Reducing Timing Jitter in OFDM Systems Using Oversampling

J. Harshitha
M.Tech in CESP, RVR & JC College of Engineering, Chandramoulipuram, Chowdavaram, Guntur Dist, Andrapradesh, India.

N. Renuka
Associate Professor, Dept of ECE, RVR & JC College of Engineering, Chandramoulipuram, Chowdavaram, Guntur Dist, Andrapradesh, India.

ABSTRACT:

Timing jitter is a major issue in very high speed OFDM systems, which causes serious impairments that degrade the performance of overall system. In order to reduce the effect caused by timing jitter, here in this proposal, we introduced a novel scheme that reduces the noise caused by the effect of timing jitter by using integral oversampling under different channel environments such as AWGN, Rayleigh and Rician channels. Simulation results have shown that the average jitter noise which is affected by timing jitter occurrence has reduced under various channel conditions. Also compared the performance of channels in terms of sub carrier index versus powerIndex Terms: OFDM, Timing Jitter, Integral oversampling, AWGN, Rayleigh and Rician channels.

I.INTRODUCTION:

Due to the multi path fading effect and receiver complexity of wireless channels, the traditional modulation techniques which are based on single carrier can achieve limited data rates. In many wireless multime dia network applications [1], the high data-rates were desirable. However, the symbol duration will get reduced whenever there is an increment in communication systems data rates. Therefore, the communication systems using single carrier modulation suffer from severe inter symbol interference (ISI) caused by dispersive channel impulse response, thereby needing a complex equalization mechanism. Orthogonal Frequency Division Multiplexing (OFDM) is a special form of multi carrier modulation technique, in which the total frequency selective fading channel will be divided into many orthogonal narrow band flat fading sub channels. In OFDM system high-bit-rate data stream is transmitted in parallel over a number of lower data rate subcarriers and do not undergo ISI due to the long symbol duration[2]. Major advantages of OFDM systems are

- High spectral efficiency
- Simple digital realization by using the FFT operation
- Due to the ISI avoidance, the complexity in the receiver will be reduced
- Various modulation schemes will be used to achieve the best performance of the system

Due to the above mentioned advantages, OFDM has been used in many wireless applications such as Wireless Personal Area Network (WPAN), Wireless Metropolitan Area Network (WMAN), Digital Audio Broadcasting (DAB) [3], Wireless Local Area Network (WLAN) [4] and Digital Video Broadcasting (DVB) [5]. It is also useful for 3GPP-LTE, IEEE 802.20 and 802.16 [6], [7]. With the use of cyclic prefix for eliminating the effect of ISI, there is a need for a simple one tap equalizer at the OFDM receiver. OFDM brings in unparalleled bandwidth savings, which leads to high spectral efficiency.

In Recent years optical systems uses OFDM for higher data rates (see [8] and the references therein) since, data rates in fiber optic systems are typically much higher than the Radio Frequency (RF) wireless systems. Timing jitter is a major problem and it causes serious impairments at these very high data rates, which in results the poor efficiency and degraded performance of the system. Sampling clock is the major source of jitter in the very high speed analog-to-digital converters (ADCs) which are required in these systems. It causes many serious issues in OFDM radios which uses high frequency band pass sampling [9]. In [10] and [11-15] authors analyzed the effect of timing jitter. These papers focus on the colored low pass timing jitter which is typical of systems using phase lock loops (PLL).
They consider only integral oversampling. By leaving some band-edge sub carriers which will be unused, the fractional oversampling can be achieved in OFDM. Here in this proposal we investigated the integral oversampling to reduce the timing jitter. Very high speed ADCs typically uses parallel pipeline architecture not a PLL\cite{13} and for these the white jitter which is the focus of this paper is a more appropriate model.

**SYSTEM MODEL:**

Fig.1 shows the block diagram of a high speed OFDM system, $T$ denotes the OFDM symbol period, in each $T$ up to $N$ complex values representing the constellation points are used to modulate up to $N$ sub carriers. In Practical OFDM systems, the jitter can be introduced at a number of points. But, here in this proposal we considered that the jitter is introduced at sampler block of the receiver ADC. The definition of timing jitter has been shown in fig.2. Ideally the OFDM signal is sampled at uniform intervals of $T/N$. In fig.2 (a) the uniform sampling intervals has been represented by dashed lines, where the actual sampling times represented by solid lines. The deviation between the actual sampling times and uniform sampling times is caused by the effect of timing jitter, it denoted by $\tau_n$. Discrete timing jitter example is shown in fig.2 (b).

In \cite{5}, \cite{14} and \cite{15} we showed that the timing jitter matrix can describe the effect of timing jitter in OFDM systems. The constrict matrix form for OFDM with timing jitter is

$$Y=WHX^T+N$$

Where,

$Y$= received signal

$X$= transmitted signal

$N$= Additive white Gaussian signal

$$Y=[Y_{((-N)/2+1)}...Y_0...Y_{(N/2)}]^T$$

$$X^T=[X_{((-N)/2+1)}...X_0...X_{(N/2)}]^T$$

$$H=\text{diag}(H_{((-N)/2+1)}...H_0...H_{(N/2)})$$

Timing jitter causes an added noise like component in the received signal.

$$Y = HX^T + (W-I)HX^T + N$$

(3)

Where $I$ is the $N \times N$ identity matrix. In eq. (3), the first term represents wanted term while the second term indicates jitter noise. In it was shown that the elements of the timing jitter matrix $W$ are given by

$$W_{i,k} = \frac{1}{N} \sum_{n=-N/2}^{N/2-1} e^{j2\pi kn/T} e^{j2\pi (k-i)n}$$

(4)

Where $n$ is the time index, $k$ is the transmitted subcarrier index and $l$ is the received subcarrier index.

**ANALYSIS OF TIMING JITTER:**

We now analyze the effect integral oversampling in OFDM and show that it will reduce the impairments caused by timing jitter. The received signal will be sampled with $MN/T$ to achieve the integral oversampling, where $M$ is an integer. Some band edge sub carriers are unused in the transmitted signal for fractional oversampling. The band width of
the OFDM signal is $N/2T$, when all $N$ subcarriers are modulated. So as shown in fig.2 sampling at interval of Nyquist sampling $T/N$.

The bandwidth of the signal is $N_{u}+N_{d}/2T$, if instead, only subcarriers with indices between $-N_{d}$ and $N_{u}$ are non zero. In this case the sampling Interval is above Nyquist rate. In general, where both integral and fractional oversampling is applied, the signal samples after the ADC receiver are given by,

$$y_{nm} = y_{nmT} + \frac{1}{\sqrt{N}} \sum_{k=-N_{d}}^{N_{u}} H_{k} X_{k} e^{j2\pi k n_{m} T/N} + \eta \left( \frac{n_{mT}}{N_{d}} \right)$$

where $n_{m}$ is the time index of discrete oversampling and $\eta$ is the AWGN. The N-point FFT in the receiver is replaced by an ‘oversized’ $N M$-point FFT in proposed algorithm. The output of this FFT is a vector of length $N M$ with elements

$$y_{nm} = \frac{1}{\sqrt{N_{d}}} \sum_{n_{m}=-N_{d}+1}^{N_{d}} y_{nmT} e^{j2\pi n_{m} T/n_{m}} + \eta \left( \frac{n_{mT}}{N_{d}} \right)$$

where $I_{nm}$ is the $N M$-point FFT output index.

Then combining the equations (4), (5) and (6), the modified equation for the proposed case is obtained by,

$$y_{nm} = \frac{1}{\sqrt{N_{d}}} \sum_{n_{m}=-N_{d}+1}^{N_{d}} y_{nmT} e^{j2\pi n_{m} T/n_{m}} + \eta \left( \frac{n_{mT}}{N_{d}} \right)$$

We now calculate the each sub carrier average jitter noise power for the case of white jitter. From (3) and (6),

$$Y_{I_{nm}} = H_{I_{nm}} X_{I_{nm}} + \sum_{k=-N_{d}}^{N_{u}} \left( w_{I_{nm}} - I_{(m,k)} H_{k} X_{k} \right) + N(l)$$

Where the second term represents the jitter noise. In the following, we consider a flat channel with $H_{k} = 1$, and consider that the power of transmitted signal has been distributed equally across the used subcarriers so that for each used subcarrier $E\left[I_{k}^{2}\right] = \alpha_{l}^{2}$.

Then the average jitter noise power, $P_{j}(l)$ to received signal power of $I_{nk}$ subcarrier is given by,

$$P_{j}(l) = \frac{E\left[I_{k}^{2}\right]}{\sigma_{l}^{2}} = \frac{E\left[I_{k}^{2}\right]}{\sigma_{l}^{2}}$$

For a system in which there are equal number of unused subcarriers at each band edge, so $N_{d} = N_{d}/2$, $N_{d} = N_{d}/2 + 1$.

Where $N_{d}$ is number of used subcarriers. Then the eq.(9) becomes

$$P_{j}(l) = \frac{E\left[I_{k}^{2}\right]}{\sigma_{l}^{2}} = \sum_{k=-N_{d}/2+1}^{N_{d}/2} E\left[I_{k}^{2}\right]$$

Reducing the numbers of subcarriers will results in reduction of over all data, the transmitted power and even bandwidth when the symbol period in unchanged. To make a fair comparison the symbol period is also reduced so that $T_{p} = N_{s} T_{N}/N$, where $T_{p}$ is the used sub carriers symbol period and $T_{N}$ is the symbol period of N used subcarriers.

$$P_{j}(l) = \sum_{k=-N_{d}/2+1}^{N_{d}/2} E\left[I_{k}^{2}\right]$$

Using $\Sigma_{k=-N_{d}/2+1}^{N_{d}/2} k^{2} = \frac{1}{12} N_{d} (2 + N_{d}^{2})$, we
obtain
\[
\frac{P_J(t)}{\sigma^2} = \frac{2}{3M} E\{\tau_n^2\} \left(\frac{N_u}{N}\right)^2 + \frac{1}{3M} E\{\tau_n^2\} \left(\frac{N_u^2}{N}\right)^2 \tag{12}
\]

Since the first term in eq. (12) is very small, so it can be ignored. Then the equation will be rewritten as
\[
\frac{P_J(t)}{\sigma^2} = \frac{\pi^2}{3M} E\{\tau_n^2\} \left(\frac{N_u N}{N}\right) \tag{13}
\]

If we consider that there is no oversampling then \(M=1\) and \(N_u = N\), therefore
\[
\frac{P_J(t)}{\sigma^2} = \frac{\pi^2}{3} E\{\tau_n^2\} \left(\frac{N^2}{N}\right) \tag{14}
\]

Comparing the eq. (13) and (14), it can be seen that the combination of integral and fractional oversampling reduces the jitter noise power by a factor of \(N_u / NM\).

IV. SIMULATION RESULTS:
In this section we present the simulations, which have been tested in MATLAB 2011a version. Here we had considered 2000 OFDM symbols, \(N = 512\) subcarriers and \(E\{\tau_n^2\} = (0.37N_u / N)^2\) jitter variance, which will not vary when we apply oversampling. Fig. 3 shows that the variance of the noise due to the jitter as a function of index of received subcarriers when band-edge subcarriers are unused, which shows that the index of subcarrier and that removing the band-edge subcarriers reduces the noise equally across the all subcarriers. Fig. 4 shows that the simulation and theoretical results of average jitter noise power as a function of the oversampling factor. When you observe the graph, we can conclude that for every doubling of sampling rate reduces the jitter noise power by 3dB.
V. CONCLUSIONS:

It has been shown that the proposed algorithm for timing jitter noise power reduction shows better performance and it reduces the impairments caused by the timing jitter in OFDM systems. In this, we investigated oversampling mathematically by deriving the equations to them in the proposed oversampling technique, which was implemented by receiver sampling rate increment. Finally, it shows that the proposed algorithm has given reduced noise power of 3dB for every doubling of sampling rate under various channel conditions such as AWGN, Rayleigh and Rician.

REFERENCES:


