

Analysis of MIMO Systems with MMSE and ZF_SIC Receive as for different Modulation under various Channels

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Abstract

Consider a multiuser system where an arbitrary number of users communicate with a distributed receive array over independent Rayleigh fading paths. The receive array performs minimum mean squared error (MMSE) or zero forcing (ZF) combining and perfect channel state information is assumed at the receiver. This scenario is well-known and exact analysis is possible when they receive antennas are located in a single array. However, when the antennas are distributed, the individual links all have different average signal to noise ratio (SNRs) and this is a much more challenging problem. In this paper, we provide approximate distributions for the output SNR of a ZF receiver and the output signal to interference plus noise ratio (SINR) of an MMSE receiver. In addition, simple high SNR approximations are provided for the symbol error rate (SER) of both receivers assuming M -PSK or M -QAM modulations. These high SNR results provide array gain and diversity gain information as well as a remarkably simple functional link between performance and the link powers.

Index Terms—Macrodiversity, MMSE, ZF, outage probability, optimum combining, zero-forcing, Network MIMO, CoMP.

INTRODUCTION

WITH the advent of space diversity systems, decoupling users through channel aware signal processing techniques in the presence of multiple access interference (MAI) and noise has become an

integral part of the system design. There are various processing techniques now widely adopted in research and standards [1]. Among them, linear combining methods are popular for their simplicity despite the fact that they are not optimum in a maximum likelihood sense. Twokey linear combiners are zero forcing (ZF) and minimummean squared-error (MMSE). Although they are not optimal, the MMSE receiver satisfies an alternative criterion, i.e., it minimizes the mean squared error (MSE) and ZF is known to eliminate MAI completely. The performance analysis of such linear receivers is of great interest in wireless communication [2] as it provides a baseline link level performance metric for the system. Today, performance results for MMSE/ZF receivers are well known for microdiversity systems where co-located diversity antennas at the base station communicate with distributed users [3], [4], [7].

Macro-scale diversity combining has recently become more common from a variety of perspectives [8], [9]. Any system where both transmit and receive antennas are widely separated can be interpreted as a macrodiversity multiple input multiple output (MIMO) system. They occur naturally in network MIMO systems [9], [10] and collaborative MIMO concepts [11, p. 69] and [12].The performance of macrodiversity systems has been investigated via simulation [13], but very few analytical results appear to be available. The reason for the lack of results is the complexity of the channel matrix that arises in macrodiversity systems. When the receive antennas are colocated, classical models and Kronecker correlation matrix has a Wishart form. Here, extensive results in

Multivariate statistics can be leveraged and performance analysis is now well advanced. In contrast, the macrodiversity case violates the Wishart assumptions and there is no such distribution in the literature for macrodiversity channel matrices for finite size systems. This makes most of the analytical work extremely difficult.

The analytical complexity is clearly evident even in the simplest case of a dual source scenario [14]. Despite this complexity, some analytical results are available for the dual user case in [14] for macrodiversity MMSE and ZF receivers. In [14], they consider the statistical properties of the output signal to interference plus noise ratio (SINR)/signal to noise ratio (SNR) of MMSE and ZF receivers respectively and obtain high SNR approximations of the symbol error rate (SER). In [15], the SER performance of macrodiversity maximal ratio combining (MRC) has been exactly derived for arbitrary numbers of users and antenna configurations. The ergodic sum capacity of the macrodiversity MIMO multiple access channels is considered in [16] where tight approximations of ergodic sum capacity are derived in a compact form. Rayleigh fading is assumed for finite system sizes in [14], [15] and [16]. One of the analytical techniques used in [16] is also used here. In [16], sum capacity in logarithmic form is expressed as an exponential and ergodic sum capacity is then written as the mean of a ratio of quadratic forms. In this work, we have a very different starting point and consider the characteristic function (CF) of the SNR/SINR. The exponential in the CF also leads to a mean of a ratio of quadratic forms.

Hence, the two studies produce similar ratios at this point in the analysis and the same technique, namely a Laplace type approximation [27], is employed both here and in [16] to simplify the result. Note that the analysis leading up to the ratio of quadratic forms and following the Laplace approximation is quite specific to the individual problems considered. As a result, [16] gives approximate results for ergodic capacity and here, approximate SER results and SNR/SINR

distributions are obtained. Note that some of the quadratic forms encountered here are of a different form to those in [16]. On another front, an asymptotic large random matrix approach is employed to derive a deterministic equivalent to the ergodic sum capacity in [17]. Similarly, an asymptotic approach is used to study cellular systems with multiple correlated base station (BS) and user antennas in [18], [19]. In this paper, we extend the results in [14] to more general user and antenna configurations. In particular, the contributions made are as follows:

1. We derive the approximate probability distribution function (PDF) and cumulative distribution function (CDF) of the output SINR/SNR of MMSE/ZF receivers. The approximate cumulative distribution functions are shown to have a remarkably simple form as a generalized mixture of exponentials.
2. High SNR approximations for the SER of MMSE/ZF receivers are derived for a range of modulations and these results are used to derive diversity order and array gain results. The high SNR results are simple, have a compact form and can be used to gain further insights into the effects of channel distribution information (CDI) on the performance of macrodiversity MIMO systems.

The rest of the paper is laid out as follows. Motivational example for this work is given in Sec. II. Sec. III describes the system model and receiver types. Sec. IV provides preliminary results which will be used throughout the paper. The main analysis is given in Secs. V and VI. Secs. VII and VIII give numerical results and conclusions, respectively.

MODULATION OF DATA

The data to be transmitted on each carrier is then differentially encoded with previous symbols, then mapped into a Phase Shift Keying (PSK) format. Since differential encoding requires an initial phase reference an extra symbol is added at the start for this purpose. The data on each symbol is then mapped to a phase angle based on the modulation method. For example, for QPSK the phase angles used are 0, 90, 180, and 270 degrees. The use of phase shift keying produces a

constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading.

Inverse Fourier Transform:

After the required spectrum is worked out, an inverse fourier transform is used to find the corresponding time waveform. The guard period is then added to the start of each symbol.

Guard Period:

The guard period used was made up of two sections. Half of the guard period time is a zero amplitude transmission. The other half of the guard period is a cyclic extension of the symbol to be transmitted. This was to allow for symbol timing to be easily recovered by envelope detection. However it was found that it was not required in any of the simulations as the timing could be accurately determined position of the samples.

After the guard has been added, the symbols are then converted back to a serial time waveform. This is then the base band signal for the OFDM transmission.

Channel:

A channel model is then applied to the transmitted signal. The model allows for the signal to noise ratio, multipath, and peak power clipping to be controlled. The signal to noise ratio is set by adding a known amount of white noise to the transmitted signal. Multipath delay spread then added by simulating the delay spread using an FIR filter. The length of the FIR filter represents the maximum delay spread, while the coefficient amplitude represents the reflected signal magnitude.

Receiver:

The receiver basically does the reverse operation to the transmitter. The guard period is removed. The FFT of each symbol is then taken to find the original transmitted spectrum. The phase angle of each transmission carrier is then evaluated and converted back to the data word by demodulating the received

phase. The data words are then combined back to the same word size as the original data.

Preamble:

The preamble is used for both frequency synchronization and channel estimation. For the proposed frequency synchronization algorithm a repetition of the training symbol is required. For the estimation of the MIMO channel, it is important that the subchannels from the different TX antennas to every RX antenna can be uniquely identified. To achieve that, the preambles on the different TX antennas have to be orthogonal and shift-orthogonal for at least the channel length.

Frame detection:

The task of the frame detection (FD) is to identify the preamble in order to detect packet arrival. This preamble detection algorithm can also be used as a coarse timing (CT) algorithm, since it inherently provides a rough estimate of the starting point of the packet.

Symbol timing:

The symbol timing in an OFDM system decides where to place the start of the FFT window within the OFDM symbol. Although an OFDM system exhibits a guard interval (GI), making it somewhat robust against timing offsets, a non-optimal symbol timing will cause more ISI and inter-carrier interference (ICI) in delay spread environments. This will result in a performance degradation.

Frequency synchronization:

The frequency synchronization has to correct for the frequency offset, which is caused by the difference in oscillator frequencies at the transmitter and the receiver. We estimate this frequency offset and compensate the received signals for it. The frequency offset can be estimated using the phase of the complex correlation between the two consecutive received training symbols

Channel estimation:

When time synchronization is performed at the receiver and after the received signals are corrected for the frequency offset, the channel can be estimated using the known training symbols within the preamble. When the timing is performed correctly, we know which received samples correspond to the training part.

MIMO COMMUNICATION SYSTEM

The use of multiple antennas so that the transmitter and the receiver, which is commonly referred as MIMO, is a popular research area in wireless communications literature because of its reliability and spectral efficiency. With the growth of applications that demand better quality of services, higher throughput and bandwidth, MIMO communication has emerged as a promising technology. The ideas behind the MIMO communication are either creating a multiple data pipes to increase the data rate and/or adding diversity to improve the reliability. The former idea is achieved through use of SM technique, which offers multiplexing gain, with effective detection algorithm at the receiver.

Analytical MIMO Channel Models:

It is important to know the characteristic behaviour of the MIMO channel in order to design good detection algorithms at the receiver. For a $N_t \times N_r$ MIMO systems where N_t and N_r denote the number of transmit and receive antennas, respectively, the MIMO channel matrix at a given time instant given as below,

$$H = \begin{bmatrix} h_{1,1} & h_{2,1} & \dots & h_{N_t,1} \\ h_{2,1} & h_{2,2} & \dots & h_{N_t,2} \\ \dots & \dots & \dots & \dots \\ h_{1,N_r} & h_{2,N_r} & \dots & h_{N_t,N_r} \end{bmatrix} \quad (2.1)$$

Where $h_{i,j}$ is the channel coefficient between the i^{th} transmit and j^{th} receive antenna. In most of the works, those coefficients are chosen independent identically distributed (i.i.d) complex Gaussian random variables, however antenna spacings and scattering

properties of the environment introduce correlation. Whereas rich scattering in the environment and adequate antenna spacings ensures the decorrelation of the MIMO channel elements.

Rayleigh Fading Channels:

When the spatial distance between antennas and angular spreading is large enough, the channel coefficients are assumed to be uncorrelated. Also, if all the channel elements have the same average power, the correlation matrix is proportional to unity. In this case, the complex fading coefficients $h_{i,j}$'s are assumed to be a zero-mean unit variance complex Gaussian random variable with independent real and imaginary parts. Equivalently, $h_{i,j}$'s have uniform phase and Rayleigh amplitude.

However, as it is pointed out in limited angular spread and limited distance between antennas cause the channels become correlated. Furthermore, if there is a strong LOS component, the channel statistics become Rician distributed. In addition, the use of polarization diversity creates gain imbalances between elements of the MIMO channel matrix since the vertical and horizontal polarizations have different propagation characteristics.

Based on the measurement results, a simple stochastic MIMO channel model is developed in, referred as Kronecker product model in the literature, that takes into account the correlation effects both at the transmitter and the receiver side. However, we assume that all antenna elements have the same polarization and radiation pattern which is not the case in polarization diversity. They separate the correlation effects at both link ends and show in a matrix form as R_{TX} and R_{RX} for the transmitter and the receiver sides, respectively. And the general spatial correlation for the MIMO radio channel is shown as Kronecker product of the R_{TX} and R_{RX} and is given by

$$R_{MIMO} = R_{TX} \otimes R_{RX} \quad (2.2)$$

The elements of the $N_T \times N_T$ transmit and the $N_R \times N_R$ receive correlation matrices are given as

$$\rho_{ij}^{tx} = \langle h_{m,i}, h_{m,j} \rangle, \\ \rho_{ij}^{rx} = \langle h_{i,m}, h_{j,m} \rangle. \quad (2.3)$$

Where ρ_{ij}^{tx} and ρ_{ij}^{rx} denote the complex transmit and receive correlation coefficients, respectively. $\langle x, y \rangle$ is the power correlation coefficient between the complex random variables x and y given as

$$\langle x, y \rangle = \frac{E\{|x|^2|y|^2\} - E\{|x|^2\}E\{|y|^2\}}{\sqrt{E\{|x|^4 - (E\{|x|^2\})^2\}}E\{|y|^4 - (E\{|y|^2\})^2\}}}$$

Rician Fading Channels:

When there is not any obstruct between the transmitter and the receiver, typically a LOS case, received signal consists of a direct wave and number of waves. In this case, channel coefficient would have non-zero mean because of the strong direct path. The sum of direct signal together with a Rayleigh distributed scattered signal results in a signal with a Rician envelope distribution. Under the assumption that the total average signal power is normalized to unity, the pdf of the Rician distribution is given as,

$$p(a) = 2a(1+K)e^{-K-(1+K)a^2 I_0(2a\sqrt{K(K+1)})} a \geq 0$$

Where I_0 is the modified Bessel function of the first kind and zero order and K is the Rician factor which denotes the power ratio of the direct and scattered signal components. For $K=0$ Rician distribution becomes the Rayleigh distribution. For higher values of K , the channel approaches to the LOS channel. When K goes to infinity, the channel is more like AWGN channel

Thus, MIMO channel matrix can be decomposed into a LOS and NLOS components as

$$H = \sqrt{\frac{K}{K+1}} \bar{H} + \sqrt{\frac{1}{K+1}} \tilde{H}$$

where \bar{H} stands for the fixed component of the channel (LOS component) and \tilde{H} stands for the variable component to the channel (NLOS component).

Dual-Polarized Channels:

As mentioned in section 2.1.1, low levels of physical separation between antennas introduce correlation that results in a degradation in system performance. Physical separation must be up to λ and 30λ between the transmitter and the receiver antenna elements, respectively, in order to achieve significant multiplexing and diversity gain. On the other hand, with recent amendments in the wireless technology, there is much attention to small hand held devices. Thus, polarization diversity scheme is first proposed in, as a promising alternative to space diversity systems. The technique has recently become of interest, due to the fact that it does not require any increase in bandwidth and any physical separation between the antenna elements.

In polarization diversity scheme, signals are transmitted through horizontal and vertical polarization at the same time and frequency without any requirement of spatial separation. Typically, two polarization schemes are used: horizontal/vertical ($0^\circ/90^\circ$) or slanted ($+45^\circ/-45^\circ$) as shown in Figure 2.1.



Figure 2.1. Schematic of dual-polarized slanted antenna setup both at the transmitter (T_X) and the receiver (R_X) side.

CHANNEL STATE INFORMATION

Channel state information:

In wireless communications, channel state information (CSI) refers to known channel properties of a communication link. This information describes how a signal propagates from the transmitter to the

receiver and represents the combined effect of, for example, scattering, fading, and power decay with distance. The CSI makes it possible to adapt transmissions to current channel conditions, which is crucial for achieving reliable communication with high data rates in multi antenna systems.

CSI needs to be estimated at the receiver and usually quantized and fed back to the transmitter (although reverse-link estimation is possible in TDD systems). Therefore, the transmitter and receiver can have different CSI. The CSI at the transmitter and the CSI at the receiver are sometimes referred to as CSIT and CSIR, respectively.

Different kinds of channel state information:

Here are basically two levels of CSI, namely instantaneous CSI and statistical CSI. Instantaneous CSI (or short-term CSI) means that the current channel conditions are known, which can be viewed as knowing the impulse response of a digital filter. This gives an opportunity to adapt the transmitted signal to the impulse response and thereby optimize the received signal for spatial multiplexing or to achieve low bit error rates.

Statistical CSI (or long-term CSI):

It means that a statistical characterization of the channel is known. This description can include, for example, the type of fading distribution, the average channel gain, the line-of-sight component, and the spatial correlation. As with instantaneous CSI, this information can be used for transmission optimization.

The CSI acquisition is practically limited by how fast the channel conditions are changing. In fast fading systems where channel conditions vary rapidly under the transmission of a single information symbol, only statistical CSI is reasonable. On the other hand, in slow fading systems instantaneous CSI can be estimated with reasonable accuracy and used for transmission adaptation for some time before being outdated.

In practical systems, the available CSI often lies in between these two levels; instantaneous CSI with some

estimation/quantization error is combined with statistical information.

Mathematical description:

In a narrowband flat-fading channel with multiple transmit and receive antennas (MIMO), the system is modeled as

$$y = Hx + n$$

where y and x are the receive and transmit vectors, respectively, and H and n are the channel matrix and the noise vector, respectively. The noise is often modeled as circular symmetric complex normal with

$$n \sim CN(0, S)$$

where the mean value is zero and the noise covariance matrix S is known.

Instantaneous CSI:

Ideally, the channel matrix H is known perfectly. Due to channel estimation errors, the channel information can be represented as

$$\text{vec}(H_{\text{estimate}}) \sim CN(\text{vec}(H), R_{\text{error}})$$

where H_{estimate} is the channel estimate and R_{error} is the estimation error covariance matrix. The vectorization $\text{vec}()$ was used to achieve the column stacking of H , as multivariate random variables are usually defined as vectors.

Statistical CSI:

In this case, the statistics of H are known. In a Rayleigh fading channel, this corresponds to knowing that $\text{vec}(H) \sim CN(0, R)$ for some known channel covariance matrix R .

Estimation of CSI:

Since the channel conditions vary, instantaneous CSI needs to be estimated on a short-term basis. A popular approach is so-called training sequence (or pilot sequence), where a known signal is transmitted and the channel matrix H is estimated using the combined knowledge of the transmitted and received signal.

Let the training sequence be denoted, P_1, \dots, P_N where the vector P_i is transmitted over the channel as

$$y_i = HP_i + n_i$$

By combining the received training signals y_i for, $i = 1, \dots, N$ the total training signalling becomes

$$Y = [y_1, \dots, y_N] = HP + N$$

with the training matrix $P = [p_1, \dots, p_N]$ and the noise matrix $N = [n_1, \dots, n_N]$

With this notation, channel estimation means that H should be recovered from the knowledge of Y and P .

Least-square estimation:

If the channel and noise distributions are unknown, then the least-square estimator (also known as the minimum-variance unbiased estimator) is

$$H_{LS\text{-estimate}} = YP^H(PP^H)$$

where $()^H$ denotes the conjugate transpose. The estimation Mean Square Error (MSE) is proportional to $\text{tr}(PP^H)^{-1}$

where tr denotes the trace. The error is minimized when PP^H is a scaled identity matrix. This can only be achieved when N is equal to (or larger than) the number of transmit antennas. The simplest example of an optimal training matrix is to select P as a (scaled) identity matrix of the same size that the number of transmit antennas.

MMSE estimation:

If the channel and noise distributions are known, then this a priori information can be exploited to decrease the estimation error. This approach is known as Bayesian estimation and for Rayleigh fading channels it exploits that

$$\text{vec}(H) \sim \text{CN}(0, R), \quad \text{vec}(N) \sim \text{CN}(0, S)$$

The MMSE estimator is the Bayesian counterpart to the least-square estimator and becomes

$$\begin{aligned} & \text{vec}(H_{MMSE\text{-estimate}}) \\ &= \left(R^{-1} \right. \\ & \left. + (P^T \times I)^H S^{-1} (P^T \times I) \right)^{-1} (P^T \\ & \times I)^H S^{-1} \text{vec}(Y) \end{aligned}$$

where \times denotes the Kronecker product and the identity matrix I has the dimension of the number of receive antennas. The estimation Mean Square Error (MSE) is

$$\text{tr} \left(R^{-1} + (P^T \times I)^H S^{-1} (P^T \times I) \right)^{-1}$$

and is minimized by a training matrix P that in general can only be derived through numerical optimization. But there exist heuristic solutions with good performance based on water filling. As opposed to least-square estimation, the estimation error for spatially correlated channels can be minimized even if N is smaller than the number of transmit antennas. Thus, MMSE estimation can both decrease the estimation error and shorten the required training sequence. It needs however additionally the knowledge of the channel correlation matrix R and noise correlation matrix S . In absence of an accurate knowledge of these correlation matrices, robust choices need to be made to avoid MSE degradation.

CONCLUSION

From the above simulation results, the energy efficiency of the MIMO system good for high amount of power transmission, compared to less power transmission system under equal iterations i.e. from our discussions at equal number of iteration the MIMO system which has more number of active achieves high energy efficiency than with less antenna MIMO system under available unique power resource only. In simple words, by efficiently allocating the power among all the active antennas and by gradually increasing the outage capacity using proposed iterative scheme the energy efficiency of the MIMO-OFDMA system increased which is our "An Efficient Resource Allocation in MIMO-OFDMA Network".

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