparison of LQG and LQGI Controllers:

From the below graph it can be observed that the response of the system with LQG controller was 6 seconds. This is further reduced to 3.5 seconds. It can be included that the response of the TRMS is better with LQGI controller when compared to LQG and LQGI controllers.

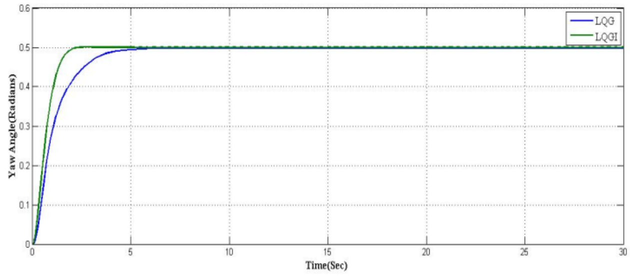

Fig. 5 Comparison Of Yaw Angle Response of TRMS with LQG and LQGI Controllers

Pitch Control Input Comparison for LQG and LQGI Controllers:

Following graph shows the input to the system for the reference step signal U1=0.3. It was observed that system requires same control input of 0.9 volts to run main rotor which is responsible for pitch control. So LQGI controllers shown the better performance with less settling time for the same control input.

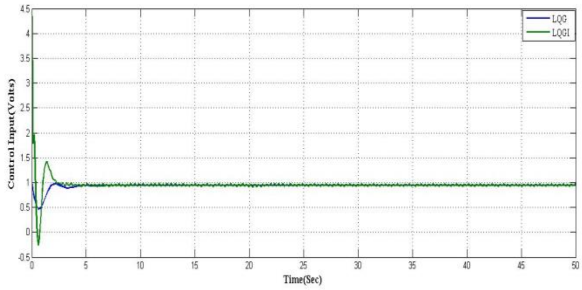

Fig. 6 Pitch Control Input Comparison of LQG and LQGI Controllers

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Input Yaw Control Comparison for LQG and LQGI Controllers:

Below graph is showing the control input given to tail rotor to attain 0.5 radians. It was observed with both control techniques. Both need same control voltage of -3v to run it at 0.5 volts.

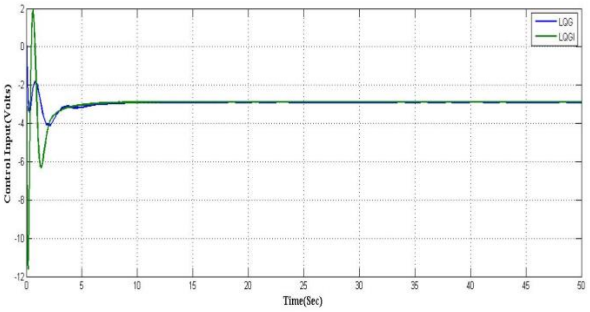

Fig. 7 Yaw Control Input Comparison of LQG and LQGI Controllers

CONCLUTION

In the present work an LQG controller and LQGI controllers were designed for TRMS model. This work involves the implementation and analysis of Linear Quadratic Gaussian controller and Linear Quadratic Gaussian controller with integral action for Twin Rotor MIMO system. It was already observed that Linear Quadratic Gaussian controller is giving the optimum response when compared to LQR controller. In the present work it was observed that the LQGI controller for TRMS results better response (in terms of time domain specifications) when compared to LQG controller for the same control energy.

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