

Vibrational Behaviour of Cantilever Beam with Piezo Electric Patches

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INTRODUCTION

Smart structures are a rapidly advancing field with the range of support and enabling technologies having significant advances, notable optics and electronics. The definition of smart structure was a topic of controversy from the late 1970 to 1980. In order to define this a special workshop was organized by the US army research office in 1988 in which Sensors, Actuators, Control mechanism and Timely response were recognized as the four qualifying features of any smart system or structures. In this workshop Smart structure is defined as "A system or material which has built in intrinsic Sensor, actuator and control mechanism whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short time and reverting to its original state as soon as the stimulus is removed." According to Spilman a smart structure is defined as "a physical structure having a definite purpose, means of imperative to achieve that purpose and the pattern of functioning of a computer. "Smart structure contains a host structure, a sensor to gauge its internal state, an actuator to affect its internal state and, a controller whose purpose is to process the sensors and appropriately send signals to actuators.

Vibration control is an important area of interest in several industrial applications. Unwanted vibration can have a detrimental and sometimes catastrophic effect on the serviceability or structural integrity of mechanical systems. To control the vibrations in a system, different techniques have been developed. Some of these techniques and methods use piezoelectric materials as sensors or actuators. A vibration isolation system is called active if it uses external power to perform its

function. It consists a servomechanism with a sensor, actuator, and signal processor. Active control systems are required in applications where passive vibration control is not possible because of material constraints or simply not sufficient for the level of control required. Active control is a favourable method of control because it works in a wide frequency range, reducing resonant vibrations within that range and because it is adaptive to changes in the nature of the disturbance. Active smart materials are those materials which possess the capacity to modify their geometric or material properties under the application of electric, thermal or magnetic field, thereby acquiring an inherent capacity to transducer energy. The active smart materials are piezoelectric material, Shape memory alloys, Electro-rheological fluids and Magneto-structive materials. Being active they can be used as force transducers and actuators. The materials which are not active under the application of electric, thermal or magnetic field are called Passive smart materials. Fibre optic material is good example of passive smart material. Such materials can act as sensors but not as actuators and transducers.

Piezoelectric Material:

Piezoelectricity is the ability of a material to develop an electric charge when subjected to a mechanical strain, this effect is called Direct Piezoelectric Effect (DPE) and Conversely material develop mechanical strain in response to an applied electric field, this effect is called Converse Piezoelectric Effect (CPE). Due to this coupled mechanical and electrical properties, piezoelectric materials make them well suited for use as sensors and actuators. Sensors use Direct Piezoelectric Effect (DPE) and actuators use Converse Piezoelectric Effect (CPE).

As a sensors, deformations cause by the dynamic host structure produce an electric change resulting in an electric current in the sensing circuit. While as an actuators, a high voltage signal is applied to piezoelectric device which deforms the actuator and transmit mechanical energy to the host structure. Piezoelectric materials basically divided into two group Piezo-ceramics and piezo-polymers.

Piezo-Ceramics:

The most common commercial piezo-polymer is Barium Titanate (BaTiO₃), Lead Titanate (PbTiO₃), Lead Zirconate ((PbZrO₃) Lead metaniobate (PbNb₂O₆) and Lead (plumbum) Zirconate Titanate (PZT) [Pb(ZrTi)O₃].

Among these materials last Lead (plumbum) Zirconate Titanate (PZT) became the dominant piezo-electric ceramic material for transducer due to its high coupling coefficient (0.65). When this PZT plate subjected to static or dynamic loads, it can generate voltages as high as 20,000 volts.

Examples: Microphones, headphones, loudspeakers, buzzers, wrist watches, clocks, calculators, hydrophones and projectors.

Research Objectives:

There are three main objectives of this research:

- 1) Obtaining an accurate analysis of a repaired notched cantilever beam by piezoelectric material;
- 2) Establish an effective control in the repair of damaged cantilever beam and
- 3) Comparing the damaged, healthy repaired cantilever beam frequencies.
- 4) Establish an effective control on natural frequency reduction.

THEORETICAL FORMULATION

The constitutive equations of a linear piezoelectric material read (IEEE std, 1988).

$$\{T\} = [c_E]\{S\} - [e]_T\{E\} \quad (1)$$

$$\{D\} = [e]\{S\} + [\epsilon_S]\{E\} \quad (2)$$

Where $\{T\} = \{T_{11} \ T_{22} \ T_{33} \ T_{23} \ T_{13} \ T_{12}\}^T$ is the stress vector, $\{S\} = \{S_{11} \ S_{22} \ S_{33} \ 2S_{23} \ 2S_{13} \ 2S_{12}\}$ the deformation vector, $\{E\} = \{E_1 \ E_2 \ E_3\}$ the electric field, $\{D\} = \{D_1 \ D_2 \ D_3\}$ the electric displacement, $[c]$ the elasticity constants matrix, $[\epsilon]$ the dielectric constants matrix, $[e]$ the piezoelectric coupling coefficients matrix (superscripts indicate values at E and S constant respectively).

The dynamic equations of a piezoelectric continuum can be derived from the Hamilton principle, in which the Lagrangian and the virtual work are properly adapted to include the electrical contributions as well as the mechanical ones. The potential energy density of a piezoelectric material includes contributions from the strain energy and from the electrostatic energy (Tiersten, 1967).

$$H = \left(\frac{h}{2}\right) \{S\}^T \{T\} - \{E\} \{D\} \quad (3)$$

Similarly, the virtual work density reads

$$\delta W = \{\delta u\}^T \{F\} - \delta \phi \sigma \quad (4)$$

Where $\{F\}$ is the external force and σ is the electric charge. From Equ.(3) and (4), the analogy between electrical and mechanical variables can be deduced (Table 1).

Mechanical		Electrical	
Force	$\{F\}$	σ	Charge
Displ.	$\{u\}$	ϕ	Voltage
Stress	$\{T\}$	$\{D\}$	Electric Displ.

Table 3.1: Electro mechanical analogy

The vibrational principle governing the piezoelectric materials follows from the substitution of H and δW into the Hamilton principle (Allik and Hughes, 1970).

$$\begin{aligned} 0 = & - \int_V [P\{U\}_T \ddot{u}] - \{\delta S\}^T [C^E] \{S\} + \{\delta S\}^T [e]^T \{E\} + \{\delta E\}^T [e] \{S\} \\ & + \{\delta E\}^T [\epsilon^E] \{E\} + \{\delta u\}^T [Pb] dV + \int_{S_1} \{\delta u\}^T \{P_s\} dS + \{\delta u\}^T \{P_c\} \\ & - \int_{S_2} \delta \phi \sigma dS - \delta \phi Q \end{aligned} \quad (5)$$

In the finite element formulation, the displacement field $\{u\}$ and the electric potential ϕ over an element are related to the corresponding node values $\{u_i\}$ and $\{\phi_i\}$ by the mean of the shape functions $[N_u]$, $[N_\phi]$.

$$\{u\} = [N_u]\{u_i\} \quad (6)$$

$$\{\Phi\} = [N_\phi]\{\phi_i\} \quad (7)$$

And therefore, the strain field $\{S\}$ and the electric field $\{E\}$ are related to the nodal displacements and potential by the shape functions derivatives $[B_u]$ and $[B_\phi]$ defined by

$$\{S\} = [D][N_u]\{u_i\} = [B_u]\{u_i\} \quad (8)$$

$$\{E\} = -\nabla [N_\phi]\{\phi_i\} = -[B_\phi]\{\phi_i\} \quad (9)$$

Where ∇ is the gradient operator and $[D]$ is the derivation operator defined such as $\{S\} = [D]\{u\}$.

$$\begin{aligned} 0 = & -\{\delta U\}^T \int_V \rho [N_u]^T [N_u] dV \{u_i\} - \{\delta U\}^T \int_V \{B_u\} [C^E] dV \{u_i\} \\ & - \{\delta U\}^T \int_V \{B_u\}^T \{e\} \{B_\phi\} dV \{\phi_i\} - \{\delta \phi\}^T \int_V \{B_\phi\}^T \{e\}^T \{B_u\} dV \{u_i\} \\ & + \{\delta \phi\}^T \int_V \{B_\phi\}^T \{\epsilon^T\} \{B_\phi\} dV \{\phi_i\} + \{\delta U\}^T \int_V [N_u]^T \{P_b\} dV \\ & + \{\delta U\}^T \int_{S_1} [N_u]^T \{P_s\} dS + \{\delta U\}^T \{N_u\}^T \{P_c\} \\ & - \{\delta \phi\}^T \int_{S_2} [N_\phi]^T \sigma dS - \{\delta \phi\}^T [N_\phi]^T Q \end{aligned} \quad (10)$$

Which must be verified for any arbitrary variation of the displacements $\{u_i\}$ and electrical potentials $\{\phi_i\}$ compatible with the essential boundary conditions.

For an element, Equ.(10) can be written under the form

$$[M]\{u_i\} + [K_{uu}]\{u_i\} + [K_{u\phi}]\{\phi_i\} = \{f_i\} \quad (11)$$

$$[K_{\phi u}]\{u_i\} + [K_{\phi\phi}]\{\phi_i\} = \{g_i\} \quad (12)$$

$$[M] = \int_V \rho [N_u]^T [N_u] dV \quad (13)$$

$$[K_{uu}] = \int_V [B_u]^T [C^E] [B_u] dV \quad (14)$$

$$[K_{u\phi}] = \int_V [B_u]^T \{e\} [B_\phi] dV \quad (15)$$

$$[K_{\phi\phi}] = -\int_V [B_\phi]^T \{\epsilon^T\} [B_\phi] dV \quad (16)$$

$$[K_{\phi u}] = [K_{u\phi}]^T \quad (17)$$

$$\{f_i\} = \int_V [N_u]^T \{P_b\} dV + \int_{S_1} [N_u]^T \{P_s\} dS + [N_u]^T \{P_c\} \quad (18)$$

$$\{g_i\} = -\int_{S_2} [N_\phi]^T \sigma dS - [N_\phi]^T Q \quad (19)$$

Each element k of the mesh is connected to its neighbouring elements at the global nodes and the displacement is continuous from one element to the next.

Based on that formulation, piezoelectric finite elements of type multi-layered Mindlin shell (Piefort and Preumont, 2000) and volume have been derived. For shell elements, it is assumed that the electric field and displacement are uniform across the thickness and aligned on the normal to the mid-plane. The electrical degrees of freedom are the voltages ϕ_k across the piezoelectric layers; it is assumed that the voltage is constant over each element (this implies that the finite element mesh follows the shape of the electrodes). One electrical degree of freedom of type voltage per piezoelectric layer is defined. The assembly takes into account the equipotentiality condition of the electrodes; this reduces the number of electric variables to the number of electrodes.

State space model:

The idea behind modelling structures embedding piezoelectric actuators and sensors using finite elements is indeed to gather the necessary information's to design a good control strategy. It is therefore necessary to interface the structural analysis software (finite element package) with control design software.

The assembled system of equations can be complemented with a damping term $[C]\{U\}$ to obtain the full equation of dynamics and the sensor equation:

$$\{0\} = [M]\{U''\} + [C]\{U'\} + [K_{UU}]\{U\} + [K_{U\phi}]\{\Phi\} \quad (20)$$

$$\{G\} = [K_{\phi U}]\{U\} + [K_{\phi\phi}]\{\Phi\} \quad (21)$$

where $\{U\}$ represents the mechanical *dof*, $\{\Phi\}$ the electric potential *dof*, $[M]$ the inertial matrix, $[C]$ the damping matrix, $[K_{UU}]$ the mechanical stiffness matrix, $[K_{U\phi}] = [K_{\phi U}]^T$ the electromechanical coupling matrix and $[K_{\phi\phi}]$ the electric capacitance matrix. Voltage actuation and charge sensing are considered. Actuation is done by imposing a voltage $\{\Phi\}$ on the actuators and sensing by imposing a zero voltage ($\{\Phi\} = \{0\}$) and measuring the electric charges $\{G\}$ appearing on the sensors.

Using a truncated modal decomposition (n decoupled modes) $\{U\} = [Z]\{x\}(t)$, where $[Z]$ represents the n modal shapes and $\{x\}(t)$ the n modal amplitudes, Equ (20) and (21) become and left-multiplying Equ (22) by $[Z]^T$ using the orthogonality properties of the mode shapes.

$$\{0\} = [M][Z]\{\ddot{x}\} + [C][Z]\{\dot{x}\} + [K_{uu}][Z]\{x\} + [K_{u\phi}]\{\phi\} \quad (23)$$

$$\{G\} = [K_{\phi u}][Z]\{x\} + [K_{\phi\phi}]\{\phi\} \quad (24)$$

$$[Z]^T[M][Z] = \text{diag}(\mu_k) \quad (25)$$

$$[Z]^T[K][Z] = \text{diag}(\mu_k \omega_k^2) \quad (26)$$

$$[Z]^T[C][Z] = \text{diag}(2\zeta_k \mu_k \omega_k) \quad (27)$$

The dynamic equations of the system in the state space representation finally read

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\Omega^2 & -2\xi\Omega \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} - \begin{bmatrix} 0 \\ -\mu Z^T K_{u\phi}^T \end{bmatrix} \{\phi\} \quad (28)$$

$$\{G\} = [K_{\phi u}^T Z \quad 0] \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \{D_{HF}\}\{\phi\} \quad (29)$$

where the modal shapes $[Z]$, the modal frequencies $[\Omega] = \text{diag}(\omega_k)$, the modal masses $[\mu] = \text{diag}(\mu_k)$, the modal electric charge on the $[K_{\phi u}^T][Z]$ and the modal electric charge on the actuators, transposed (by reciprocity) $[Z]^T [K_{\phi u}]$ representing the participation factor of the actuators to each mode, are obtained from a dynamic finite element analysis. $[\xi] = \text{diag}(\zeta_k)$ are the modal classical damping ratios of the considered structure and $[D_{HF}]$ is the static contribution of the high frequency modes (Preumont, 1997; Loix, 1998; Loix et al., 1998); Its elements are given by

$$D_{lm} = d_{lm} - \frac{\sum_{k=1}^n [K_{\phi u}^T Z]_l [Z_k^T K_{\phi u}]_m}{\mu_k \omega_k^2} \quad (29)$$

Where d_{lm} is the charge appearing on the l^{th} sensor when a unit voltage is applied on the m^{th} actuator and is obtained from a static finite element analysis.

Such a state space representation is easily implemented in control oriented software allowing the designer to extract the various transfer functions and use the control design tools.

FINITE ELEMENT ANALYSIS

Problem of steel beam:

Cantilever beam model was created in software for finite element analysis ANSYS 14.5 as shown in figures [4.1.1-4.1.3]. The beam model is based from laboratory set-up experiment for cantilever aluminium beam with following dimensional properties in the table:

Table4.1.1 Properties of Steel Beam

Thickness b	0.25 m
Height h	0.25 m
Length from fixed end l	2 m
Young's modulus E	$207 \times 10^9 \text{ N/m}^2$
Density ρ	7800 kg/m^3
Poisson ratio μ	0.3
Piezo patch dimensions	$0.5\text{m} \times 0.25\text{m} \times 0.25\text{m}$

Table4.1.2: Anisotropic Properties

Linear Elastic Anisotropic properties (N/m^2)	
D_{11}	1.26×10^{11}
D_{12}	8.41×10^{10}
D_{13}	7.95×10^{10}
D_{22}	1.17×10^{11}
D_{23}	8.41×10^{10}
D_{33}	1.2×10^{11}
D_{44}	2.3×10^{10}
D_{55}	2.3×10^{10}
D_{66}	2.35×10^{10}

Table4.1.3: Electromagnetic Properties.

Electromagnetic Relative Permittivity (F/m)	
ϵ_{11}	1.151×10^{-3}
ϵ_{22}	1.043×10^{-3}
ϵ_{33}	1.151×10^{-3}

Table 4.1.4: Piezoelectric Properties.

Piezoelectric constant stress matrix(C/m ²)	
e ₁₂	-5.4
e ₂₂	15.8
e ₃₂	-5.4
e ₄₁	12.3
e ₅₃	12.3
Pzt Density ρ	7800 kg/m ³

The beam is designed in Ansys by using element plane183 for steel beam and plane223 for Pzt patch. The above figures [4.1.1-4.1.7] show the beam without patch and Pzt patch at different lengths scenarios ($l_1=0.2m$, $l_2=0.8m$ & $l_3=1.5m$) with patch length of 0.5m and a Pzt patch of 0.1m length at $l_1=0.3m$, $l_2=0.9m$, $l_3=1.2m$, $l_4=1.6m$ & $l_5=1.9m$. The natural frequency response for the beam with and without Pzt patches is studied.

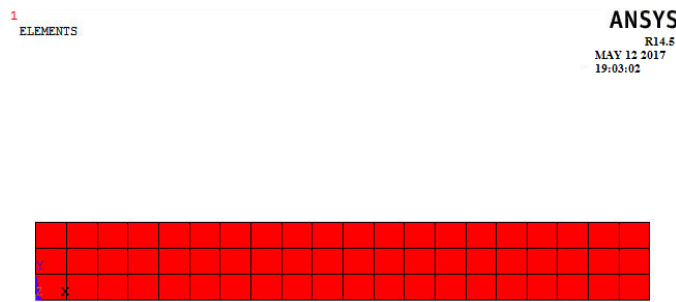


Figure 4.1.1 Cantilever steel beam without PZT patch

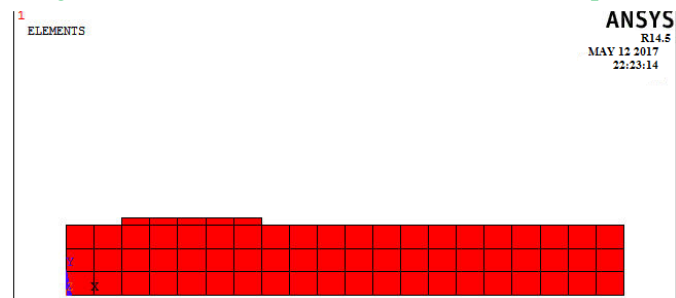


Figure 4.1.2 Cantilever steel beam with PZT patch at $l_1=0.2m$

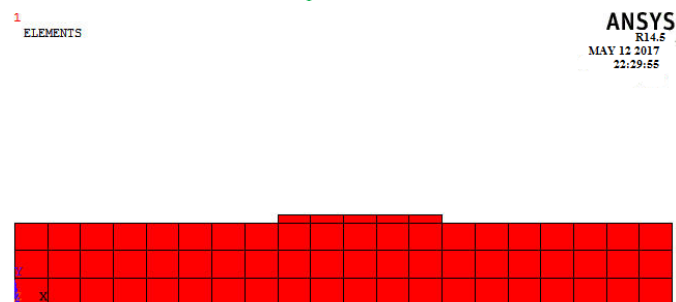


Figure 4.1.3 Cantilever steel beam with PZT patch at $l_2=0.8m$

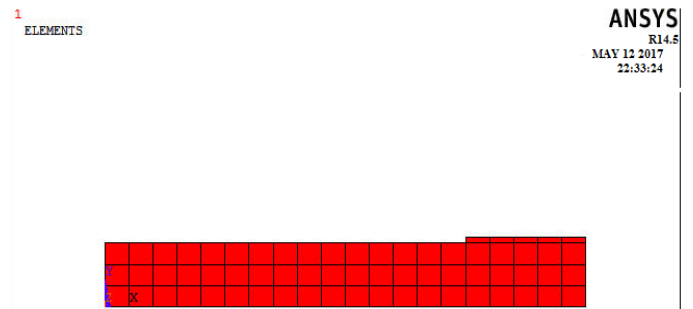


Figure 4.1.4 Cantilever steel beam with PZT patch at $l_3=1.5m$

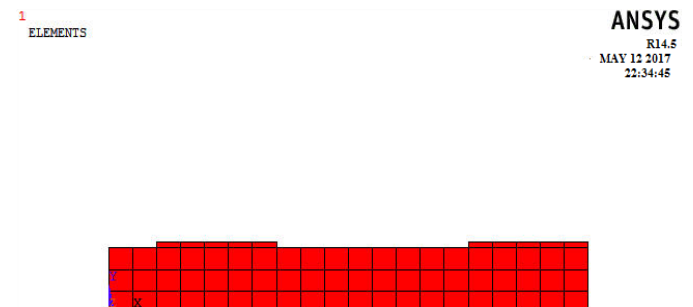


Figure 4.1.5 Cantilever steel beam with PZT patch's at $l_1=0.2m$ & $l_3=1.5m$

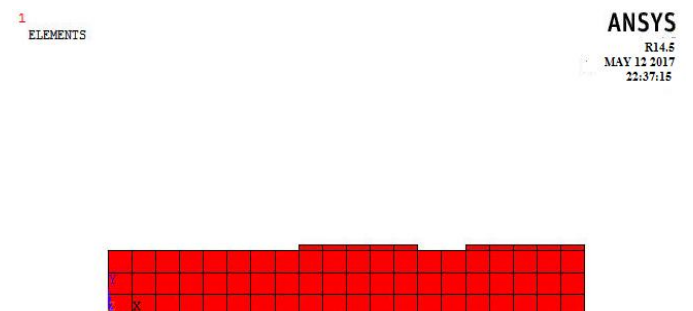


Figure 4.1.6 Cantilever steel beam with PZT patch at $l_2=0.8m$ & $l_3=1.5m$

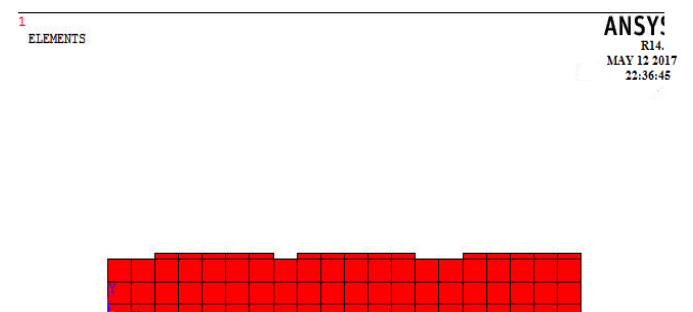


Figure 4.1.7 Cantilever steel beam with PZT patch at $l_1=0.2m$, $l_2=0.8m$ & $l_3=1.5m$

ELEMENTS

ANSYS
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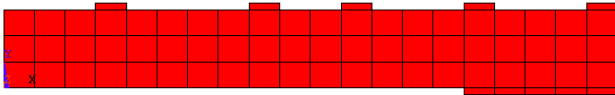


Figure 4.1.7 Cantilever steel beam with PZT patch's at $l_1=0.3m, l_2=0.9m, l_3=1.2m, l_4=1.6m$ & $l_5=1.9m$

NUMERICAL RESULTS

Results of steel beam:

Vibration behaviour of the beam simulated in FEA software ANSYS. The natural frequencies are reduced when the PZT patch is placed on the beam at different scenarios compared to the natural frequencies of the beam without PZT patches. The natural frequencies at all ten modes have effectively dropped when the patch is introduced at three locations when compared to the natural frequencies of the beam without Pzt patches as shown in table 5.1.1

Table 5.1.1 Natural frequencies at different scenarios of Pzt patches

Mode	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
1	51.488	50.769	49.673	48.237	49.193	48.127	49.089	49.055
2	302.15	296.13	297.673	294.08	292.33	293.64	291.98	293.93
3	644.98	631.94	623.73	616.65	617.89	610.92	612.58	621.64
4	775.99	762.49	768.76	763.44	755.8	760.81	753	760.54
5	1374.9	1358	1365.9	1360.7	1348.6	1356.2	1343.6	1348.3
6	1932.7	1880.1	1895.7	1887.6	1857.9	1873.2	1845.5	1887.7
7	2052.5	2026.8	2023.9	2034.9	2016.1	2021.7	2002.6	2012.6
8	2776.4	2739.6	2744.4	2741.9	2719.6	2724.7	2702.5	2714.1
9	3213.2	3147.8	3161.3	3182.5	3132.1	3146.4	3094.3	3152.3
10	3527.9	3471.6	3479.7	3469.6	3439.7	3448	3418.1	3436.8

f_1 - Natural frequencies of Cantilever steel beam without Pzt patch

f_2 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=0.2m$

f_3 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=0.8m$

f_4 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=1.5m$

f_5 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=0.2m$ & $l_2=1.5m$

f_6 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=0.8m$ & $l_2=1.5m$

f_7 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=0.2m, l_2=0.8m$ & $l_3=1.5m$.

f_8 - Natural frequencies of Cantilever steel beam with Pzt patch at $l_1=0.3m, l_2=0.9m, l_3=1.2m, l_4=1.6m$ & $l_5=1.9m$

CONCLUSIONS

In this a comprehensive study of smart materials and smart structures is done, for the effect of the piezoelectric actuator placement on controlling the structural vibrations. Cantilever beam with piezo-electric patches at different locations is used for this study, a cantilever steel beam with PZT actuator at different positions. The systems are modelled in ANSYS. The cantilever structure shows that the actuator locations influence the change in the first three natural frequencies.

In future, the application of PZT actuators for the Composite smart structures can be carried out.

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25. Modelling and analysis of laminate composite plates with embedded active-passive piezoelectric networks: T.C. Godoy ¹, M.A. Trindade ¹ 1 Department of Mechanical Engineering, S˜ao Carlos School of Engineering, University of S˜ao Paulo, Av. Trabalhador S˜ao-Carlense, 400, S˜ao Carlos, SP 13566-590, Brazil.
26. Simultaneous Sensing and Actuating for Path Condition Monitoring of a Power Wheel Chair: Hossein Mousavi Hondori, PhamQuoc Trung, Ling Shih-Fu School of Mechanical and Aerospace Engineering Nanyang Technological University Singapore.
27. Use of a Collocated Sensor/Actuator for Dynamic Control and Structural Health Monitoring: Molly Nelis¹, Kenneth Ogorzalek², Alberto Vázquez Ramos³, Gyuhae Park⁴. 1Department of Electrical & Computer Eng. and Department of Mechanical Eng., Rose- Hulman Inst. Technology, Terre Haute, IN 47803, 2Department of Civil Eng. and Mechanics, University of Wisconsin-Milwaukee, Milwaukee, WI 53201, 3Department of Mechanical Eng., University of Turabo, Gurabo, PR 00787, 4The Engineering Institute, Los Alamos National Laboratory, Los Alamos, NM 87545.
28. Modelling and analysis of smart piezoelectric beams using simple higher order shear deformation theory M Adnan Elshafei¹ and Fuzy Alraies² Department of Aeronautics, Military Technical Collage, Cairo, Egypt.
29. Numerical and experimental study on integration of control actions into the finite element solutions in smart structures: L. Malgaca* and H. Karag˘ulle Department of Mechanical Engineering, Dokuz Eyl˘ul University, 35100, Bornova / Izmir, Turkey.
30. Multiphysics Modelling and Experimental Validation of the Active Reduction of Structure-Borne Noise: Tomasz G. Zielinski Department of Intelligent Technologies, Institute of Fundamental Technological Research, ul. Pawinskiego 5B, 02-106 Warszawa, Poland.
31. Vibration Analysis of Cantilever Smart Structure by using Piezoelectric Smart Material: K. B. Waghulde, Dr. Bimlesh Kumar Mechanical Engineering Department J. T. Mahajan College of Engineering, Faizpur, (Maharashtra State) India, 425503.
32. Continuous model for flexural vibration analysis of a Timoshenko cracked beam: M. HEYDARI, A. EBRAHIMI, M. BEHZAD Sharif University of Technology Mechanical Engineering Department P.O. Box 11365-9567, Azadi Ave. Tehran, Iran.
33. Vibration Analysis of Cantilever Beam for Damage Detection: Marjan Djidrov Viktor Gavriloski Jovana Jovanova Teaching and Research Assistant Ss. Cyril and Methodius University in Skopje Faculty of Mechanical Engineering Republic of Macedonia.