

Design, Load Analysis and Optimization of Epicyclic Gear Trains (EGT) and Planet Gear Trains (PGT) By Using Graph Theory



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ABSTRACT:

In this paper the primary presented on graph concept is used for the have a look at of epicyclic tools trains or planet gear trains (E.G.T. or P.G.T.). A PGT has been represented by means of its practical schematic, structural and kinematic graph. benefits of kinematic graph illustration of E.G.T. have been indexed. Structural analysis of P.G.T. has been finished. a survey of works linked with the hassle of the modeling of gears by means of versatile graph concept fashions. the following issues had been taken into consideration: derivation of systems of equations describing the behaviour of gear subsystems and detection of a redundant wheel. The survey is based on over 60 papers posted specifically inside closing 10 years, a number of them in global-huge high level clinical magazines. a few current papers claim that graph representations and derived techniques belong to the branch of synthetic intelligence due to the opportunity of acquiring mechanically versatile re-sults, e.g. di erent constructional layout answers of mechanisms.

INTRODUCTION:

equipment trains are used to transmit motion and / or power from one rotating shaft to any other. A ancient evaluate of gear trains, from 3000 B.C. to the 1960s, can be located in Dudley [1]. A gear train is known as an normal gear teach if all the rotating shafts are set up on a commonplace stationary frame and a planetary gear educate (PGT if some gears not only rotate approximately their very own joint axes, however also

revolve around some other gears. The graph idea is a branch of arithmetic which has flexible appli-cations to many elds of engineering, which turned into summarized in well-known monographs (Deo, 1974; Wojnarowski, 1977a, Murota, 1987). since the edition of these books, many new results had been done, particularly within the eld of the modeling of mechanisms. The goal of this paper is to present a particular range of applications of gra-aphs for the modeling of gears. these days, this vicinity of application of graphs has been substantially advanced all around the world. The time period "graphs", used inside the identity of the paper, covers a wide range of gadgets, i.e. graphs and hypergraphs taken into consideration by using the graph concept (i.e. linear graphs, digraphs, weighted graphs), in addition to bond graphs.

The term "gears" is attached with di erent styles of gears: vehicle tools bins, planetary gears, blended geared trains in addition to components of machines enclosing geared pairs or geared mechanisms (e.g. geared robot wrists). In truth, as has been simply referred to, bond graphs have robust theoretic connections with graphs in the feel of a natural graph concept, but for their application purpose there isn't always any need to research them. nevertheless, this direction of investigations is evolved as nicely (Ort and Martens, 1974). The strategies described underneath are based no longer only on graphs

themselves, but also on different algebraic objects assigned to the graphs: matrices, structural numbers, vector spaces and structures of equations. In widespread, the concept of software of graphs to the modeling of technical objects consists in: automation of analysis, era of all possible constructional paperwork or answers, synthesis of unique mechanical structures, finding precise parts internal technical structures (i.e. a redundant wheel), optimization of particular engineering issues, e.g. the shortest path, the maximum flow or transmission machine containing a equipment. This method is diagnosed as a subarea of the artificial intelligence which must be added to its fashionable factors like pattern popularity, gadget translation or expert systems. This idea arose from the notion of Shai and Preiss (1999) and it's miles supported through the present paper. Inside the opinion of the authors of the existing work, the direction of investigation regarding using artificial intelligence techniques is worth wearing on and propagating Shai's assertion that strategies based totally upon graphs are simply such ones.

another scientist who independently started works in this field is Schmidt et al. (2000) who known as the methodology advanced by using her team as "graph grammars". It consists in algorithmization of engineering obligations in a unique meta-language (quasi-language). A simple graph G (Bondy and Murty, 1977; Deo, 1974) is a couple $(V;E)$, where V a set of vertices, E a hard and fast of edges and E is a subset of the cartesian made from the set V . A weighted graph is a triple $(V;E;W)$, wherein W is a weight function having the area E and a selected set of values, e.g. R set of actual numbers. the load can be a actual quantity, e.g. a distance in [km] whilst we model a street community, however it can be also a vector of two or more components. this will be illustrated inside the following sections. A hypergraph H is a pair $(V;HE)$ in which he's a set of hyperedges i.e. the own family of subsets of V . A bondgraph constitutes the concept of representing relationships between different physical parameters of the system in shape of a pictogram or an icon coupled with good

enough algebraic formulas and becoming a member of guidelines. The linear graphs and bond graphs are matched with different algebraic items to create powerful models of engineering structures based on the regulations of undertaking "device-graph" and derived clever technique regarding different issues listed underneath. The graphs may be considered as fashions of such different technical gadgets like e.g.: (a) vibrating structures (Wojnarowski, 1977a), (b) trusses (Shai and Preiss, 1999a,b), (c) mechanisms (Chen and Yao, 2000) (d) gears (Wojnarowski, 1977b; Choi and Bryant, 2002), (e) robotic wrists (Lin, 1990), (f) networks of precise media (e.g. water, gasoline, railway, electric current, and many others.) (Deo, 1974).

The issues considered by way of manner of those modeling are as follows:

analysis of vibrating systems (determination of eigen-frequencies) finding the redundant gear wheel, speed calculations, dynamical analysis enumeration of all possible constructional kinds of a equipment optimization of a particular problem (layout, ratios, weight of the tools-container) synthesis of a machine with specific residences, e.g. diploma of freedom, transmission characteristic. The query arises how it is able to be viable to model such different gadgets and bear in mind such different troubles by way of the identical magnificence of items? it's miles specially feature for Polish scientists because hobby on this field of know-how is pretty slender, simply opposite to the scenario in western international locations, the united states and The a ways East.

The solution to this sort of formulated question is composed in modeling manner, i.e. what's assigned to a vertex, what's diagnosed as an area or hyper-area, what's a set of weights or how is a bond graph built as well as the way to assign more advanced algebraic structures as matrices and matrix-based systems of equations or generalized networks. Moreover, the thriller is veiled in these matrix formulas and systems of equations which can be robotically created in

devoted laptop packages where a graph (representing a mechanical machine) is the initial facts. these wellknown packages are broadly used, e.g. MODELICA, 20-Sim, SIMULINK or others. In popular, strategies derived from the modeling of graphs are algorithmizable and programmable, that is an important benefit bobbing up from the effort of gaining knowledge of and applying those graphs.

NOTATIONS USED: The following notations have been used. E.G.T.: Epicyclic Gear Train

c_i : degrees of constraint on relative motion imposed by joint i . F or dof : degrees of freedom of a mechanism.

f_i : degrees of relative motion permitted by joint i .

j : number of joints in a mechanism, assuming that all joints are binary.

j_i : number of joints with i dof; namely, j_1 denotes the number of 1-dof joints, j_2 denotes the number of 2-dof joints, and so on.

L : number of independent loops in a mechanism.

n : number of links in a mechanism, including the fixed link.

λ : degrees of freedom of the space in which a mechanism is intended to function. S: spherical kinematic pair (dof =3).

E: plane kinematic pair (dof =3). G: gear pair (dof =2).

j_g : no. of gear pair (dof =2).

j_t :no.of turning (revolute) pair.

f_p : number of passive dof in a mechanism

λ is called motion parameter $\lambda = 6$ (for spatial mechanisms kinematic chain) and $\lambda = 3$ (for planar and spherical mechanism kinematic chain).

d_i : no. of joints on link i .

Functional Schematic Representation:

For clarity and simplicity, only those functional elements that are necessary to the structural topology of a mechanism are shown [see Figure-3(a)]. Two E.G.T. with internal and external gear mesh may have identical structural topology as shown in see Figure-4.

STRUCTURAL REPRESENTATION:

Each link of a mechanism is represented by a polygon whose vertices represent the kinematic pairs. A binary, ternary, quaternary, ---links are represented by a line with two end vertices, across-hatched triangle with three vertices, a quaternary link with cross-hatched quadrilateral with four vertices, and so on. Figure-6 shows the structural representation of links. The plain and solid vertices of links denote revolute pairs and gear pairs respectively. Figure-3 illustrates the side view of the P.G.T. shown in Figure-2. Figure-6 shows the schematics and structural representations of link assortments used in geared kinematic chains

KINEMATIC GRAPH REPRESENTATION:

In a kinematic graph representation, the vertices denote links and the edges denote joints of a mechanism. The edge connection between vertices corresponds to the pair between links. To differentiate between various pairs, the edges can be labeled or colored. For example, the gear pairs and turning pair in a gear train are represented by thick edges and thin edges respectively. Figure-5 illustrates the graph representation of the P.G.T. shown in Figure-2. Figure-4 represents a gear set with two external gear and internal meshes. Figure-7 shows three different kinematic representations of four E.G.Ts.

Advantages of Using Graph Representation:

The advantages of using the graph representation are:

1. Euler's equation can be applied to obtain the *loop mobility criterion* of mechanisms directly.

2. The similarity and difference between two different mechanisms can be easily identified.
3. Graphs may be used as an aid for computer-aided kinematic and dynamic analysis of mechanisms. For example, Freudenstein and Yang [2] applied the theory of fundamental circuits for the kinematic and static force analysis of planar spur gear trains. The theory was subsequently extended to the kinematic analysis of bevel-gear robotic mechanisms [3]. Recently, a systematic methodology for the dynamic analysis of gear coupled robotic mechanisms was developed [4].

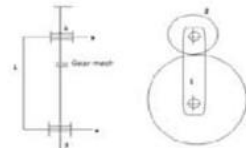


Figure 4(a): Functional schematic of a spur gear with an external gear mesh

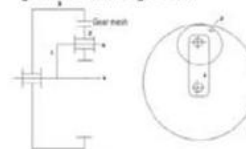


Figure 4(b): Functional schematic of a spur gear with an internal mesh.
Figure 4: Functional schematics of two gear sets that share the same structural topology

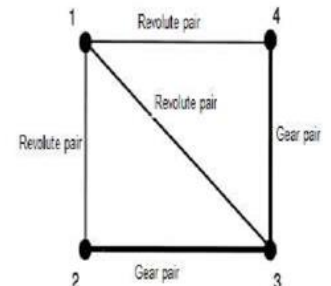


Figure 5: Kinematic graph representation of the P.G.T. shown in Figure-2.

4. Graph theory may be used for systematic enumeration of mechanisms. [5,6,7,8,9,10,11,12,13].
5. Graphs can be used for systematic classification of mechanisms. A single collection of graphs can be used to generate a number of mechanisms [14, 8, 16].
6. Graphs may use as help in automated sketching of mechanisms [17].

Link type	Functional schematic

Figure-6 Link assortment used in gear kinematic chain

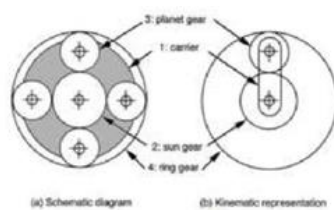
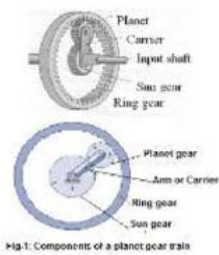


Figure-2: Schematic diagram and kinematic representation of E.G.T.

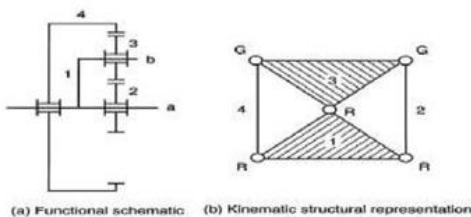


Figure-3: Functional and structural representations of the P.G.T. shown in Figure-2

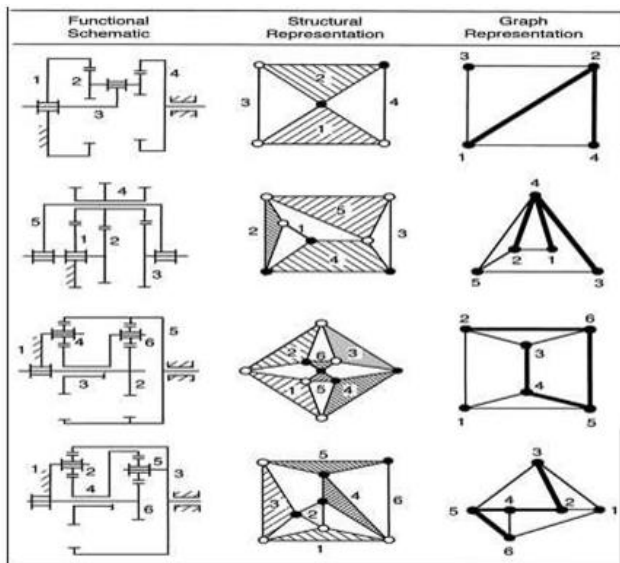


Figure-7: Kinematic representation of 4 P.G.T.

STRUCTURAL CHARACTERISTICS:

There are revolute (R), prismatic (P) and gear (G) pairs in the planar geared mechanisms. If j_i denote the number of i -dof joints are connected by then total number of joints are given by eq.(1).

$$j = j_1 + j_2 \dots \dots \dots (1)$$

Because the revolute and prismatic pairs have one-dof and the gear pair two-dof. So, the sum of the dof in all pairs is given by eq.(2).

$$\sum_{i=1}^j (f_i) = j_1 + 2j_2$$

Subtracting eq.(1) from (2), we get eq.(3).

$$\sum_{i=1}^j (f_i) = j_2$$

We also know that

$$F = \lambda (n - j - 1)$$

Eq.(4) is called Grübler or Kutzbach criterion. Eq.(4) can be represented by introducing the term f_p .

$$F = \lambda (n - j - 1) + \sum_{i=1}^j (f_i - f_p) \dots \dots \dots (5)$$

In general, $F > 0$, the mechanism has F dof, $F = 0$, the mechanism becomes a structure with zero dof, $F < 0$, the mechanism becomes an over-constrained structure. An equation among L , n and j in a KC can be developed. For a single-loop mechanisms KC (planar,

spherical, or spatial), $n = j$, and all the links are binary. By increasing a mechanism KC from 1 to L loops, the difference between the j and n is increased by $L - 1$. So,

$$L = j - n + 1 \dots \dots \dots (6)$$

Eq. (6) is Euler's equation. From eq. (6) and (4), we get

$$\sum_{i=0}^j f_i = F + \lambda L \dots \dots \dots (7)$$

Eq.(7) is known as the loop mobility criterion. Substituting $\lambda=3$ into eq.(7), we get

$$j + j_2 = F + 3L \dots \dots \dots (8)$$

Hence, the no. of joints and of links in a geared mechanism depends on the gear pairs apart from the dof and L .

It has been shown that the number of gear pairs in a geared mechanism cannot exceed the number of independent loops [10]; that is,

$$j_2 \leq L \dots \dots \dots (9)$$

By using eq.(6),(8),and(9),geared mechanisms can be designed and classified according to the number of dof, L , and n .

In addition to satisfying the above structural characteristics, the following constraints are imposed [5]:

1. All links of an epicyclic gear train are capable of unlimited rotation.
2. For each gear pair, there exists a carrier, which keeps the center distance between the two meshing gears constant.

We study the effects of these two constraints on the structural characteristics of epicyclic gear trains. For all links to possess unlimited rotation, the prismatic joint is excluded from design consideration. Hence only revolute joints and gear pairs are allowed for structure synthesis of EGTs. For convenience, we use a thin edge to represent a revolute joint (or turning pair) and a thick edge to stand for a gear pair. For this reason, thin edges are sometimes called turning-pair edges and thick edges are called geared edges. Let j_g denote the number of gear pairs and j_t

represent the number of revolute joints. It is clear that the total number of joints is given by equation (10).

$$j = j_t + j_g \tag{10}$$

Substituting Eq. (2) and (10) into Eq. (4), we obtain eq.(11).

$$F = 3(n-1) - 2j_t - j_g \tag{11}$$

The first constraint implies that there should not be any circuit formed exclusively by turning pairs. Otherwise, either the circuit will be locked or rotation of the links will be limited. The second constraint implies that all vertices should have at least one incident edge that represents a turning pair. Hence, we have

The sub graph obtained by removing all geared edges from the graph of an EGT is a tree.

Since a tree of v vertices contains $v-1$ edges, we further conclude that

$$j_t = n - 1 \tag{12}$$

Substituting Equations (10) and (12) into Equation (6), we obtain

$$j_g = L \tag{13}$$

Substituting Equations (12) and (13) into Equation (11) yields

$$L = n - 1 - F = j_t - F \tag{14}$$

Eliminating j_t and j_g from Equations (10), (13), and (14), we get

$$j = F + 2L \tag{15}$$

Summarizing Equations (12), (13), and (14) in words, we have

“In epicyclic gear trains, the number of gear pairs is equal to the number of independent loops; the number of turning pairs is equal to the number of links diminished by one; and the number of degrees of freedom is equal to the difference between the number of turning pairs and the number of gear pairs”.

We know that the minimum no. of joints on each link

of a closed loop chain is 2. Therefore,
 $d_i \geq 2$ ----- (16)

Using the fact that the no. of loops of which a vertex is a part is equal to its degree, and the maximum degree of a vertex is equal to the total no. of loops. Then

$$L + 1 \geq d_i \tag{17}$$

Combining eq. (16) and (17), we get

$$L + 1 \geq d_i \geq 2 \tag{18}$$

We can say that the minimum no. of joints on each link of a closed loop chain is 2 and maximum no. is limited by the total no. of loops. In other words, the degree of any vertex in the graph of an EGT lies between 2 and $L+1$.

In general, the graph of an EGT should not contain any circuit that is made up of only geared edges. Otherwise, the gear train may rely on special link length proportions to achieve mobility. In the case where geared edges form a loop, the number of edges must be even. Otherwise, the mechanism will not function properly.

Example -1:

A differential gear educate has been shown in Figure-8 in which gears 3, 4, 5, and 6 bureaucracy a loop. The pitch diameter of gear three is identical to that of tools 5, and the pitch diameter of gear four is same to that of equipment 6. otherwise, the mechanism will no longer characteristic well. truly, we may additionally keep in mind either equipment four or 6 as a redundant link. that is, casting off both tools 4 or 6 from the mechanism does now not affect the mobility of the mechanism. this is a regular fractionated mechanism in that hyperlinks 2,3,4, five, and six form a one-dof equipment teach and the second one diploma of freedom comes from the fact that the tools teach itself can rotate as a rigid frame about the “a-a” axis.

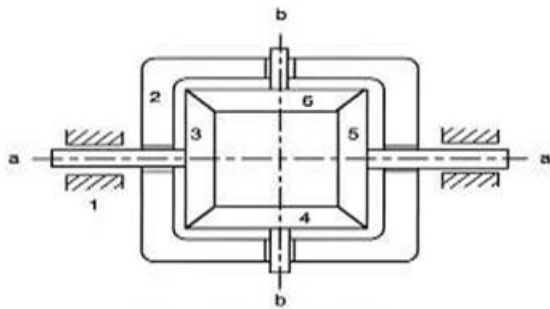


Figure-8 A Differential gear train

CONCLUSION:

in this paper, the author made the application of graph principle in the representation of P.G.T. it's been proven that in a geared mechanism, the no. of gear pairs cannot be greater than the no. of independent loops. If there's limitless rotation of all hyperlinks in a geared mechanism then it's miles called equipment train. A P.G.T. is an assemblage of hyperlinks and kinematic pairs. The study consists of the purposeful schematic illustration, structural illustration, graph representation of P.G.Ts.

The structural traits of EGTs are identified. It turned into proven that an n -link EGT includes $(n-1)$ turning pairs and $n-1-F$ equipment pairs. The study is useful for the U.G. /P.G. students and architects in their early age of mastering on the conceptual level of layout.

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