

Dynamic Analysis Using Response Spectrum Seismic Loading

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INTRODUCTION

The basic mode superposition method, which is restricted to linearly elastic analysis, produces the complete time history response of joint displacements and member forces. In the past there have been two major disadvantages in the use of this approach. First, the method produces a large amount of output information that can require a significant amount of computational effort to conduct all possible design checks as a function of time. Second, the analysis must be repeated for several different earthquake motions in order to assure that all frequencies are excited, since a response spectrum for one earthquake in a specified direction is not a smooth function.

Further, the seismic analysis of structures cannot be carried out simply based on the peak value of the ground acceleration as the response of the structure depend upon the frequency content of ground motion and its own dynamic properties. To overcome the above difficulties, earthquake response spectrum is the most popular tool in the seismic analysis of structures.

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There are computational advantages in using the response spectrum method of seismic analysis for prediction of displacements and member forces in structural systems. The method involves the calculation of only the maximum values of the displacements and member forces in each mode using smooth design

spectra that are the average of several earthquake motions.

This chapter deals with response spectrum method and its application to various types of the structures. The codal provisions as per IS:1893 (Part 1)-2002 code for response spectrum analysis of multi-story building is also summarized.

Static and Dynamic Analysis

The purpose of this chapter is to summarize the fundamental equations used in the response spectrum method and to point out the many approximations and limitations of the method. For, example it cannot be used to approximate the nonlinear response of a complex three-dimensional structural system. The recent increase in the speed of computers has made it practical to run many time history analyses in a short period of time. In addition, it is now possible to run design checks as a function of time, which produces superior results, since each member is not designed for maximum peak values as required by the response spectrum method.

Response Spectra

Response spectra are curves plotted between maximum response of SDOF system subjected to specified earthquake ground motion and its time period (or frequency). Response spectrum can be interpreted as the locus of maximum response of a SDOF system for given damping ratio.

Response spectra thus helps in obtaining the peak structural responses under linear range, which can be used for obtaining lateral forces developed in structure

due to earthquake thus facilitates in earthquake-resistant design of structures.

Usually response of a SDOF system is determined by time domain or frequency domain analysis, and for a given time period of system, maximum response is picked. This process is continued for all range of possible time periods of SDOF system.

Final plot with system time period on x-axis and response quantity on y-axis is the required response spectra pertaining to specified damping ratio and input ground motion. Same process is carried out with different damping ratios to obtain overall response spectra.

Definition Of A Response Spectrum

For three dimensional seismic motions, the typical modal Equation is rewritten as,

$$\ddot{y}(t)_n + 2\zeta_n \omega_n \dot{y}(t)_n + \omega_n^2 y(t)_n = p_{nx} \ddot{u}(t)_{gx} + p_{ny} \ddot{u}(t)_{gy} + p_{nz} \ddot{u}(t)_{gz}$$

where the three Mode Participation Factors are defined

by $p_{mi} = -\phi_n^T \mathbf{M}_i$ which i is equal to x, y or z.

Two major problems must be solved in order to obtain an approximate response spectrum solution to this equation. First, for each direction of ground motion maximum peak forces and displacements must be estimated. Second, after the response for the three orthogonal directions is solved it is necessary to estimate the maximum response due to the three components of earthquake motion acting at the same time. This section will address the modal combination problem due to one component of motion only.

For input in one direction only, Equation is written as

$$\ddot{y}(t)_n + 2\zeta_n \omega_n \dot{y}(t)_n + \omega_n^2 y(t)_n = p_{ni} \ddot{u}(t)_g$$

Given a specified ground motion $\ddot{u}(t)_g$, damping value and assuming $\zeta_n = -1.0$, it is possible to solve Equation

(15.2) at various values of ζ_n and plot a curve of the

maximum peak response $y(\omega)_{MAX}$. For this acceleration input, the curve is by definition the displacement response spectrum for the earthquake motion.

A different curve will exist for each different value of damping. A plot of $\omega y(\omega)_{MAX}$ is defined as the

pseudo-velocity spectrum and a plot of $\omega^2 y(\omega)_{MAX}$ is defined as the pseudo-acceleration spectrum. These three curves are normally plotted as one curve on special log paper. However, these pseudo values have minimum physical significance and are not an essential part of a response spectrum analysis. The true values for maximum velocity and acceleration must be calculated from the solution of Equation. There is a mathematical relationship, however, between the pseudo-acceleration spectrum and the total acceleration spectrum. The total acceleration of the unit mass, single degree-of-freedom system, governed by Equation below,

$$\ddot{u}(t)_T = \ddot{y}(t) + \ddot{u}(t)_g$$

Therefore, for the special case of zero damping, the total acceleration of the system is equal to $\omega^2 y(t)$. For this reason, the displacement response spectrum curve is normally not plotted as modal displacement $y(\omega)_{MAX}$ vs ω . It is standard to present the curve

in terms of $S(\omega)$ vs. a period T in seconds. Where,

$$S(\omega)_a = \omega^2 y(\omega)_{MAX} \quad \text{and} \quad T = \frac{2\pi}{\omega}$$

The pseudo-acceleration spectrum $S(\omega)_a$, curve has the units of acceleration vs. period which has some physical significance for zero damping only. It is apparent that all response spectrum curves represent the properties of the earthquake at a specific site and are not a function of the properties of the structural system. After estimation is made of the linear viscous damping

properties of the structure, a specific response spectrum curve is selected.

Factor Influencing Response Spectra

The response spectral values depends upon the following parameters,

- I) Energy release mechanism
- II) Epicentral distance
- III) Focal depth
- IV) Soil condition
- V) Richter magnitude
- VI) Damping in the system
- VII) Time period of the system

Calculation of Modal Response

The maximum modal displacement, for a structural model, can now be calculated for a typical mode n with period T_n and corresponding spectrum response

value $S(\omega_n)$. The maximum modal response associated with period T_n is given by

$$y(T_n)_{MAX} = \frac{S(\omega_n)}{\omega_n^2}$$

The maximum modal displacement response of the structural model is calculated from

$$\mathbf{u}_n = y(T_n)_{MAX} \phi_n$$

The corresponding internal modal forces f_{kn} , are calculated from standard matrix structural analysis using the same equations as required in static analysis.

Design of Earthquake Resistant Structure

Based on Codal Provisions General principles and design philosophy for design of earthquake-resistant structure are as follows:

- a) The characteristics of seismic ground vibrations at any location depends upon the magnitude of earth quake, its depth of focus, distance from epicenter, characteristic of the path through which the waves travel, and the soil strata on which the structure stands. Ground motions are predominant in horizontal direction.
- b) Earthquake generated vertical forces, if significant, as in large spans where differential settlement is not allowed, must be considered.

c) The response of a structure to the ground motions is a function of the nature of foundation soil, materials size and mode of construction of structures, and the duration and characteristic of ground motion.

d) The design approach is to ensure that structures possess at least a minimum strength to withstand minor earthquake (DBE), which occur frequently, without damage; resist moderate earthquake without significant damage though some nonstructural damage may occur, and aims that structures withstand major earthquake (MCE) without collapse.

Actual forces that appeared on structures are much greater than the design forces specified here, but ductility, arising due to inelastic material behavior and detailing, and over strength, arising from the additional reserve strength in structures over and above the design strength are relied upon to account for this difference in actual and design lateral forces.

e) Reinforced and pre-stressed members shall be suitably designed to ensure that premature failure due to shear or bond does not occur, as per IS:456 and IS:1343.

f) In steel structures, members and their connections should be so proportioned that high ductility is obtained.

g) The soil structure interaction refers to the effect of the supporting foundation medium on the motion of structure. The structure interaction may not be considered in the seismic analysis for structures supporting on the rocks.

h) The design lateral forces shall be considered in two orthogonal horizontal directions of the structures. For structures, which have lateral force resisting elements in two orthogonal directions only, design lateral force must be considered in one direction at a time.

Structures having lateral resisting elements in two directions other than orthogonal shall be analyzed according to clause 2.3.2 IS 1893 (part 1): 2002. Where both horizontal and vertical forces are taken into account, load combinations must be according to clause 2.3.3 IS 1893 (part 1): 2002.

i) When a change in occupancy results in a structure being re-classified to a higher importance factor (I), the structure shall be confirm to the seismic requirements of the new structure with high importance factor.

Typical Response Spectrum Curves

A ten second segment of the Loma Prieta earthquake motions, recorded on a soft site in the San Francisco Bay Area, is shown in Figure 1.1. The record has been corrected, by use of an iterative algorithm, for zero displacement, velocity and acceleration at the beginning and end of the ten second record. For the earthquake motions given in Figure 1.1a, the response spectrum curves for displacement and pseudo-acceleration are summarized in Figure 1.2a and 1.2b. The velocity curves have been intentionally omitted since they are not an essential part of the response spectrum method. Furthermore, it would require considerable space to clearly define terms such as peak ground velocity, pseudo velocity spectrum, relative velocity spectrum and absolute velocity spectrum.

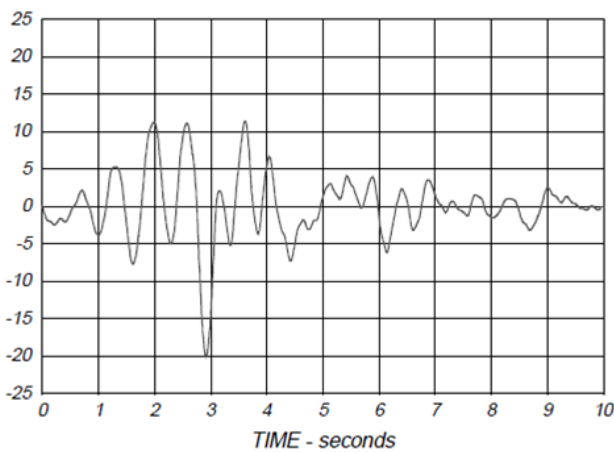


Figure 1.1a. Typical Earthquake Ground Acceleration - Percent of Gravity

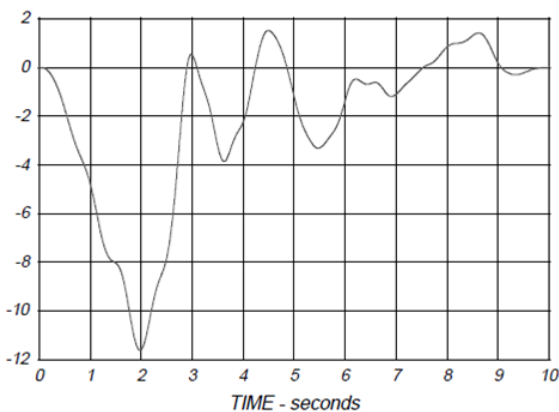


Figure 1.1b. Typical Earthquake Ground Displacements - Inches

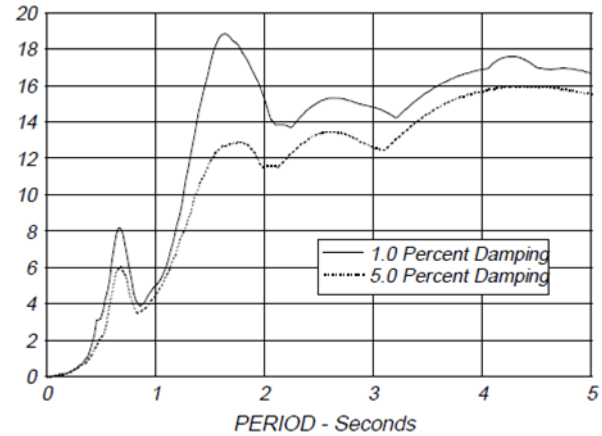


Figure 1.2a. Relative Displacement Spectrum

$y(\omega)_{MAX}$ - Inches

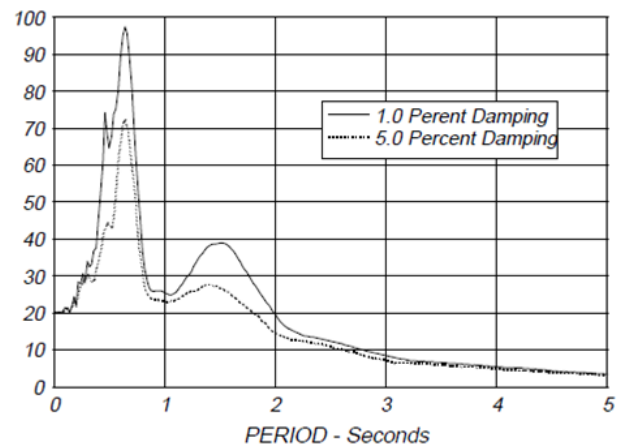


Figure 1.2b. Pseudo Acceleration Spectrum

$S_a = \omega^2 y(\omega)_{MAX}$ - Percent of Gravity

The maximum ground acceleration, for the earthquake defined by Figure.1.1a, is 20.01 percent of gravity at 2.92 seconds. It is important to note that the pseudo acceleration spectrum, shown in Figure, has the same value for a very short period system. This is due to the physical fact that a very rigid structure moves as a rigid body and the relative displacements within the structure are equal to zero as indicated by Figure 1. Also, the behavior of a rigid structure is not a function of the viscous damping value. The maximum ground displacement shown in Figure 1 is -11.62 inches at 1.97 seconds.

The CQC Method Of Modal Combination

The most conservative method that is used to estimate a peak value of displacement or force within a structure is to use the sum of the absolute of the modal response values. This approach assumes that the maximum modal values, for all modes, occur at the same point in time. Another very common approach is to use the Square Root of the Sum of the Squares, SRSS, on the maximum modal values in order to estimate the values of displacement or forces. The SRSS method assumes that all of the maximum modal values are statistically independent. For three dimensional structures, in which a large number of frequencies are almost identical, this assumption is not justified.

The relatively new method of modal combination is the Complete Quadratic Combination, CQC, method [2] that was first published in 1981. It is based on random vibration theories and has found wide acceptance by most engineers and has been incorporated as an option in most modern computer programs for seismic analysis.

Because many engineers and building codes are not requiring the use of the CQC method, one purpose of this chapter is to explain by example the advantages of using the CQC method and illustrate the potential problems in the use of the SRSS method of modal combination. The peak value of a typical force can now be estimated, from the maximum modal values, by the CQC method with the application of the following double summation equation:

$$F = \sqrt{\sum_n \sum_m f_n \rho_{nm} f_m} \tag{15.9}$$

where, f_n is the modal force associated with mode n . The double summation is conducted over all modes. Similar equations can be applied to node displacements, relative displacements and base shears and overturning

moments. The cross-modal coefficients ρ_{nm} , for the CQC method with constant damping are

$$\rho_{nm} = \frac{8\zeta^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\zeta^2r(1+r)^2} \tag{15.10}$$

where $r = \omega_n / \omega_m$ and must be equal to or less than 1.0. It is important to note that the cross-modal coefficient array is symmetric and all terms are positive

Basic Equations for Calculation of Spectral Forces

The stated design criterion implies that a large number of different analyses must be conducted in order to determine the maximum design forces and stresses. It will be shown, in this section, that maximum values for all members can be exactly evaluated from one computer run in which two global dynamic motions are applied.

Furthermore, the maximum member forces calculated are invariant with respect to the selection system.

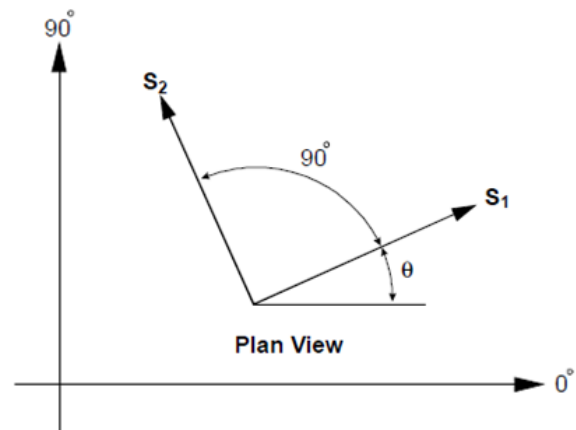


Figure 1.7. Definition of Earthquake Spectra Input

Figure 1.7 indicates that the basic input spectra S_1 and S_2 are applied at an arbitrary angle θ . At some typical point within the structure, a force, stress or displacement F is produced by this input. In order to simplify the analysis, it will be assumed that the minor input spectrum is some fraction of the major input spectrum. Or,

$$S_2 = aS_1 \tag{15.11}$$

Where, a is a number between 0 and 1.0.

Recently, Menun and Der Kiureghian [3] presented the CQC3 method for the combination of the effects of orthogonal spectrum.

The fundamental CQC3 equation for the estimation of a peak value is

$$F = [F_0^2 + a^2 F_{90}^2 - (1 - a^2)(F_0^2 - F_{90}^2) \sin^2 \theta + 2(1 - a^2) F_{0-90} \sin \theta \cos \theta + F_z^2]^{\frac{1}{2}} \quad (15.12)$$

Where,

$$F_0^2 = \sum_n \sum_m f_{0n} \rho_{nm} f_{0m} \quad (15.13)$$

$$F_{90}^2 = \sum_n \sum_m f_{90n} \rho_{nm} f_{90m} \quad (15.14)$$

$$F_{0-90} = \sum_n \sum_m f_{0n} \rho_{nm} f_{90m} \quad (15.15)$$

$$F_z^2 = \sum_n \sum_m f_{zn} \rho_{nm} f_{zm} \quad (15.16)$$

in which f_{0n} and f_{90n} are the modal values produced by 100 percent of the lateral spectrum applied at 0 and 90 degrees respectively and f_{zn} is the modal response from the vertical spectrum which can be different from the lateral spectrum.

It is important to note that for equal spectra $a = 1$, the value F is not a function of θ and the selection of the analysis reference system is arbitrary. Or,

$$F_{MAX} = \sqrt{F_0^2 + F_{90}^2 + F_z^2} \quad (15.17)$$

This indicates that it is possible to conduct only one analysis, with any reference system, and the resulting structure will have all members that are designed to equally resist earthquake motions from all possible directions. This method is acceptable by most building codes.

The General CQC3 Method

For $a=1$ the CQC3 method reduces to the SRSS method. However, this can be over conservative since real ground motions of equal value in all directions have not been recorded. Normally, the value of θ in Equation (15.12) is not known; therefore, it is necessary to calculate the critical angle that produces the maximum response.

Differentiation of Equation (15.12) and setting the results to zero yields

$$\theta_{\sigma} = \frac{1}{2} \tan^{-1} \left[\frac{2F_{0-90}}{F_0^2 - F_{90}^2} \right] \quad (15.17)$$

Two roots exist for Equation (15.17) that must be checked in order that the following equation is maximum,

$$F_{MAX} = [F_0^2 + a^2 F_{90}^2 - (1 - a^2)(F_0^2 - F_{90}^2) \sin^2 \theta_{\sigma} - 2(1 - a^2) F_{0-90} \sin \theta_{\sigma} \cos \theta_{\sigma} + F_z^2]^{\frac{1}{2}} \quad (15.18)$$

At the present time no specific guidelines have been suggested for the value of a . Reference [3] presented an example with values a between 0.50 and 0.85.

Limitations Of The Response Spectrum Method

It is apparent that use of the response spectrum method has limitations, some of which can be removed by additional development. However, it will never be accurate for nonlinear analysis of multi-degree of freedom structures. The author believes that in the future more time history dynamic response analyses will be conducted and the many approximations associated with the use of the response spectrum method will be avoided.

RESULT

In this chapter it has been illustrated that the response spectrum method of dynamic analysis must be used carefully. The CQC method should be used to combine modal maxima in order to minimize the introduction of avoidable errors. The increase in computational effort, as compared to the SRSS method, is small compared to the total computer time for a seismic analysis. The CQC method has a sound theoretical basis and has been accepted by most experts in earthquake engineering. The use of the absolute sum or the SRSS method for modal combination cannot be justified.

In order for a structure to have equal resistance to earthquake motions from all directions, the CQC3 method should be used to combine the effects of earthquake spectra applied in three dimensions. The percentage rule methods have no theoretical basis and are not invariant with respect to the reference system. Engineers, however, should clearly understand that the response spectrum method is an approximate method used to estimate maximum peak values of displacements and forces and that it has significant limitations. It is restricted to linear elastic analysis in which the damping

properties can only be estimated with a low degree of confidence. The use of nonlinear spectra, which are commonly used, has very little theoretical background and should not be used for the analysis of complex three dimensional structures. For such structures, true nonlinear time-history response should be used.

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