

Fluid Dynamics: MHD Flows Through or Past a Porous Media



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ABSTRACT:

The study of fluid dynamics is core of the most important branches of research in and applied sciences because of its wide range of applications such as in astrophysical, geophysical, aero dynamic problems. In meteorology hydrology, and oceanography the study of fluids is basic since the atmosphere and the ocean are fluids. The study of fluids through or past porous medium assumed importance because of its importance applications in diverse fields of science, engineering and technology. The practical applications are in the percolation of water through soil extraction and filtration of oils from wells, the drainage of water, irrigation sanitary engineering and also in the inter disciplinary fields such as biomedical engineering etc. The flow in as porous medium is governed by Darcy's law or Brinkman model.

The classical Darcy's law [Muskat[2]] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as $V = \frac{-K}{\mu} \nabla P$. The flow gives good results in the solutions when the flow is unidirectional or the flow is at low speed. In general the specific discharge increases the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortion in the velocity field in the case of highly porous media such as fiber glass, papers of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by Beavers and Joseph[6] saffman[10] and others. A generalized Darcy's law proposed by Brinkman is given by $0 = -\nabla p - \left(\frac{\mu}{K}\right)v + \mu \nabla^2 v$ where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium. The generalized equation for the flow through the porous medium is $p \left(\frac{\partial v}{\partial t} + (v \cdot \nabla)v\right) = \nabla p + \mu \nabla^2 v - \left(\frac{\mu}{K}\right)v$. The classical Darcy's law helps in studying flows through porous medium. In the case of highly porous medium such as porous of dandelion etc, the Darcy's law fails to explain the flow near the surface in the absence of pressure gradient. The non-Darcy approach is employed to study the problem of flow through highly porous medium by several investigation [12, 15, 9 and 14]

The study of magneto hydrodynamic flows through or past porous media is of considerable interest because of its abundant application in several branches of science and technology, such as Astrophysical, Geophysical, ground water flow, petroleum engineering problems and in developing magnetic generators for obtaining electrical energy at minimum cost. The development of MHD generators needs the study of the effect of magnetic field on various flow patterns. Hartmann [1] studied the problem of steady magneto hydro dynamic channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall.

Hughes and young [5] considered the problem of flow through a rectangular channel bounded by walls which are infinitely conduction or perfectly insulation. The genera problem of the rectangular channel flow has been discussed considering various cases of insulating conducting walls. Jagadeesed [4] studied the hydro-magnetic coquette flow between two conducting porous walls. Chandrasekhara investigated the study MHD flow of viscous incompressible fluid in a squeeze film bounded above by a porous thin plate. Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems, etc. buoyancy is also of importance in an environment where difference between land and air temperatures can give rise to complicated flow patterns. Magneto hydro-dynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures interstellar matter, radio propagation through the ionosphere, etc. in engineering it finds its application in MHD pumps, MHD bearing etc.

The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. The thermal physics of hydro magnetic Radiative flows are encountered in countless industrial and environment processes, e.g., heating and cooling chambers, fossil fuel combustion energy process, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. In view of the applications in industries, science and engineering fields the applicant plans to study some MHD flows through or past porous media.

I. INTRODUCTION:

The viscous flow through porous media occurs in many industrial situations and has got several important scientific and engineering applications such

as flow through packed beds and ion-exchange beds, extraction of energy from the geothermal regions, filtration of solids from liquids. Literature survey reveals that most of the research works available in flow through porous media is confined to undeformable porous media and the work on deformable porous media is very limited. The coupled phenomenon of fluid flow and deformation of porous materials is a problem of prime importance in geomechanics and biomechanics. One such application of interaction of free flow and deformable porous media is the study of hemodynamic effect of the endothelial glycocalyx. In view of these applications Terzaghi [1] was the first among others who initiated the study of flow through deformable porous materials and subsequently Biot [2–4] continued the work of Ref. [1] and proposed a successful theory of soil consolidation and acoustic propagation. Further, Atkin and Craine [5], Bowen [6] and Bedford and Drumheller [7] made some important contributions to the theory of mixtures. Jayaraman [8] extended the work of Biot [2] to water transport in the artery wall.

Mow et al. [9,10] and Holmes and Mow [11] developed a similar theory for the study of biological tissue mechanics and rectilinear cartilages. Sreenadh et al. [12] analyzed the Couette flow of a viscous fluid in a parallel plate channel in which a finite deformable porous layer is attached to the lower plate. It is found that the increase in the volume fraction component of fluid phase reduces the magnitude of velocity in the free flow region of the horizontal channel. All the above mentioned researchers restricted their analyses to Newtonian fluid flow through deformable porous media. It is essential to note that most of the technological indus-tries prefer non-Newtonian fluids. Prasad et al. [13,16] have done extensive work on porous media considering non-Newtonian fluid with different physical situation. Further, it is evinced from surveys that biofluids are classified as non-Newtonian fluids. Numerous researchers conveniently used Jeffrey model to explain the biological fluid flow in living organisms.

Peristaltic transport of a Jeffrey fluid under the influence of transverse magnetic field in an asymmetric channel was analyzed by Kothandapani and Srinivas [17] and Nadeem and Akbar[18] whereas Hayat and Ali [19] analyzed the same effects in a tube. Nadeem et al. [20] examined the effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface. Vajravelu et al. [21] explained the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Hayat et al. [22] studied the boundary layer flow of a Jeffrey fluid with convective boundary conditions. The effect of magnetic field on the peristaltic pumping of a Jeffrey fluid in an inclined channel was analyzed by Krishna Kumari et al. [23]. Recently, Bhaskara Reddy et al. [24] studied the flow of a Jeffrey fluid between torsionally oscillating disks and Santhosh [25] examined the flow of a Jeffrey fluid through a porous medium in narrow tube. Most recently, Vajravelu et al. [26] analyzed the influence of free convection on nonlinear peristaltic transport of a Jeffrey fluid in a finite vertical porous stratum using the Brinkman model and established that the effect of viscous and Darcy dissipations is to reduce the rate of heat transfer in the finite vertical porous channel under peristalsis. In view of the above studies, the present paper deals with the effect of deformable porous layer on the classical Couette flow of a Jeffrey fluid between two parallel plates. MHD flow of a Jeffrey fluid between a deformable porous layer and a moving rigid plate is investigated. The fluid velocity, displacement of the solid matrix, mass flux and its fractional increase are obtained. The effects of various physical parameters on the flow quantities are discussed through graphs and Tables.

II. MATHEMATICAL FORMULATION

Consider, a steady, fully developed Couette flow through a channel with solid walls at $y = -L$ and $y = h$ and deformable porous layer of thickness L attached to the lower wall as shown in Fig. 1. The flow over the deformable layer is bounded above by a rigid plate moving with velocity U_0 .

The flow region between the plates is divided into two regions. The flow region between the lower plate $y = -L$ and the interface $y = 0$ is termed as deformable porous layer whereas the flow region between the interface $y = 0$ and the upper plate $y = h$ is the free flow region. The fluid velocity in the free flow region and in the porous flow region are assumed respectively as $(q, 0, 0)$ and $(v, 0, 0)$. The displacement due to the deformation of the solid matrix is taken as $(u, 0, 0)$. A pressure gradient $\frac{\partial p_x}{\partial x} = G_0$ is applied, producing an axially directed flow in the channel. Further, a uniform transverse magnetic field of strength B_0 is applied perpendicular to the walls of channel. The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = -\bar{p}\bar{I} + \bar{s}, \bar{s} = \frac{\mu}{1+\lambda_1} \left(\dot{\bar{\gamma}} + \lambda_2 \ddot{\bar{\gamma}} \right)$$

where T and s are the Cauchy's stress tensor and extra stress tensor respectively, p is the pressure, I is the identity tensor, λ_1 is the ratio of relaxation to retardation time, λ_2 is the retardation time, γ is shear rate, and dots over the quantities indicate differentiation with respect to time. In view of the assumptions mentioned above, the equations of motion in the deformable porous layer and free flow region are (See for details Barry et al. [27] and Ranganatha et al. [28]).

$$\mu \frac{\partial^2 u}{\partial y^2} - \phi^f G_0 + Kv = 0, \tag{1}$$

$$\frac{2\mu_a}{1+\lambda_1} \frac{\partial^2 v}{\partial y^2} - \phi^f G_0 - Kv - \sigma B_0^2 v = 0 \tag{2}$$

$$\frac{\mu_f}{1+\lambda_1} \frac{\partial^2 q}{\partial y^2} - \sigma B_0^2 q = G_0 \tag{3}$$

The boundary conditions are

$$\text{at } y = -L: v = 0, u = 0$$

$$\text{at } y = 0: q = \phi^f v, \phi^f \mu_f \frac{dq}{dy} = 2\mu_a \frac{dv}{dy}, \mu_f \frac{dq}{dy} = \frac{\mu}{\phi^f} \frac{du}{dy}$$

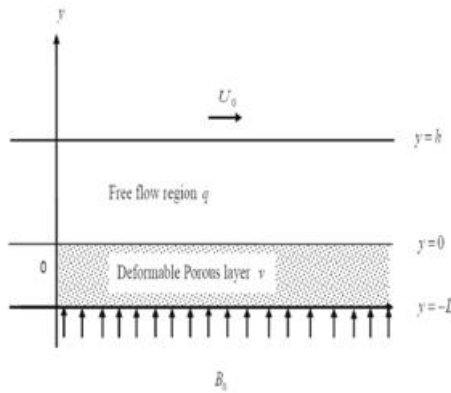


Fig.1 Physical Model

$$\text{at } y = h: q = U_0 \quad (4)$$

III. NONDIMENSIONALIZATION OF THE FLOW QUANTITIES

It is convenient to introduce the following non-dimensional quantities.

$$y = h\hat{y}, u = -\frac{h^2 G_0}{\mu} \hat{u}, v = -\frac{h^2 G_0}{\mu_f} \hat{v}, q = -\frac{h^2 G_0}{\mu_f} \hat{q},$$

$$\varepsilon = \frac{L}{h}, U_0 = -\frac{h^2 G_0}{\mu} \hat{U}_0, \hat{\tau} = -\frac{\tau}{h G_0}$$

In view of the above dimensionless quantities, after neglecting the hats (\wedge), the Eqs. (1)–(4) take the following form

$$\frac{d^2 u}{dy^2} = -\phi^f - \delta v \quad (5)$$

$$\frac{d^2 v}{dy^2} = (1 + \lambda_1) \eta \left[(\delta + M^2) v - \phi^f \right] \quad (6)$$

$$\frac{d^2 q}{dy^2} - M^2 (1 + \lambda_1) q = -(1 + \lambda_1) \quad (7)$$

where $M^2 = \frac{\sigma B_0^2 h^2}{\mu_f}, \delta = \frac{K h^2}{\mu_f}, \hat{G} = \frac{G}{G_0}, \eta = \frac{\mu_f}{2\mu}, G_0 = \frac{dp}{dx}$.

The parameter δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium.

The parameter η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer. The boundary conditions are

$$\text{at } y = -\varepsilon: v = 0, u = 0; \quad (8a)$$

$$\text{at } y = 0: q = \phi^f v, \frac{dq}{dy} = \frac{1}{\eta \phi^f} \frac{dv}{dy}, \frac{dq}{dy} = \frac{1}{\phi^f} \frac{du}{dy} \quad (8b)$$

$$\text{at } y = 1: q = U_0 \quad (8c)$$

IV. SOLUTION OF THE PROBLEM

Equations (5)–(7) are coupled with differential equations that can be solved by using the boundary conditions (8a). The solid displacement and fluid velocities in the free flow region and deformable porous layer are obtained as below,

$$u(y) = -\phi^f \frac{y^2}{2} - \frac{\delta c_3 e^{by}}{b^2} - \frac{\delta c_4 e^{-by}}{b^2} - \frac{\phi^f \delta}{\delta + M^2} \frac{y^2}{2} + c_5 y + c_6 \quad (9)$$

$$q(y) = c_1 e^{ay} + c_2 e^{-ay} + \frac{1}{M^2} \quad (10)$$

$$v(y) = c_3 e^{by} + c_4 e^{-by} + \frac{\phi^f}{\delta + M^2} \quad (11)$$

where $a = M\sqrt{1 + \lambda_1}$ and $b = \sqrt{((1 + \lambda_1)\delta + a)}$ the constants c_1, c_2, c_3, c_4, c_5 and c_6 found from the boundary conditions, are

$$c_1 = \frac{(a_2 b_2 - b_1) + a_2 a_3 (c_3 + c_4)}{a_2 - a_1}, c_2 = \frac{b_1 - a_1 c_1}{a_2}$$

$$c_3 = \frac{b_3 [(a_7 a_4 (a_2 - a_1) + a a_2 a_3 (a_2 + a_1)) + a_8 [a (a_2 + a_1) (a_2 b_2 - b_1) - a b_1 (a_2 - a_1)]]}{a_7 [(a_7 a_4 (a_2 - a_1) + a a_2 a_3 (a_2 + a_1)) + a_8 [(a_7 a_4 (a_2 - a_1) - a a_2 a_3 (a_2 + a_1))]]}$$

$$c_4 = \frac{b_3 - a_7 c_3}{a_8}, c_5 = \frac{a(c_1 - c_2) + a_6(c_3 - c_4)}{a_{12}}, c_6 = a_9 c_3 + a_{10} c_4 + a_{11} c_5 - b_4$$

$$a_1 = e^a, a_2 = e^{-a}, b_1 = U_0 - \frac{1}{M^2}, a_3 = \phi^f, a_4 = \frac{b}{\eta \phi^f}, b_2 = \frac{(\phi^f)^2}{\delta + M^2} - \frac{1}{M^2},$$

$$a_6 = \frac{\delta}{\phi^f b}, a_7 = e^{-b\varepsilon}, a_8 = e^{b\varepsilon}, b_3 = \frac{-\phi^f}{\delta + M^2}, a_9 = \frac{\delta e^{-b\varepsilon}}{b^2}, a_{10} = \frac{\delta e^{b\varepsilon}}{b^2},$$

$$a_{11} = \varepsilon \text{ and } b_4 = -\left(\frac{\phi^f \varepsilon^2}{2} + \frac{\phi^f \delta}{\delta + M^2} \left(\frac{\varepsilon^2}{2} \right) \right).$$

V. MASS FLOW RATE

(i) Mass flow rate with deformable porous layer

The dimensionless mass flow rate M_d per unit width of the channel in the free flow region

($0 \leq y \leq 1$) is given by:

$$M_d = \int_0^1 q dy = \frac{(c_1 e^a - c_2 e^{-a} - (c_1 - c_2))}{a} + \frac{1}{M^2} \quad (12)$$

(ii) Mass flow rate in absence of deformable porous layer

The fluid velocity q_r for the MHD Couette flow of a Jeffrey fluid between parallel plates $y = 0$ and $y = 1$ is obtained on solving equation (7) subject to the boundary conditions

$$\text{at } y = 0 : q_r = 0$$

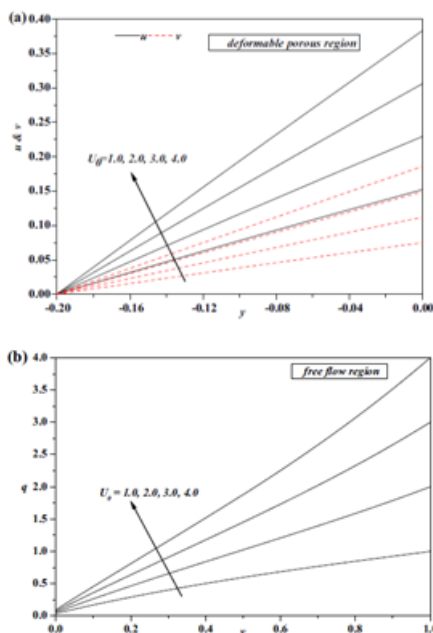


Fig. 2 a Velocity and displacement profiles for different values of U_0 with $\delta = 2.0$, $\varepsilon = 0.2$, $\lambda_1 = 0.5$, $M = 1.0$, $\eta = 0.5$, $\phi f = 0.5$, b velocity profile for different values of U_0 with $\delta = 2.0$, $\varepsilon = 0.2$, $\lambda_1 = 0.5$, $M = 1.0$, $\eta = 0.5$, $\phi f = 0.5$

$$\text{at } y = 1 : q_r = U_0$$

It can be seen that $q_r = Ae^{ay} + Be^{-ay} + \frac{1}{M^2}$ where $A = \frac{-1}{M^2} - B$, $B = \frac{1 - e^a - M^2 U_0}{M^2(e^a - e^{-a})}$.

The dimensionless mass flow rate M_r per unit width of the channel in the free flow region ($0 \leq y \leq 1$) is given by:

$$M_r = \int_0^1 q_r dy = \frac{(1 + \lambda_1) \left(\sqrt{(1 + \lambda_1) M^2 + (M^2 U_0 - 2)} \right) \tanh \left(\frac{1}{2} \sqrt{(1 + \lambda_1) M^2} \right)}{((1 + \lambda_1) M^2)^{3/2}} \quad (13)$$

Let F denote the fractional increase in mass flow rate due to deformable porous layer and it is defined by;

$$F = \frac{M_d - M_r}{M_r} \quad (14)$$

Shear Stress

The shear stress in the free flow region in non-dimensional form is given by

$$\tau = \frac{1}{1 + \lambda_1} \left(\frac{dq}{dy} \right)$$

and the shear stress at the upper plate is

$$\tau_1 = (\tau)_{y=1} = \frac{a}{1 + \lambda_1} (c_1 e^a - c_2 e^{-a}) \quad (15)$$

VI. RESULTS AND DISCUSSIONS

The solutions for the fluid velocities q , v , in the free flow region and deformable porous layer and solid displacement of solid matrix u are evaluated numerically for different values of physical parameters such as the volume fraction of component ϕf , the viscous drag parameter δ , the viscosity parameter η , the thickness of lowerwall ε , magnetic field parameter M , Jeffrey parameter λ_1 and upper plate velocity U_0 . In order to understand the mathematical model, we present the numerical results graphically for fluid velocities q , v , in the free flow region and deformable porous layer and solid displacement of solid matrix u

with y in Figs. 2 to 6. Variations of M_d , F and τ_1 with ϕ^f are tabulated in Tables 1, 2 and 3. Figures 2a, b elucidate the variation of fluid velocities q , v and solid displacement u in the channel which is calculated from Eqs. (9)–(11), for different values of U_0 . It is observed that the increment in the upper plate velocity enhances the fluid velocities q , v and displacement of the solid matrix u in the channel. The variation of fluid velocities q , v in the channel is calculated for different values of viscosity parameter η and is exhibited in Fig. 3a, b. Here, the effect of the viscosity parameter is dominant in the lower half of the free flow region and is not

Table 1 Variation of M_d with ϕ^f for different values of U_0 and M for fixed values of $\delta = 2.0$, $\varepsilon = 0.2$, $\lambda_1 = 0.5$, $\eta = 0.5$, $M = 1$ and $\delta = 2.0$, $\varepsilon = 0.2$, $\lambda_1 = 0.5$, $\eta = 0.5$, $U_0 = 1$

| ϕ^f | M_d | | | | | | | |
|----------|-----------|-----------|-----------|-----------|----------|----------|----------|----------|
| | $U_0 = 1$ | $U_0 = 2$ | $U_0 = 3$ | $U_0 = 4$ | $M = 1$ | $M = 2$ | $M = 3$ | $M = 4$ |
| 0.2 | 0.557112 | 1.004110 | 1.451108 | 1.898106 | 0.557112 | 0.167963 | 0.086183 | 0.053692 |
| 0.3 | 0.560509 | 1.009183 | 1.457858 | 1.906532 | 0.560509 | 0.172616 | 0.090807 | 0.058205 |
| 0.4 | 0.565185 | 1.016168 | 1.467150 | 1.918133 | 0.565185 | 0.178948 | 0.097021 | 0.064194 |
| 0.5 | 0.571066 | 1.024951 | 1.478836 | 1.932721 | 0.571066 | 0.186796 | 0.104596 | 0.071381 |
| 0.6 | 0.578060 | 1.035396 | 1.492733 | 1.950070 | 0.578060 | 0.195963 | 0.113276 | 0.079464 |
| 0.7 | 0.586060 | 1.047349 | 1.508636 | 1.969922 | 0.586062 | 0.206239 | 0.122794 | 0.088147 |
| 0.8 | 0.594960 | 1.060639 | 1.526371 | 1.991995 | 0.594960 | 0.217405 | 0.132892 | 0.097159 |
| 0.9 | 0.604635 | 1.075088 | 1.545541 | 2.015995 | 0.604635 | 0.229246 | 0.143333 | 0.106268 |
| 1.0 | 0.614964 | 1.090516 | 1.566067 | 2.041619 | 0.614964 | 0.241559 | 0.153908 | 0.115284 |

Table 2 Variation of M_d and F with ϕ^f for different values of λ_1 for fixed values of $M = 1$, $\delta = 2.0$, $\varepsilon = 0.2$, $\eta = 0.5$, $U_0 = 1$

| ϕ^f | M_d | | | | F | | | |
|----------|-----------------|-------------------|-----------------|-------------------|-------------------|-----------------|-------------------|-----------------|
| | $\lambda_1 = 0$ | $\lambda_1 = 0.5$ | $\lambda_1 = 1$ | $\lambda_1 = 1.5$ | $\lambda_1 = 0.5$ | $\lambda_1 = 1$ | $\lambda_1 = 1.5$ | $\lambda_1 = 2$ |
| 0.2 | 0.540430 | 0.557112 | 0.572397 | 0.586456 | 2.692474 | 4.685919 | 6.947359 | 9.448118 |
| 0.3 | 0.543577 | 0.560509 | 0.576005 | 0.590243 | 2.714987 | 4.721755 | 6.998679 | 9.516801 |
| 0.4 | 0.547916 | 0.565185 | 0.580965 | 0.595443 | 2.745980 | 4.771022 | 7.069141 | 9.610987 |
| 0.5 | 0.553384 | 0.571066 | 0.587190 | 0.601959 | 2.784957 | 4.832865 | 7.157440 | 9.728827 |
| 0.6 | 0.559903 | 0.578060 | 0.594578 | 0.609674 | 2.831311 | 4.906249 | 7.262000 | 9.868101 |
| 0.7 | 0.567382 | 0.586062 | 0.603009 | 0.618459 | 2.884352 | 4.990002 | 7.381047 | 10.026314 |
| 0.8 | 0.575725 | 0.594960 | 0.612356 | 0.628172 | 2.943327 | 5.082853 | 7.512670 | 10.200806 |
| 0.9 | 0.584828 | 0.604635 | 0.622487 | 0.638667 | 3.007447 | 5.183483 | 7.654899 | 10.388842 |
| 1.0 | 0.594584 | 0.614964 | 0.633266 | 0.649800 | 3.075910 | 5.290558 | 7.805758 | 10.587707 |

Table 3 Variation of τ_1 at the upper wall $y = 1$ with ϕ^f for different values of λ_1 and M for fixed values of $M = 1$, $\delta = 2.0$, $\varepsilon = 0.2$, $\lambda_1 = 0.5$, $\eta = 0.5$, $U_0 = 1$ and $\delta = 2.0$, $\varepsilon = 0.2$, $\lambda_1 = 0.5$, $\eta = 0.5$, $U_0 = 1$

| ϕ^f | τ_1 | | | | | | | |
|----------|-------------------|-----------------|-------------------|-----------------|----------|----------|----------|----------|
| | $\lambda_1 = 0.5$ | $\lambda_1 = 1$ | $\lambda_1 = 1.5$ | $\lambda_1 = 2$ | $M = 1$ | $M = 2$ | $M = 3$ | $M = 4$ |
| 0.2 | 0.521925 | 0.362934 | 0.269741 | 0.209220 | 0.521925 | 1.524090 | 2.302655 | 3.109985 |
| 0.3 | 0.517923 | 0.359872 | 0.267271 | 0.207162 | 0.517923 | 1.520241 | 2.300433 | 3.108892 |
| 0.4 | 0.512412 | 0.355662 | 0.263879 | 0.204339 | 0.512412 | 1.515001 | 2.297446 | 3.107442 |
| 0.5 | 0.505482 | 0.350378 | 0.259629 | 0.200808 | 0.505482 | 1.508508 | 2.293804 | 3.105702 |
| 0.6 | 0.497240 | 0.344108 | 0.254597 | 0.196634 | 0.497240 | 1.500923 | 2.289631 | 3.103745 |
| 0.7 | 0.487809 | 0.336952 | 0.248867 | 0.191893 | 0.487809 | 1.492421 | 2.285056 | 3.101643 |
| 0.8 | 0.477323 | 0.329018 | 0.242531 | 0.186665 | 0.477323 | 1.483182 | 2.280201 | 3.099460 |
| 0.9 | 0.465922 | 0.320420 | 0.235685 | 0.181030 | 0.465922 | 1.473384 | 2.275182 | 3.097255 |
| 1.0 | 0.453749 | 0.311271 | 0.228424 | 0.175071 | 0.453749 | 1.463197 | 2.270099 | 3.095072 |

Significant in the upper half of the flow region. It is also found that the velocity v increases with increasing viscosity parameter η . This is because increasing viscosity parameter $\mu f / 2\mu a$ gives rise to an increase in the velocity in the porous layer (which may be due to reduction in apparent viscosity). Figure 4a, b explains the effect of velocities q , v and solid displacement u in the channel which is calculated for different values of volume fraction of component ϕ^f . It is observed that at the interface $y = 0$, the velocities q , v increases with the increase in ϕ^f and is reverse in the case of solid displacement u . The effect of increasing values of Jeffrey parameter λ_1 is observed from Fig. 5a, b. It is clear from governing Eqs. (2) and (3), that an increase in Jeffrey parameter λ_1 results in the decrease in the viscosity of the fluid.

So the velocities q , v and solid displacement increases with the increase in λ_1 . The effect of different values of magnetic field parameter M on q , v and u is shown in Fig. 6a, b. It is observed that v and u decreases with the increase in the magnetic field parameter M and in the case of free flow velocity q opposite behavior are reported. This is due to the fact that with the increasing value of M , the Lorentz force associated with the magnetic field increases and it produces more resistance to the transport phenomena in the free flow region. The influence

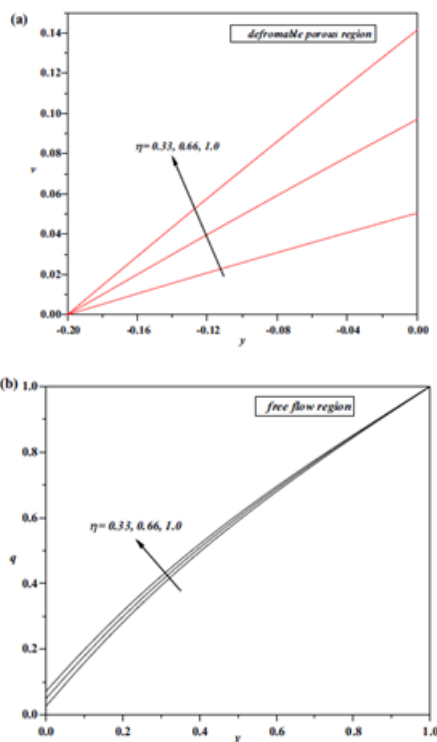


Fig. 3 a Velocity profile for different values of η in deformable porous region with $\delta = 2.0, \epsilon = 0.2, U_0 = 1.0, \lambda_1 = 0.5, M = 1.0, \phi f = 0.5$, **b** velocity profile for different values of η in free flow region with $\delta = 2.0, \epsilon = 0.2, U_0 = 1.0, \lambda_1 = 0.5, M = 1.0, \phi f = 0.5$.

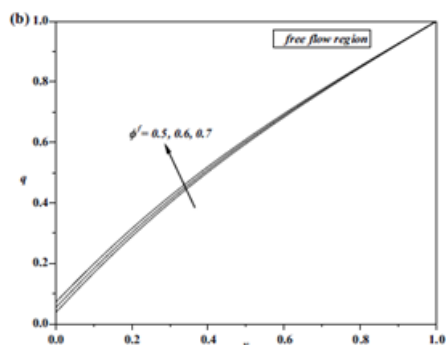


Fig. 4 a Velocity and displacement profiles for different values of ϕf with $\delta = 2.0, \epsilon = 0.2, \lambda_1 = 0.5, M = 1.0, \eta = 0.5, U_0 = 1.0$, **b** velocity profile for different values of η with $\delta = 2.0, \epsilon = 0.2, U_0 = 1.0, \lambda_1 = 0.5, M = 1.0, \eta = 0.5, \phi f = 0.5$

the thickness of the deformable porous media on the flow velocity and solid is placement is depicted in Fig. 7. It is clear that the increment in the thickness of the deformable porous layer enhances the velocity and displacement. This is similar to the behavior observed by Channabasappa et al. [30] for the undeformable porous layer.

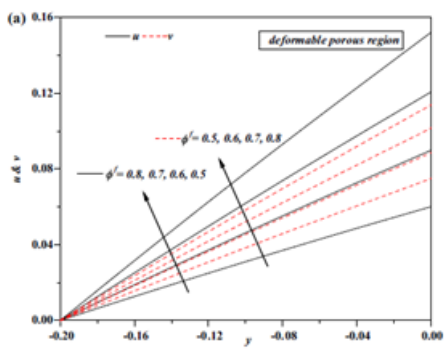


Fig. 5 a Velocity and displacement profile for different values of λ_1 with $\delta = 2.0, \epsilon = 0.2, U_0 = 1.0, M = 1.0, \eta = 0.5, \phi f = 0.5$, **b** velocity profile for

different values of λ_1 with $\delta = 2.0, \epsilon = 0.2, U_0 = 1.0, \eta = 0.5, M = 1.0, \phi f = 0.5$

The variation of mass flow rate for M_d in the free flow region is calculated using Eq. (12) for different values of upper plate velocity U_0 and magnetic field parameter and are tabulated in Table 1. It is observed that the mass flow rate increases with increase in the upper plate velocity U_0 . Further, the effect of magnetic field is to reduce the mass flow rate, depending on the strength of the magnetic field, which is similar to the observation made by Rudraiah et al. [29] for the Hartmann flow over a non-deformable permeable bed. Table 2 explains the variation of mass flow rate M_d and fractional increase F with λ_1 which is calculated using Eq. (14). It is clear from the table that both M_d and F increases with increase in Jeffrey parameter λ_1 . Thus the effect of non-Newtonian Jeffrey parameter λ_1 enhance the flux in the free flow region. The variation of shear stress τ_1 with λ_1 and M is calculated using Eq. (15)

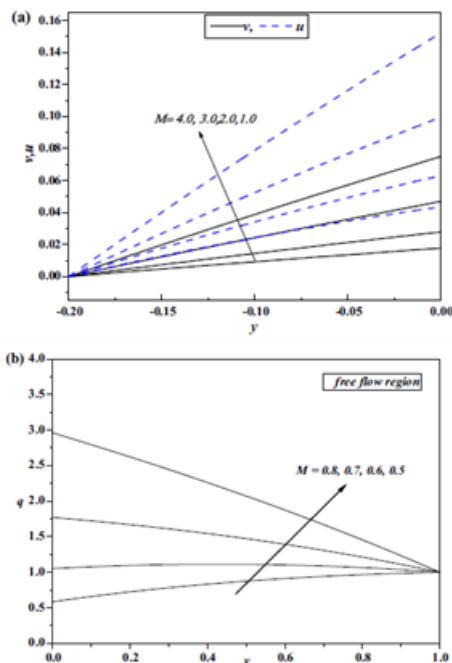


Fig. 6 a Velocity and displacement profiles for different values of M in deformable porous region with $\delta = 2.0, \epsilon = 0.2, \lambda_1 = 0.5, \eta = 0.5, U_0 = 1.0, \phi f = 0.5$, b velocity profile for different values of M with $\delta = 2.0, \epsilon = 0.2, \lambda_1 = 0.5, U_0 = 1.0, \eta = 0.5, \phi f = 0.5$

And are tabulated in Table 3. It is evident from Table 3 that the shear stress at the upper plate decreases with the increase in Jeffrey parameter λ_1 and increases for increasing magnetic field parameter M .

VII. CONCLUSIONS

The present study deals with MHD Couette flow of a Jeffrey fluid over a deformable porous layer. The results are analyzed for different values of the pertinent parameters, namely, Jeffrey

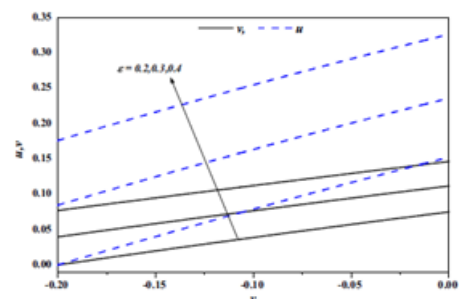


Fig. 7 Velocity and displacement profiles for different values of ϵ in deformable porous region with $\delta = 2.0, \eta = 0.5, U_0 = 1.0, \phi f = 0.5, M = 1.0$

Parameter, upper plate velocity, volume fraction component. The findings of the problem are helpful in understanding the blood (modeled as Jeffrey fluid) flow behavior near the tissue layer (modeled as a deformable porous layer). Some of the interesting findings are as follows:

- The velocity of the fluid in the free flow region and the deformable porous layer and solid displacement increases with an increase in the upper plate velocity.
- The effect of increase in the volume fraction component ϕf enhances the fluid velocity between the parallel plates. But opposite behavior is observed in the case of solid displacement.
- The effect of magnetic field reduces the fluid velocity in the free flow region. In the deformable porous layer, both the fluid velocity and displacement

of the solid matrix increase with increase in magnetic field.

– The flux in the free flow region increases with an increase in the Jeffrey parameter. Also opposite behavior is noticed in case of magnetic field.

– The effect of increase in the magnetic field parameter enhances the shear stress at the upper plate.

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