Multi Objective Optimization of Planetary Gear Train

A.Sri Harsha
PG Scholar,
UCEK, JNTUK University,
Kakinada, E. G. Dt., AP India.

Dr.K.Mallikarjunarao
Professor in Mechanical Engineering,
JNTUK University,
Kakinada, E. G. Dt., AP India.

Abstract:
Planetary Gear Trains are extensively used for the power transmission. Large reduction in a small volume is possible with planetary gear trains. High efficiency – 95% per stage is common in planetary gear trains. For the automation of car parking systems helical gear box is used. But requirements of the gear box of these kind shall be compact in size and should have high efficiency. For these reasons in this paper a three stage planetary gear train is designed and optimized using Genetic Algorithm. Two objectives functions are considered, minimization of volume and maximization of efficiency. This process helps in finding the optimized design for planetary gear train which can give the same gear ratio within less volume and high efficiency.

Index Terms:
Planetary gears, Volume, Efficiency, Car parking system, Genetic Algorithm.

I. Introduction:
Planetary gear train normally consists of a centrally mounted sun gear, ring gear and several planet gears that are found in between sun gear and ring gear. The planets are connected by a carrier. The main advantage of planetary gear train is that it can maintain high torque density. This torque density can be increased by adding more number of planets thus making multiple mesh points. That means a planetary gear with three planet gears can transfer there times torque of a similar sized, fixed axis standard spur gear set. The Load distribution on multiple gear mesh points implies that the load is supported by N contacts (Where N is the number of planets) increases the torsional stiffness of gearbox by N.

II Literature review:
A literature review on planetary gear train design, optimization and analysis was carried out and a brief report of the same is presented as below: Kalyanmoy Deb et. al. [1] optimized a multi-speed gearbox involving different types of decision variables and objective functions. They demonstrated the use of multi-objective evolutionary algorithm which is capable of solving the multi-objective optimization problem. Tianpei Chen et. al. [2] used differential evolution algorithm based on mathmatica to optimize a 2 stage planetary gear train. They have proved that the optimum design of the planetary gear train could make the volume minimum (as well as the weight) under the conditions of carrying capacity. JelenStefanovic et. al. [3] presented a paper on optimization possibilities of planetary gear trains. They has given original model original model for multi criteria optimization of planetary gear train.
This mathematical model for optimization is defined by the variables, objective functions and conditions required for the proper functioning of the planetary gear train. K Akhil et al. [4] designed modeled and analyzed a 3 stage planetary reduction gear unit which is being used for a flight vehicle. It is designed to meet the output specifications and modeled using CATIA to check the interference. The modelled components are checked using ANSYS to check for their strength. Anjali Gupta et al. [5] optimized a spur gear set using genetic algorithm. They have taken central distance as an objective function and module, number of teeth on pinion are taken design parameters. The bending stress limit and surface stress limit are considered as constraints.

III Methodology of Multi-Objective Optimization:

The Optimization of multi stage planetary gear train contains of the following steps:

a) Determination of reduction ratio for each stage
b) Defining the objective functions, design variables and constraints
c) Solving the optimization problem using NSGA II

Determination of reduction ratio:

The reduction ratio for planetary gear train is calculated by multiplying the reduction ratio of all the stages present in the gear train. Let us assume that R be the overall reduction ratio of the gear train which is having n stages and R1, R2, R3…, Rn be the reduction ratios of the individual stages then overall reduction ratio of the planetary gear train will be,

\[ R = R_1 \times R_2 \times R_3 \times \ldots \times R_n \]

The number of teeth on the ring gear is determined as below:

\[ \text{Reduction Ratio} = \frac{\text{PCD of Ring Gear} + \text{PCD of Sun Gear}}{\text{PCD of Sun Gear}} \]

\[ \frac{t_1 + t_3}{t_1} \]

Where \( PCD = \text{Pitch Circle Diameter} \)

\( t_1 = \text{Number of teeth on sun gear} \)

\( t_3 = \text{Number of teeth on ring gear} \)

The number of teeth on the ring gear is determined by the equation below:

\[ t_3 = t_1 + 2t_2 \quad \text{(1)} \]

Where \( t_2 = \text{Number of teeth on planet gear} \)

The helical gearbox which is being used for automation of Car parking system is having 1410 rpm as input and 18 rpm as output. Input power of this gear train is 32 kW. In this paper a 3 stage planetary gear train is designed which is going to have the same input and output rpm. So the reduction ratio should be the same. So the overall reduction ratio will be:

\[ R = \frac{1410}{18} = 78.33 \]

Total reduction ratio is split in 3 ratios R1, R2, and R3 which represent the reduction ratios for each stage in the 3 stage planetary gear train. Therefore

\[ R = R_1 \times R_2 \times R_3 \]

Here in the 3 stages in the planetary gear train are considered as identical. So the reduction ratio for each stage is same as the other. The reduction ratio for each stage will be:

\[ R = R_1 \times R_1 \times R_1 \]

\[ R_1 \times R_1 = 78.33 \]
So each stage in the gear train will consist a reduction ratio of 4.28.

Defining the Objective functions, Design variables and Constraints:
Minimization of volume and maximization of efficiency are taken as objective functions for this optimization of planetary gear train. Since all the three stages in the gear train are identical optimization of one stage is enough for reducing the complexity. So four design variables are considered namely module m, number of teeth on sun gear t1, number of teeth on planet gear t2 and face width of gear b.

Volume of planetary gear train will be:

\[ V = \frac{\pi}{4} \times m^2 \times t_3^2 \times b \]

From (1) \( t_3 = t_1 + 2t_2 \). Then

\[ V = \frac{\pi}{4} \times m^2 \times (t_1 + 2t_2)^2 \times b \]

Efficiency of planetary gear train is

\[ \eta = 1 - \left( \frac{t_3}{t_3 + t_1 + t_2} \right) \left( \frac{0.15}{t_1} + \frac{0.35}{t_2} + \frac{0.20}{t_3} \right) \]

\[ = 1 - \left( \frac{t_1 + t_2}{2t_2} \right) \left( \frac{0.15}{t_1} + \frac{0.35}{t_2} + \frac{0.20}{t_1 + 2t_2} \right) \]

Constraints:
\[ c(1) = t_1 + t_2 \times \sin \alpha - \sqrt{(t_2 + 2)^2 - (\cos \alpha \times t_2)^2} \]
\[ c(2) = b - 5m \]
\[ c(3) = 20m - b \]
\[ c(4) = (t_1 + t_2) \times \sin(60) - (t_2 + 2) \]
\[ e(1) = 2(1 + \frac{t_2}{t_1}) - 4.28 \]

Where \( t_1 = \) Number of teeth on sun gear, \( t_2 = \) Number of teeth on planet gear, \( b = \) face width, \( m = \) module, \( \alpha = \) Pressure angle.

\( c(a) \geq 0 \), where \( a = 1 \) to 4 and \( e(1) = 0 \). The constraints \( c(1) \) is for interference, \( c(2) \) and \( c(3) \) is for face width, \( c(4) \) is for Planetary condition and \( c(1) \) is for reduction ratio.

Solving the optimization problem:
The Optimization problem is obtained as follows:

Minimum Volume:
\[ V = 0.785 \times x[0] \times x[0] \times (x[1] + (2 \times x[2])) \times (x[1] + (2 \times x[2])) \times x[3] \]

Maximum Efficiency:
\[ \eta = \left( (1+ \frac{x[1]}{(2 \times x[2])}) \times (0.15/x[1]) + (0.35/x[2]) + (0.20/(x[1] + (2 \times x[2])))) \right) - 1 \]

(Since the objective function is of maximization type it is multiplied by -1)

Subjected to:
\[ c(a) \geq 0 \] where \( a = 1,2,3,4. \)
\[ e(1) = 0 \]
\[ 1 \leq x[0] \leq 10 \]
\[ 30 \leq x[1] \leq 60 \]
\[ 61 \leq x[2] \leq 110 \]
\[ 100 \leq x[3] \leq 200 \]

IV Results and Discussion:
The NSGA II method is used to compute the results. NSGA II code is obtained from KanGAL. The obtained volume of single stage of planetary gear train is 0.071 m³ and efficiency is 98.7%. But we have 3 stages and volume of 3 carriers which hold the planets and the occupied by the shafts which connect the 3 stages. Taking all of them into consideration we obtain an overall volume of 0.63 m³. The overall efficiency obtained is 96.1%. The volume of the existing helical gear box is 0.72 m³. So the designed 3 stage planetary gearbox in 13.5% less in volume when compared to the existing helical gear box.

References:

