

## Performance Analysis of Pilot Symbol Aided Channel Estimation Algorithms under Rayleigh Distribution

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### ABSTRACT

*In order to reduce the noise elements effect, pilotsymbolaided (PSA) channel estimation (CE) algorithms based on the transform domainsuch as DFT, DCT appear attractive owing to their capacity. Here, in this paper we introduced an improved channel estimation algorithm based on shortened DCT (S-DCT) with Rayleigh distribution (RD) to eliminate the energy leakage by using the symmetric property and also compared with the existing channel estimation methods such as Least Square (LS), DFT and mirror weighted DCT even that of S-DCT with additive white Gaussian noise (AWGN) distribution. Simulation results demonstrate that the S-DCT-RDcan reduce the energy leakage more efficiently, and performed far better than the existing CE algorithms.*

### INTRODUCTION

OFDM is a multiple access technique [1] in which the symbol duration will be increased by transmitting the large number of narrowband subchannels over a large bandwidth in parallel, which in results the reduction of inter symbol interference (ISI) [2]. For fighting against multipath fading while transmitting the high bit rate data over wireless mobile environments is an important task [2]. To do so OFDM is used as an effective approach. Major advantages of OFDM systems are

- High spectral efficiency
- Simple digital realization by using the FFT operation
- Due to the ISI avoidance, the complexity in the receiver will be reduced

- Various modulation schemes will be used to achieve the best performance of the system

Due to the above mentioned advantages, OFDM has been used in many wireless applications such as Wireless Personal Area Network (WPAN), Wireless Local Area Network (WLAN) [4], Wireless Metropolitan Area Network (WMAN), Digital Audio Broadcasting (DAB) [3] and Digital Video Broadcasting (DVB) [5]. It is also being considered for IEEE 802.20, 802.16 [6], [7] and 3GPP-LTE. With the use of cyclic prefix for eliminating the effect of ISI, there is a need for a simple one tap equalizer at the OFDM receiver. OFDM brings in unparalleled bandwidth savings, which leads to high spectral efficiency. The channel coefficients of OFDM should be estimated with the minimum error to achieve the more potential advantages with higher efficiency.

### System Model

The general OFDM system model has given in fig1. Firstly, the binary data has given as an input to the M-QAM/QPSK modulator. After mapping and grouping the input data the comb-pilot insertion will be done to split the transmitted data into  $N$  low rate modulated symbols  $X(k)$ , where  $k= 0,1,2, \dots, N-1$ . Then the time domain signal  $x(n)$  can be written as a frequency domain signal  $X(k)$  after performing the IFFT:

$$x(n) = IDFT\{X(k)\}$$

$$IDFT\{X(k)\} = \sum_{k=0}^{N-1} X(k) \exp(j2\pi nk/N) \quad (1)$$

Where  $N$  is the number sub carriers

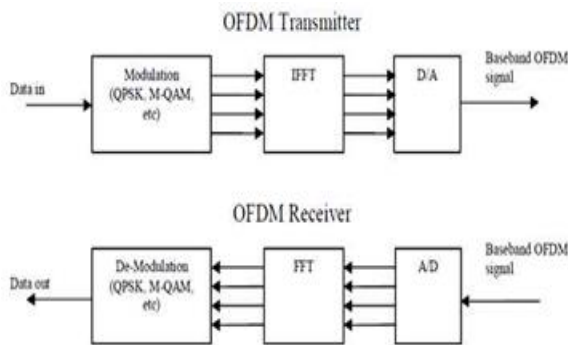


Fig1. General OFDM block diagram

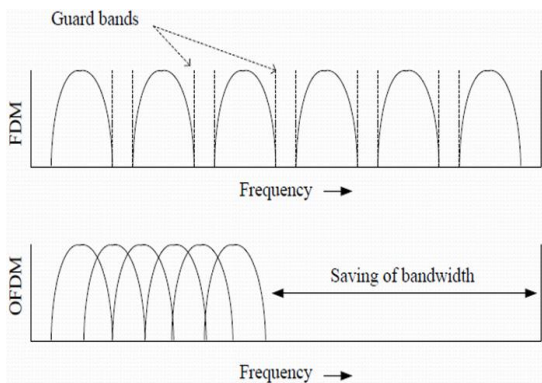


Fig2. Comparison of FDM vs OFDM

$$x_g(n) = \begin{cases} x(N_g + n) & n = -N_g, -N_g + 1, \dots, -1 \\ x(n) & n = 0, 1, \dots, N - 1 \end{cases} \quad (2)$$

Where  $N_g$  is the cyclic prefix (CP) length which reduces the ISI and as well as inter carrier interference (ICI).

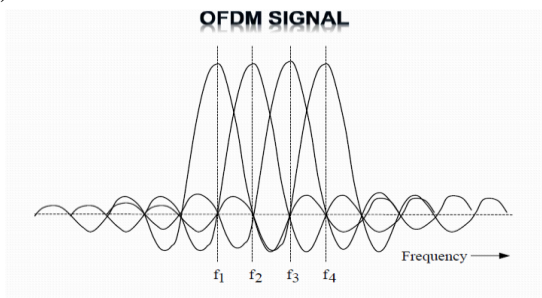


Fig3. Output of the OFDM transmitter

The impulse response of a discrete channel can be described by a formula:

$$h(n) = \frac{1}{N} \sum_{l=0}^{L-1} \alpha_l e^{-j\frac{\pi(n+(N-1)\tau_l)}{N}} \frac{\sin(\pi\tau_l/T_c)}{\sin(\pi(\tau_l/T-N)/N)} \quad (3)$$

Where  $T_c$  = sampling period

$L$  = number of multi paths and  $\tau_l$  = time delay of  $L$

### LITERATURE REVIEW

In the past decades, there are so many channel estimation schemes have been proposed. Among those, the channel estimations based on PSA will help us to increase data rate. There are two parts in PSA-CE methods, those are: No prior information of channel and Prior information of channel based methods. The authors in [8] and [9], proposes a least squares (LS) approach and Minimum Mean Square Error (MMSE) which are categorized under no prior information of channel. Among those, LS estimator is a simplest method and it has lower computational complexity when we compared with the MMSE estimator. But, however the results of MMSE are better than the LS estimator. Different interpolation methods are given in [9] and [10], which will be used in PSA-CE. Compared with the existing methods, transformation based methods will perform excellent with moderate complexity increase. In [11], the author explained DFT based CE method. It can improve the system efficiency by reducing the computational complexity by exploiting the fast Fourier transform (FFT) algorithm. However, it can't provide an excellent performance when a sample spaced path delay will be existed by the multi path fading channels. To overcome this drawback, windowed DFT-CE method has been proposed in [12] to improve the performance of the system. But, it reduces the utilization of frequency band. Then after to improve the performance of the system more accurately, the DCT based PSA-CE methods have been proposed in [13], [14] and [15].

These methods have got excellent performance at high SNR region, but at low SNR values it gives very poor performance. In order to improve the system performance even at low SNR values, here in this paper we proposed a novel PSA-CE method based on S-DCT-RD. the proposed algorithm utilizes the DCT characteristics which compresses the power to the low frequency region. By observing the simulation results, proposed algorithm has dominated the existing PSA-CE methods with higher SNR and lower bit error rate (BER) values.

**PROPOSED ALGORITHM**

If the length of the channel is less than the CP then the received signal can be expressed as follows:

$$y_g(n) = x_g(n) \otimes h(n) + w(n)$$

When  $n = 0, 1, 2, \dots, N - 1$ ,  $\otimes$  = circular convolution and  $w(n)$  = additive white gaussian noise (AWGN). After applying discrete Fourier transform (DFT)

$$Y(k) = \sum_{n=0}^{N-1} y(n) e^{-j2\pi nk/N} = X(k)H(k) + W(k) \quad (4)$$

Where  $H(k) = \sum_{l=0}^{L-1} h(k) e^{-j2\pi k\tau_l/NT_c}$  = transfer function of channel in frequency domain,  $W(k) = \sum_{k=0}^{N-1} w(n) e^{-j2\pi nk/N}$  is known as AWGN noise samples.

**LS Approach**

It is very simple and flexible approach, the channelestimation can be done by multiplying the sub symbols of received pilot carriers with the inverse of the sub symbols of reference pilot carriers, which can be described as follows:

$$\hat{H}_p^{LS} = X_p^{-1} Y_p = [X_{p_0}^{-1} Y_{p_0} \dots \dots X_{p_{p-1}}^{-1} Y_{p_{p-1}}]^T \quad (5)$$

Where  $X_p$  = reference pilot carrier sub symbols

$Y_p$  = received pilot carrier sub symbols and  $(p_0, p_1, \dots \dots p_{p-1})$  are the set of sub carriers which will be used to carry sub symbols of pilot. Then after, it estimates the channel by interpolating  $\hat{H}_{pz}^{LS}$ , which are obtained at positions of pilot, where  $z = 0, 1, \dots, P - 1$ , over the entire band to get the  $\hat{H}_{LS}(k)$ .

**DFT Approach**

The frequency response can be estimated according to eq. (4)

$$\hat{H}_p(k) = \frac{Y_p(k)}{X_p(k)} + \frac{W(k)}{X_p(k)} = H_p(k) + \frac{W(k)}{X_p(k)} \quad (6)$$

Where  $H_p(k)$  is channel estimation value in a pilot symbol,  $k = 0, 1, 2, \dots, M - 1$  and  $M$  = number of pilot sub carriers.

The phase of  $\hat{H}_p(k)$  executes pre-deflection.

$$H'_p(k) = \hat{H}_p(k) e^{j2\pi\alpha/M} \quad (7)$$

$\alpha$  is a minimal normalized integer of channel delay. After applying the IDFT to the eq. (7) then the channel response can be obtained by using

$$\hat{h}_p = IDFT\{H'_p(k)\} = \frac{1}{M} \sum_{k=0}^{M-1} H'_p(k) e^{j\frac{2\pi vk}{M}} \quad (8)$$

Performing DFT to the above equation:

$$\hat{H}(k) = \sum_{n=0}^{N-1} \hat{h}(n) e^{-j\frac{2\pi jk}{N}}, k = 0, 1, 2, \dots, N - 1 \quad (9)$$

The impulse response phase performs anti-deflection

$$\hat{H}(k) = H'(k) e^{-j\pi k\alpha/M} \quad 0 \leq k \leq M - 1 \quad (10)$$

DFT method has implicit periodicity. After DFT operation, original data will become infinite sequence due to the extension of finite non-periodic data. There should be high frequency components if the response of channel is discontinuous at both ends. Due to this interpolation operation DFT causes the aliasing error. To eliminate this error, the mirror weighted DCT approach has been proposed.

**Mirror Weighted DCT Approach (MW-DCT)**

Fig 4 shows that the block diagrams of MW-DCT channel estimator. This algorithm will remove the aliasing effect due to the symmetric property. The channel estimation through MW-DCT can be expressed as follows:

$$\hat{H}_{2M}(k) = \begin{cases} \hat{H}_p(k) & 0 \leq k \leq M - 1 \\ 0 & k = M \\ \hat{H}_p(2M - k) e^{-\frac{j\pi(2M-1)}{M}} & M + 1 \leq k \leq 2M - 1 \end{cases} \quad (11)$$

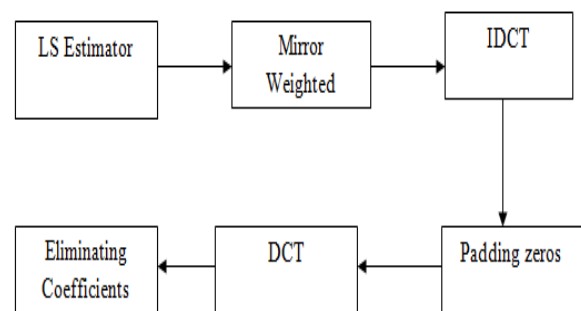


Fig4. Block diagram of MW-DCT Channel Estimator

**Algorithm:**

1. First,  $M$  points  $\hat{H}_p(k)$  will be multiplied by the weighted coefficient i.e., defined by

$$\hat{H}'_p(k) = \begin{cases} \frac{1}{\sqrt{2}} \hat{H}_p(k) & k = 0 \\ e^{\frac{j\pi k}{2M}} \hat{H}_p(k) & 1 \leq k \leq M - 1 \end{cases} \quad (12)$$



2. Now,  $\widehat{H}'_p(k)$  will be transformed to time domain by Inverse DCT

$$\widehat{h}_p(n) = \sum_{k=0}^{M-1} v(k) \widehat{H}'_p(k) \cos\left(\frac{\pi(2k+1)n}{2M}\right) \quad (13)$$

$$m = 0, 1, 2, \dots, M - 1$$

$$\text{Where } v(k) = \begin{cases} \frac{1}{\sqrt{M}} & k = 0 \\ \sqrt{\frac{2}{M}} & k \neq 0 \end{cases}$$

3. Now, the data can be extended to N points by padding the N-M zeros to the  $\widehat{h}(n)$  to get the desired time domain signal. This extended data will be transferred by N-DCT.
4. Finally, to get the interpolated channel response, the weighted coefficients will be eliminated, it can be expressed as follows:

$$H(k) = \begin{cases} \sqrt{\frac{2N}{M}} H_p(k) & k = 0 \\ \sqrt{\frac{N}{M}} H_p(k) & 1 \leq k \leq N - 1 \end{cases} \quad (14)$$

When we compared the both DFT and DCT based channel estimators, the algorithms which are based on DCT can reduce few of high frequency elements in transformation domain. With the operation of MW-DCT, the lack of continuity of a signal problem can be saved and it makes the sequence much smoother in period edges.

### S-DCT Algorithm

This section explains the proposed S-DCT algorithm with AWGN and Rayleigh distribution. Followed by the characteristics of DCT, the original sequence power in frequency domain is digested to the low frequency reign by DCT, the high frequency power reign considered as channel noise. Hence, we adopted a method which filters this surplus noise by setting the threshold as shortened coefficients. The shortened coefficients has been represented by  $N_{sh}$

$$\begin{cases} \widehat{h}_p(k) = \widehat{h}_p(k) & 1 \leq k \leq N_p \leq M \\ \widehat{h}_p(n) = 0 & k > N_p > 0 \end{cases} \quad (15)$$

Where  $\widehat{h}_p(k)$  is known as a time domain signal after applying IDCT

- First, perform the LS algorithm to get the channel response  $\widehat{H}_p(k)$ :

$$H(k) = H_p(k) + W_p(k) \quad (16)$$

Where  $W_p(k)$  = AWGN in pilot sequence

- By using eq. (14), multiply  $\widehat{H}_p(k)$  by the weighted coefficients.

$$\frac{\sum_{i=0}^{N_p} |\widehat{h}_p(n)|^2}{\sum_{i=0}^{M-1} |\widehat{h}_p(n)|^2} = \beta$$

- By using the above formula, the length of shortened coefficients will be confirmed and then the total power will be set to the range from zero to  $N_p$ .
- Now, in order to remove the noise, the estimator can be implemented by M-point DCT and finally it eliminates the coefficients by using the formula (14).
- Now, by using the impulse response of channel in pilot sequences, channel estimation can be obtained in a valid data sequences.

Here, an improved and efficient scheme for DCT based channel estimator has been implemented also compared with LS, DFT and MW-DCT. Proposed algorithm performs effectively to reduce the channel noise and achieve significant performance without any channel prior information.

### SIMULATION RESULTS

All the experiments have been done in the MATLAB 2014a environment with high speed CPU specifications. We examined all the above discussed algorithms performance and compared with each other. Here, QPSK-OFDM system has considered with a number of sub carriers  $N=64, 128, 256, 512, \dots$  and so on, which includes 32 pilot sequences. Assumed the SNR is from 1 to 40 dB, path delay  $1.0e-2$ , symbol rate for AWGN = 10000sec and for Rayleigh = 20 micro seconds, Doppler frequency = 100Hz, power delay = {0, -5, -10} dB are the system specifications for estimating the channel using the proposed and existing CE algorithms. First, randomly generated digital input data has shown in fig6 then the OFDM signal for transmission has shown in fig7. Fig8 shows that the performance of signal to noise ratio (SNR) to mean

squared error (MSE) with the proposed S-DCT and conventional LS, DFT, MW-DCT algorithms. It shows that the proposed CE algorithm with AWGN has the less MSE. Comparison of shortened coefficients in AWGN channel has given in fig9. Performance of proposed algorithm with Rayleigh distribution has displayed in fig10, which performs even better than the AWGN specifications by reducing the channel noise further with reduced error.

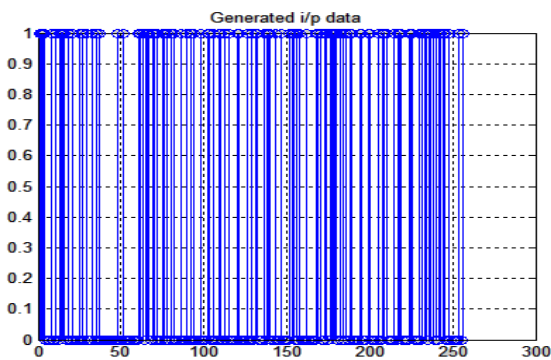


Fig6. Generated input data

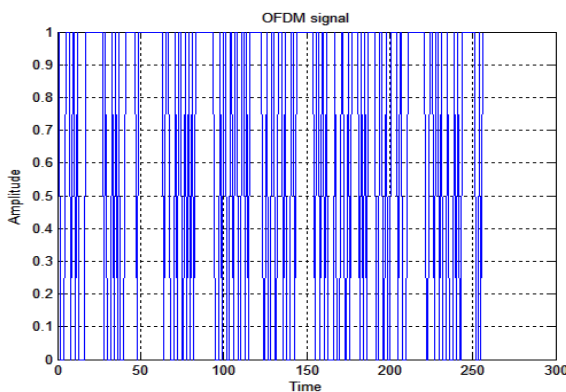


Fig7. OFDM signal for transmission

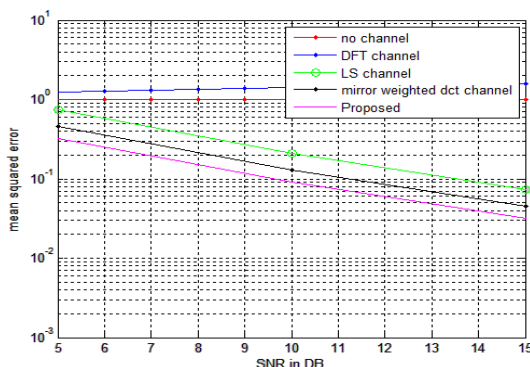


Fig8. Performance of SNR vs MSE with S-DCT-AWGN approach

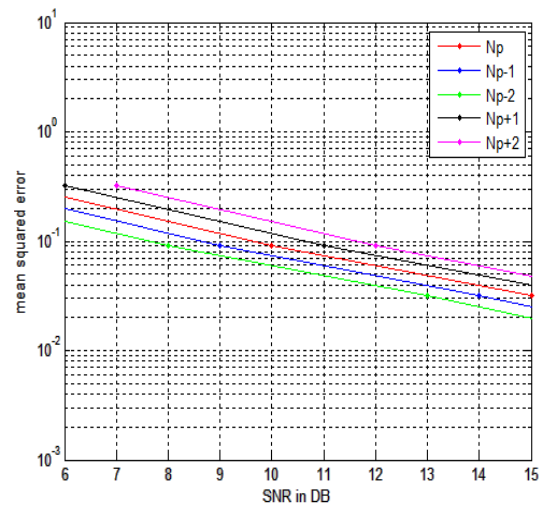


Fig9. Performance of  $N_p$  for S-DCT-AWGN approach

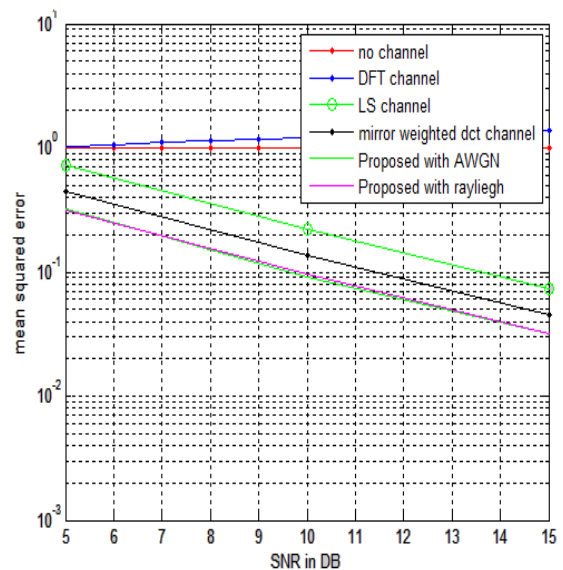


Fig10. Performance of SNR vs MSE with S-DCT-RD

### CONCLUSION

Here, a novel channel estimator algorithm has been proposed by using S-DCT under the Rayleigh distribution. Also performed the analysis of channel estimator algorithms based on LS, DFT and MW-DCT under AWGN channels. Proposed algorithm has been tested with AWGN as well as Rayleigh channel. Simulation results shown that the proposed S-DCT-RD has performed superior to the existing channel estimators in terms of channel noise and mean square error.

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