ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

Design of PID Controller for Uncertain Systems Via Optimised Reduced Order Model

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Abstract: **The authors proposed a method for order reduction of linear dynamic systems using the advantages of the Interpolation criterion and Routh method. The denominator polynomial of the reduced order model is determined by using the Routh method while the numerator coefficients are computed by minimizing the integral square error between the original and the reduced system using Interpolation method. The proposed method guarantees stability of the reduced model, if the original high order system is stable system. A PID controller is designed for the high order original systems through its low order model proposed. Some numerical examples were considered to explain the effectiveness of the method.**

Keywords: Interpolation criterion; Routh method; Stability; integral square error.

1 INTRODUCTION

The approximation of linear systems plays an important role in many engineering applications, especially in control system design, where the engineer is faced with controlling a physical system for which an analytic model is represented as a high order linear system.

In many practical situations a fairly complex and high order system is not only tedious but also not cost effective for on-line implementation. It is therefore desirable that a high system be replaced by a low order system such that it retains the main qualitative properties of the original system.

Several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-4]. Further, some methods have also been suggested by combining the features of two different methods [5-7]. To overcome these problems model order reduction techniques are implemented. It is desirable to reduce higher order transfer function to low order model which are expected to approximate the performance of original high order system.

A mixed method is proposed for the reduction of high order continuous systems. This method is developed from the method [1] available in literature and it overcomes the limitations and drawbacks of some existing methods. In the present paper, Interpolation method is used for obtaining the numerator and Routh method is used for obtaining the denominator of the reduced order model.

Then an optimum model (with minimum ISE) is obtained by varying the interpolation points. Using the proposed method a PID controller is designed for the high order original systems.

Cite this article as: P Khajamohiddin & E Anil Kumar "Design of PID Controller for Uncertain Systems Via Optimised Reduced Order Model", International Journal & Magazine of Engineering, Technology, Management and Research (IJMETMR), ISSN 2348-4845, Volume 7 Issue 7, July 2020, Page 71-78.

ISSN No: 2348-4845
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2 REDUCTION PROCEDURE

Let the transfer function of a higher order system be represented by [6], [7]

$$G(s)=\frac{d_0+d_1s+...+d_{n-1}s^{n-1}}{e_0+e_1\,s+...+e_ns^n}=\frac{d_n(s)}{e_n(s)} \quad (1)$$

Where d_i, i=0, 1… n-1and e_i, i=0, 1… n are constants.

For the high order system a reduced k th order model is proposed as given below,

$$R(s)=\frac{a_0+a_1s+...+a_{k-1}s^{k-1}}{b_0+b_1s+...+b_{k-1}s^{k-1}+b_ks^k}=\frac{a_k(s)}{b_k(s)} \quad (2)$$

Where the a_i, i=0, 1… k-1 and b_i, i=0, 1…. k are constants.

Reduced order denominator:

Step 1: The denominator $b_k(s)$ of reduced model can be obtained from the Routh Stability array of the denominator of the original system as given below:

The Routh array is formed by using the eqn. (4):

The Routh table for the denominator of the system is given below:

$$\begin{array}{llll} b_{11}=e_n & b_{12}=e_{n-2} & b_{13}=e_{n-4} & b_{14}=e_{n-6}\ ... \\ b_{21}=e_{n-1} & b_{22}=e_{n-3} & b_{23}=e_{n-5} & b_{24}=e_{n-7}\ ... \\ b_{31} & b_{32} & b_{33} & ..... \\ ..... & & & \\ b_{k-1,1} & b_{k-1,2} & & \\ b_{k,1} & & & \\ b_{k+1,1} & & & \end{array} \quad (3)$$

$$\text{Where}\quad b_{i,j}=b_{i-2,j+1}-\frac{b_{i-2,1}b_{i-1,j+1}}{b_{i-1,1}}, \quad (4)$$

$$\text{where}\ i\ge 3\ \text{and}\ 1\le j\le\left[\frac{(k-i+3)}{2}\right]$$

$b_k(S)$ may be easily constructed from the (n+1-k) th and (n+2-k) th and (n+2-k) th rows of the above to give

$$b_k(s)=b_{k+1-n.1}s^n+b_{k+2-n.1}s^{n-1}+b_{k+1-n.2}s^{n-2}+.... \quad (5)$$

Reduced order numerator:

The reduced order numerator polynomial is derived in the following sequence:

1) Choose 2k point's $s_0, s_1 ... s_{2k-1}, s_{2k} \in C$ (they can be multiple) from the location of the poles of the original systems and obtain *g(s)* as given below:

$$\begin{aligned} g(s)&=(s-s_0)(s-s_1)....(s-s_{2k-1}) \\ &=(s-s_0)^{k_0}(s-s_1)^{k_1}....(s-s_j)^{k_j} \\ &=s^{2k}+g_{2k-1}s^{2k-1}+....+g_1\ s+g_0 \end{aligned} \quad (6)$$

2) Divide $d_n(s)b_k(s)$ by g(s) to get the quotient e(s) and the remainder f(s).

3) Divide $e_n(s)a_k(s)$ by g(s) to get the quotient l(s) and the remainder h(s). It yields,

$$d_n(s)b_k(s)=g(s)e(s)+f(s) \quad (7)$$

$$e_n(s)a_k(s)=g(s)l(s)+h(s) \quad (8)$$

Where both f(s) and h(s) are polynomials of degree at most (2k-1),

$$(d_n(s)b_k(s))^{(k)}\big|_{si}=f^{(k)}\big|_{si}=(e_n(s)a_k(s))^{(k)}\big|_{si}=h^{(k)}\big|_{si}$$

$$i=0,1,...,j,\qquad k=0,1,...,k_{i-1}.\sum_{i=0}^{j}k_i=2k$$

By using the basic theorem of algebra, it is obtained that [1],

$$f(s)\equiv h(s).$$

It is found that the coefficient of each term in f(s) is the linear combination of

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
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$a_{0,}a_1....,a_{2k-1}$ and the coefficient of each term in h(s) is the linear combination of $b_0,b_1...,b_{2K-1}$. A linear system with 2k+1 unknowns and 2k equations is formed. Hence, b_k is assumed as '1' to make the no.of equations equal to the no.of unknowns.

If the coefficient matrix of the above linear system is nonsingular, then its solution $b_0,b_1...,b_k,a_1...,a_{k-1}$ can be uniquely determined by using the Cramers rule.

Numerator reduction procedure:

Step 1: Choose 2k point's s_0, s_1… s_{2k-1}, $s_{2k} \in C$ (they can be multiple) from the location of the poles of the original systems and obtain g(s) as given below:

$$g(s) = (s-s_0)(s-s_1)....(s-s_{2k-1})$$
$$= (s-s_0)^{k_0}(s-s_1)^{k_1}....(s-s_j)^{k_j}$$
$$= s^{2k} + g_{2k-1}s^{2k-1} + ... + g_1 s + g_0$$

Step 2: Compute $d_n(s)b_k(s)$ and $e_n(s)a_k(s)$, respectively,

$$d_n(s)b_k(s) = (d_{n-1}s^{n-1} + ... + d_0)(b_k s^k + ... + b_0)$$

$$= c_{k+n-1}^{(0)}s^{k+n-1} + ... + c_1^{(0)}s + c_0^{(0)}$$

$$c_0^{(0)} = b_0 d_0,$$
$$c_1^{(0)} = b_0 d_1 + b_1 d_0,$$
$$....$$
$$c_{k+n-2}^{(0)} = b_k d_{n-2} + b_{k-1} d_{n-1},$$
$$c_{k+n-1}^{(0)} = b_k d_{n-1},$$

and

$$e_n(s)a_k(s) = (e_n s^n + ... + e_0)(a_{k-1}s^{k-1} + ... + a_0)$$

$$= d_{k+n-1}^{(0)}s^{k+n-1} + d_{k+n-2}^{(0)}s^{k+n-2} + ... + d_1^{(0)}s + d_0^{(0)},$$

$$d_0^{o} = e_0 a_0$$
$$d_1^{(0)} = a_0 e_1 + a_1 e_0,$$
$$....$$
$$d_{k+n-2}^{(0)} = a_k e_{n-1} + a_{k-2} e_n,$$
$$d_{k+n-1}^{(0)} = a_{k-1} e_n.$$

Step 3: Divide $d_n(s)b_k(s)$ by g (s) to get f(s):

$$c_{k+n-1}^{(0)}s^{n-k-1} + c_{k+n-2}^{(1)}s^{n-k-2} + ...$$
$$s^{2k} + g_{2k-1}s^{2k-1} + ... + g_1 s + g_0 \sqrt{c_{k+n-1}^{(0)}s^{k+n-1} + c_{k+n-2}^{(0)}s^{k+n-2} + ... + c_{n-k-1}^{(0)}s^{n-k-1} + ... + c_0^{(0)}}$$
$$\underline{c_{k+n-1}^{(0)}s^{k+n-1} + g_{2k-1}c_{k+n-1}^{(0)}s^{k+n-2} + ... + g_0 c_{k+n-1}^{(0)}s^{n-k-1}}$$
$$c_{k+n-2}^{(1)}s^{k+n-2} + ... + c_{n-k-2}^{(1)}s^{n-k-3} + ... + c_0^{(1)}$$
$$\underline{c_{k+n-2}^{(1)}s^{k+n-2} + ... + g_0 c_{k+n-2}^{(1)}s^{n-k-2}}$$
$$c_{k+n-3}^{(2)}s^{k+n-3} + ... + c_0^{(2)}$$
$$\underline{... \quad ... \quad ... \quad ...}$$
$$c_{2k-1}^{(n-k)}s^{2k-1} + ... + c_0^{(n-k)}$$

Thus get the recursive relations:

$$c_i^{(1)} = c_i^{(0)} - c_{k+n-1}^{(0)} g_{i+k-n+1},$$
$$i = 0,1,..., k+n-2,$$
$$c_i^{(2)} = c_i^{(1)} - c_{k+n-2}^{(1)} g_{i+k-n+2},$$
$$i = 0,1,..., k+n-3,$$
$$c_i^{(3)} = c_i^{(2)} - c_{k+n-3}^{(2)} g_{i+k-n+3},$$
$$i = 0,1,..., k+n-4,$$
$$.......$$
$$c_i^{(l)} = c_i^{(l-1)} - c_{k+n-l}^{(l-1)} g_{i+k-n+l},$$

$$c_i^{(n-k)} = c_i^{(n-k-1)} - c_{2k}^{(n-k-1)} g_{i,}$$
$$i = 0,1,\ldots,2k-1,$$

Divide $e_n(s)a_k(s)$ by g (s) to get h(s):

$$\begin{array}{r} d_{k+n-1}^{(0)} s^{n-k-1} + d_{k+n-2}^{(1)} s^{n-k-2} + \ldots \\ s^{2k} + g_{2k-1}s^{2k-1} + \ldots + g_1 s + g_0 \,\overline{\big)\, d_{k+n-1}^{(0)} s^{k+n-1} + d_{k+n-2}^{(0)} s^{k+n-2} + \ldots + d_{n-k-1}^{(0)} s^{n-k-1} + \ldots + d_0^{(0)}} \\ \underline{d_{k+n-1}^{(0)} s^{k+n-1} + g_{2k-1} d_{k+n-1}^{(0)} s^{k+n-2} + \ldots + g_0 d_{k+n-1}^{(0)} s^{n-k-1}} \\ d_{k+n-2}^{(1)} s^{k+n-2} + \ldots + d_{n-k-2}^{(1)} s^{n-k-3} + \ldots + d_0^{(1)} \\ \underline{d_{k+n-2}^{(1)} s^{k+n-2} + \ldots + g_0 d_{k+n-2}^{(1)} s^{n-k-2}} \\ d_{k+n-3}^{(2)} s^{k+n-3} + \ldots + d_0^{(2)} \\ \underline{\ldots \quad \ldots \quad \ldots \quad \ldots} \\ d_{2k-1}^{(n-k)} s^{2k-1} + \ldots + d_0^{(n-k)} \end{array}$$

Thus get the recursive relations:

$$d_i^{(1)} = d_i^{(0)} - d_{k+n-1}^{(0)} g_{i+k-n+1,}$$
$$i = 0,1,\ldots,k+n-2,$$
$$d_i^{(2)} = d_i^{(1)} - d_{k+n-2}^{(1)} g_{i+k-n+2,}$$
$$i = 0,1,\ldots,k+n-3,$$
$$d_i^{(3)} = d_i^{(2)} - d_{k+n-3}^{(2)} g_{i+k-n+3,}$$
$$i = 0,1,\ldots,k+n-4,$$
$$\ldots\ldots$$
$$d_i^{(l)} = d_i^{(l-1)} - d_{k+n-l}^{(l-1)} g_{i+k-n+l,}$$

$$d_i^{(n-k)} = d_i^{(n-k-1)} - d_{2k}^{(n-k-1)} g_{i,}$$
$$i = 0,1,\ldots,2k-1,$$

When k <0, let $g_o=0$. In the above the recursive relations, the superscript n in $c_i^{(n)}$ represent the coefficients which are obtained after carrying out the algorithm n steps. And the subscript 'i' in $c_i^{(n)}$ represents the corresponding degree about the variables.

Step 4: By using the basic theorem of algebra, it is obtained that [1], [10]

$$f(s) \equiv h(s).$$

It is found that the coefficient of each term in *f(s)* in is the linear combination of $a_{0,} a_1 \ldots, a_{2k-1}$ and the coefficient of each term in *h(s)* is the linear combination of $b_0, b_1 \ldots, b_{2K-1}$.

A linear system with *2k+1* unknowns and *2k* equations is formed assuming $b_k = 1$. Thus, the reduced model is obtained as given equation *(2).*

4 NUMERICAL EXAMPLES

Example 1: consider the sixth order system as given

$$G_6(s) = \frac{s^4 + 13s^3 + 63s^2 + 133s + 102}{s^6 + 14.5s^5 + 81s^4 + 223s^3 + 318s^2 + 212.5s + 50} = \frac{d_n(s)}{e_n(s)}$$

A Second order reduced model is obtained for the above higher order system, in following steps, using the proposed method given in section-3.

$$R_2(s) = \frac{a_0 + a_1 s}{b_0 + b_1 s + b_2 s^2} = \frac{a_k(s)}{b_k(s)}$$

Step 1: Reduced Order denominator is obtained by using Routh method as below:

$$e_n(s) = s^6 + 14.5s^5 + 81s^4 + 223s^3 + 318s^2 + 212.5s + 50$$

Routh Table:

s^6	1	81	318	50
s^5	14.5	223	212.5	
s^4	65.62	303.3	50	
s^3	155.9	201.2		
s^2	218.6	50		
s^1	165.5			
s^0	50			

Hence, the reduced order denominator is:

$$b_k(s) = 218.6s^2 + 165.5s + 50$$

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
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By normalizing the above, the reduced order denominator is:

$$b_k(s) = s^2 + 0.7574s + 0.2288$$

Step 2: The numerator of reduced order model is obtained by the interpolation method as given in proposed procedure

For a 2nd order model the 4 required interpolation points are selected randomly as:

0, 0.6, 2, 3

$$g(s) = s^4 - 5.6s^3 + 9s^2 - 3.6s$$

Where g(s) is the polynomial obtained by the selected interpolation points.

From original order numerator and reduced order denominator,

$$d_n(s)b_k(s) = (s^4 + 13s^3 + 63s^2 + 133s + 102)(s^2 + 0.7574s + 0.2288)$$

$$d_n(s)b_k(s) = 4.372s^6 + 60.15s^5 + 319.5s^4 + 803s^3 + 949.2s^2 + 470.6s + 102$$

Divide $d_n(s)b_k(s)$ by g (s) to get the quotient e(s) and the remainder f(s).

Thus,

$$f(s) = 4276.42s^3 - 5523.93s^2 + 3181.04s + 102 \qquad (1)$$

From original order numerator and reduced order denominator,

$$e_n a_k = (s^6 + 14.5s^5 + 81s^4 + 223s^3 + 318s^2 + 212.5s + 50)(a_1 s + a_0$$

$$e_n a_k = a_1 s^7 + (14.5a_1 + a_0)s^6 + (81a_1 + 14.5a_0)s^5 + (223a_1 + 81a_0)s^4 + (318a_1 + 223a_0)s^3 + (212.5a_1 + 318a_0)s^2 + (50a_1 + 21.5a_0)s + 50a_0$$

Divide $e_n a_k$ by g(s) to get the quotient l(s) and the remainder h(s).

Thus,

$$h(s) = (1079.23a_0 + 4773a_1)s^3 + (1270.68a_0 + 8296.1a_1)s^2 + (876.91a_0 + 3935.228a_1)s + 50a_0 \qquad (2)$$

From equn (1) & equn (2), we get

$$a_0 = 0.0809$$
$$a_1 = 0.466$$

Hence, the reduced model is:

$$R_2(s) = \frac{0.0809s + 0.466}{s^2 + 0.75838s + 0.2287}$$

The corresponding ISE = 0.148

The procedure is repeated with another set of interpolation points and the process is repeated until optimum value of ISE is attained.

The optimum reduced order model is obtained as, [11]

$$R_2(s) = \frac{0.261988s + 0.46629}{s^2 + 0.75838s + 0.2287}$$

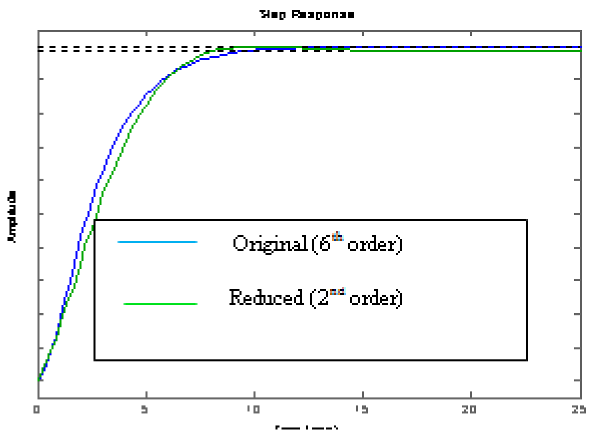

Fig1. Comparison of step response of original and

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Example 2: Consider the 8th order system as follows:

$$G(s) = \frac{d_n(s)}{e_n(s)}$$

$$d_n(s) = 0.01141s^7 + 0.5119s^6 + 2.4152s^5 + 4.918s^4 + 6.2164s^3 + 4.6146s^2 + 1.7134s + 0.261$$

$$e_n(s) = s^8 + 9.83s^7 + 36.616s^6 + 65.852s^5 + 73.018s ... + 50.03s^3 + 17.104s^2 + 1.919s + 0.25$$

A Second order reduced model is obtained for the above higher order system, in following steps, using the proposed method given in section-3.

$$R_2(s) = \frac{a_0 + a_1 s}{b_0 + b_1 s + b_2 s^2} = \frac{a_k(s)}{b_k(s)}$$

By normalizing the above, the reduced order denominator is(by applying routh method):

$$b_k(s) = s^2 + 0.07881s + 0..0186$$

Reduced order numerator (by applying Interpolatic method):

Using the procedure the optimum reduced order model is obtained as,

$$R_2(s) = \frac{0.05244s + 0.0194}{s^2 + 0.07881s + 0.0186}$$

Original Order system	Interpolation points	Reduced order system	ISE
$G(s) = \frac{s^4 + 13s^3 + 63s^2 + 133s + 102}{s^6 + 14.5s^5 + 81s^4 + 223s^3 + 318s^2 + 21}$	0,-0.5,0.9+1.78i, 0.9-1.78i	$R_2(s) = \frac{1.34s + 0}{s^2 + 0.7583}$	0.186
$G(s) = \frac{s^4 + 13s^3 + 63s^2 + 133}{s^6 + 14.5s^5 + 81s^4 + 223s^3 + 31}$	0,0.6,2,3	$R_2(s) = \frac{0.0809}{s^2 + 0.7583}$	0.148
$G(s) = \frac{s^4 + 13s^3 + 63s^2 + 133}{s^6 + 14.5s^5 + 81s^4 + 223s^3 + 31}$	0,-3.5289, 0.11+1.78i, 0.11-1.78i	$R_2(s) = \frac{0.261988}{s^2 + 0.7583}$	0.04234

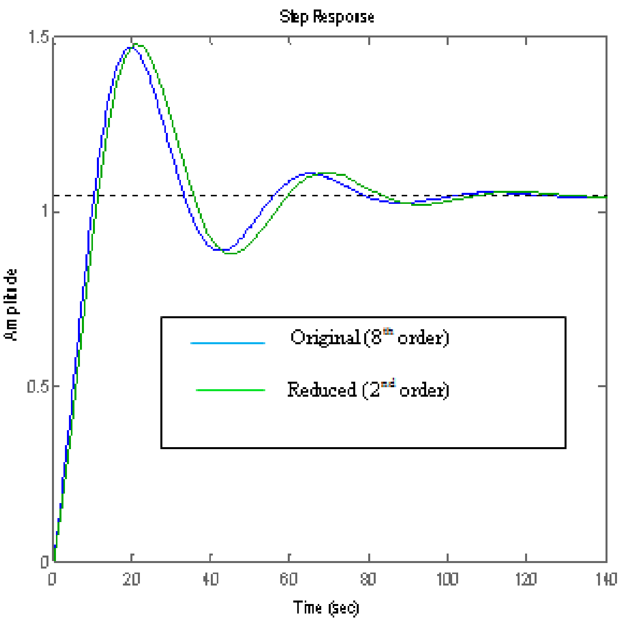

Fig 2 Step response of original and reduced order systems.

Design of PID controller for reduced order model is,

Let transfer function of the PID controller is,

$$G_c(s) = \frac{k_d s^2 + k_p s + k_i}{s}$$

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Applying ITAE performance index method to the reduced model, the values of Kp, Ki and Kd are obtained

$$R_2(s) = \frac{0.05244s + 0.0194}{s^2 + 0.07881s + 0.0186}$$

Now to obtain the closed-loop transfer function for reduced order

The tuned PID values are

$K_p = -9.1331, \quad K_i = 7.654, \quad K_d = 23.899$

Comparing characteristic equation of compensated system to the optimum ITAE characteristic equation as,

$$s^3 + 1.75w_n s^2 + 3.25w_n^2 s + w_n^3$$

The PID controller is added to the forward path and the closed loop transfer function with unity feedback of the system is given as:

$$T_c(s) = \frac{G_C(S)G(S)}{1 + G_C(S)G(S)}$$

Where *G(s)* is the high order system and *$G_c(s)$* is the PID controller transfer function.

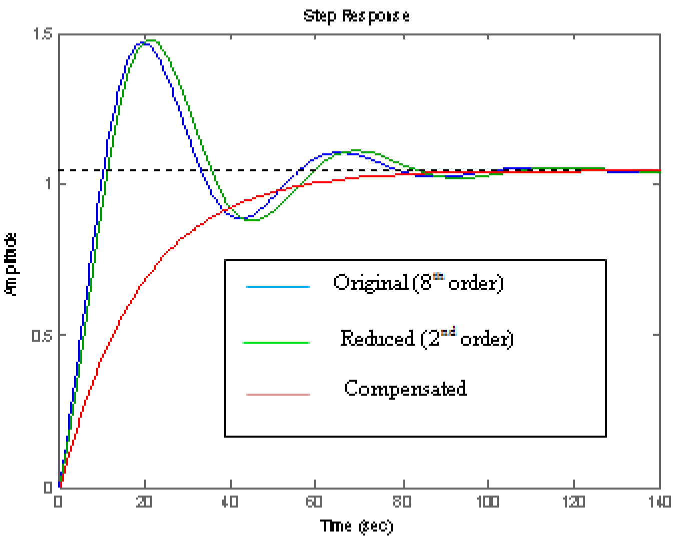

Fig 3 Step response of compensated system and uncompensated systems

CONCLUSION

In this paper, a new method is proposed for the reduction of high order continuous systems. The proposed method uses the application of Interpolation method for obtaining the numerator and Routh method for obtaining the denominator of the reduced order model. An optimum model (with minimum ISE) is derived by varying the interpolation points. Using the proposed method a PID controller is designed for the high order original systems. A computer program is developed for the proposed continuous systems reduction method.

REFERENCES

[1] GU Chuan-quig, ZHANG Ying, “AnOsculatory rational interpolation method of model reduction in linear system”, *Journal of Shanghai University (English Edition)*, 2007, 11(4):365-369...

[2] WU Bei-Bei, GU Chuan-quig. ‘A matrix Pade Type Routh Model Reduction Method for Multivariable Linear Systems’ Journal of Shanghai University (English Edition), 2006, 10(5) : 377-380.

[3] Ismail O. On multipoint Pade approximation for discrete interval systems [C]//Proceedings of the Twenty Eighth Southeastern Symposium on System Theory Washington DC, USA: IEEE Computer Society, 1996: 497-501.

[4] Bandyopadhyay B, Ismail O, Gorez R. Routh-Padeapproximation for interval systems [J]. IEEE Transaction Automatic Control, 1994, 39: 2454-2456.

ISSN No: 2348-4845
International Journal & Magazine of Engineering, Technology, Management and Research
A Peer Reviewed Open Access International Journal

[5] Mohammad J. Large –scale Systems: Modeling and Control [M]. New York: Elsevier Science Publishing Press, 1983.

[6] Hutton M F, Friedland B. Routh approximations for reducing order of linear time – invariant system [J]. IEEE Transaction Automatic Control, 1975, 20: 329-337.

[7] Baker G A, Jr, Graves – Morris P R. Pade Approximants [M]. 2nd ed. New York: Cambridge University Press, 1995.

[8] Gu Chuan-qing. Thiele-type and Lagrange-type generalized inverse rational interpolation for rectangular complex matrices [J]. Linear Algebra and Its Application, 1999, 295: 7-30.

[9] J.V.B. Jyothi, G. Raja Rao, 'Approximation of Large Time Delay MIMO Systems by using Matrix Pade-Type Routh Model' , in proceedings of National Conference on CONTEMPORARY AND SOFTCOMPUTING in Electrical Engineering (ConCon-2012) May 30-31.

[10] Interpolation among reduced-order matrices to obtain parameterized models for design, optimization and probabilistic analysis, INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS Int. J. Number. Meth. Fluids 2010; 63:207–230 published online 8 June 2009 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.2089.

[11] Parametric Model Order Reduction by Matrix Interpolation, at 8/ 2010, Heiko Panzer, Technische Universidad Munched, Jan Mohring, Fraunhofer Institute fur Techno- und Wirtschaftsmathematik, Kaiserslautern, Rudy Eid, Boris Lehmann, and Technische Universidad Munched.

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