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Tracking of a High Dynamic Projectile Using Extended Kalman Filter

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ABSTRACT:
A **radar tracker** is a component of a radar system, or an associated command and control system, that associates consecutive radar observations of the same target into tracks. It is particularly useful when the radar system is reporting data from several different targets or when it is necessary to combine the data from several different radars or other sensors. **Projectile motion** is a form of motion in which an object or particle (in either case referred to as a projectile) is thrown near the Earth's surface, and it moves along a curved path under the action of gravity only .The proposed work is to to track the Flying object through Radar Measurement thorough Ground RADAR using and Extended Kalman filter. The entire mathematical model is built and coded using MATLAB. The filter initialization and tuning is carried out and simulation results are presented. Limitations of this work and further scope of the work is also presented.

Keywords:
Radar, Tracking, Kalman Filter, Navigation, Projectile.

Nomenclature:
V_x – Velocity along x-axis,
V_{xo} - Initial velocity along x-axis,
V_y - Velocity along y-axis,
V_{yo} - Initial velocity along y-axis.
g - Acceleration due to gravity.
t - Time taken.
X – Horizontal distance travelled.
Y – Vertical Distance travelled.

1. ROLE OF THE RADAR TRACKER:
A classical rotating air surveillance radar system detects target echoes against a background of noise. It reports these detections (known as "plots") in polar coordinates representing the range and bearing of the target. In addition, noise in the radar receiver will occasionally exceed the detection threshold of the radar's constant false alarm rate detector and be incorrectly reported as targets [1]. The role of the radar tracker is to monitor consecutive updates from the radar system (which typically occur once every few seconds, as the antenna rotates) and to determine those sequences of plots belonging to the same target, whilst rejecting any plots believed to be false alarms. In addition, the radar tracker is able to use the sequence of plots to estimate the current speed and heading of the target. When several targets are present, the radar tracker aims to provide one track for each target, with the track history often being used to indicate where the target has come from.

A radar track will typically contain the following information:

- Position (in two or three dimensions)
- Heading
- Speed
- Unique track number

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Perhaps the most important step is the updating of tracks with new plots. All trackers will implicitly or explicitly take account of a number of factors during this stage, including:

- a model for how the radar measurements are related to the target coordinates
- the errors on the radar measurements
- a model of the target movement
- errors in the model of the target movement

Using these information, the radar tracker attempts to update the track by forming a weighted average of the current reported position from the radar (which has unknown errors) and the last predicted position of the target from the tracker (which also has unknown errors). The tracking problem is made particularly difficult for targets with unpredictable movements (i.e. unknown target movement models), non-Gaussian measurement or model errors, non-linear relationships between the measured quantities and the desired target coordinates, detection in the presence of non-uniformly distributed clutter, missed detections or false alarms. In the real world, a radar tracker typically faces a combination of all of these effects; this has led to the development of an increasingly sophisticated set of algorithms to resolve the problem. Due to the need to form radar tracks in real time, usually for several hundred targets at once, the deployment of radar tracking algorithms has typically been limited by the available computational power.

2. PROJECTILE MOTION:

Projectile motion is a form of motion in which an object or particle (in either case referred to as a projectile) is thrown near the Earth's surface, and it moves along a curved path under the action of gravity only. The implication here is that air resistance is negligible, or in any case is being neglected in all of these equations. The only force of significance that acts on the object is gravity, which acts downward to cause a downward acceleration. Projectile motion formula is given by:

$$X=Vxt;\ Vx=Vx0$$
$$y=Vy0t - \tfrac{1}{2}(gt^{\wedge}2),\quad Vy=Vy0 - gt$$

3. KALMAN FILTER:

“Kalman filter” is an iterative mathematical process that uses a set of equations and consecutive data inputs to quickly estimate the true value, position, velocity etc of the object being measured, when the measured values contain unpredicted or random error, uncertainty or variation [2].The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. Kalman filtering is an algorithm that uses a series of measurements observed over time, containing noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone. The algorithm works in a two-step process [3]. In the prediction step, the Kalman filter produces estimates of the current state variables, along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of error, including random noise) is observed, these estimates are updated.

3.1 Extended Kalman Filter:

In order to make state estimation on nonlinear systems, one of the possible approaches is to linearize the system around its current state and force the filter to use this linearized model. This is done by Extended Kalman Filter (EKF) [4].

3.2 Unscented Kalman Filter:

In order to improve this filter, instead of using linearization to predict the behavior of the system Unscented Transformation can be used. Hence, the Kalman Filter with the unscented transformation is called Unscented Kalman Filter (UKF).

4. EXTENDED CARTESIAN KALMAN FILTER:

The paper is presently focusing on EKF based tracking of Projectile.

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To build the model describing the tracking of High dynamic projectile as states projectile location and velocity in the downrange or x direction and projectile location and velocity in the altitude y direction [4]. Thus, the proposed states are given by

$$x=\begin{bmatrix} x_T \\ \dot{x}_T \\ y_T \\ \dot{y}_T \end{bmatrix}$$

Therefore, when the preceding Cartesian states are chosen the state-space differential equation describing projectile motion becomes

$$\begin{bmatrix} \dot{x}_T \\ \ddot{x}_T \\ \dot{y}_T \\ \ddot{y}_T \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_T \\ \dot{x}_T \\ y_T \\ \dot{y}_T \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 0 \\ -g \end{bmatrix}+\begin{bmatrix}\begin{bmatrix} 0 \\ u_s \\ 0 \\ u_s \end{bmatrix}\end{bmatrix}$$

Notice that in the preceding equation gravity g is not a state that has to be estimated but is assumed to be known in advance, we have also added process noise u_x to the acceleration portion of the equations as protection for efforts that may not be considered by the Kalman filter. From the preceding state –space equation we can see that systems dynamics matrix is given by

$$F=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Because the fundamental matrix for a time-invariant system is given by using the Taylor-series approximation, which is given by

$$\varnothing(t) = I + Ft + \frac{F^2t^2}{2!} + \frac{F^3t^3}{3!}$$

Because

$$F^2=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

All of the higher-order terms of the Taylor-series expansion must be zero, and the fundamental matrix becomes

$$\varnothing(t) = I + Ft=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}t$$

or, more simply,

$$\varnothing(t)=\begin{bmatrix} 0 & t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the discrete fundamental matrix can be found by substituting the sampling time T_S for time t and is given by

$$\varnothing_k=\begin{bmatrix} 0 & T_S & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_S \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since states have been chosen to be Cartesian, the radar measurements r and ϴ will automatically be nonlinear functions of those states. Therefore, the linearized measurement equation is

$$\begin{bmatrix} \Delta\theta^* \\ \Delta r^* \end{bmatrix}=\begin{bmatrix} \frac{\partial\theta}{\partial X_T} & \frac{\partial\theta}{\partial \dot{X}_T} & \frac{\partial\theta}{\partial Y_T} & \frac{\partial\theta}{\partial \dot{Y}_T} \\ \frac{\partial r}{\partial X_T} & \frac{\partial r}{\partial \dot{X}_T} & \frac{\partial r}{\partial Y_T} & \frac{\partial r}{\partial \dot{Y}_T} \end{bmatrix}\begin{bmatrix} \Delta x_T \\ \Delta \dot{x}_T \\ \Delta y_T \\ \Delta \dot{y}_T \end{bmatrix}+\begin{bmatrix} v_\theta \\ v_r \end{bmatrix}$$

Where V_0 and V_r represent the measurement noise on angle and range, respectively. Because the angle from the radar to the projectile is given by

$$\theta = \tan^{-1}\left(\frac{y_t - y_R}{x_T - x_R}\right)$$

Similarly, the range from radar to the projectile is given by

$$r=\sqrt{(x_T - x_R)^2 + (y_t - y_R)^2}$$

The linearized measurement matrix for the above problem is

$$H=\begin{bmatrix} \frac{-(y_t-y_R)}{r^2} & 0 & \frac{(x_t-x_R)}{r^2} & 0 \\ \frac{(x_t-x_R)}{r} & 0 & \frac{(y_t-y_R)}{r} & 0 \end{bmatrix}$$

For this problem it is assumed that we know where the radar is so that x_R and y_R are known & do not have to

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be estimated. The states required for the discrete linearized measurement matrix will be based on the projected state estimate. The discrete measurement noise matrix is given by

$$R_k = \begin{bmatrix} \boldsymbol{\sigma_0^2} & 0 \\ 0 & \boldsymbol{\sigma_r^2} \end{bmatrix}$$

Where $\boldsymbol{\sigma_0^2}$ and $\boldsymbol{\sigma_r^2}$are the variances of the angle noise and range noise measurements, respectively. Similarly the continuous process-noise matrix

matrix is

$$Q(t)=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \varnothing_s & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varnothing_s \end{bmatrix}$$

Where$\varnothing_s$ is the spectral density of the white noise sources assumed to be on the downrange and altitude accelerations acting on the projectile. The discrete process-noise matrix can be derived from the continuous process –noise matrix according to

$$Q_k = \int_0^{T_s} \varnothing(\tau)\, Q\varnothing^T(\tau)\mathrm{d}t$$

Therefore, substitution of the appropriate matrices into the preceding expression yields. Finally, after integration we obtain the final expression for the discrete process-noise matrix to be

$$Q_k=\begin{bmatrix} \frac{T_S^3\varnothing_s}{3} & \frac{T_S^2\varnothing_s}{2} & 0 & 0 \\ \frac{T_S^2\varnothing_s}{2} & T_S\varnothing_s & 0 & 0 \\ 0 & 0 & \frac{T_S^3\varnothing_s}{2} & \frac{T_S^2\varnothing_s}{2} \\ 0 & 0 & \frac{T_S^2\varnothing_s}{2} & T_s\varnothing_s \end{bmatrix}$$

We now have defined all of the matrices required to solve the Riccati equations. The next step is to write down the equations for the Kalman –filter section. First we must be able to propagate the states from the present sampling time to the next sampling time. For the linear filtering problem, the real world was represented by the state –space equation

$$\dot{x} = Fx + Gu + w$$

Where $\boldsymbol{G}$ is a matrix multiplying a known disturbance or control vector $\boldsymbol{u}$ that does not have to be estimated. We can show that the discrete linear Kalman-filtering equation is given by

$$\hat{x}_k = \varnothing_k \hat{x}_{k-1} + G_k u_{k-1} + K_k(z_k - H\varnothing_k \hat{x}_{k-1} - HG_k u_{k-1}$$

Where $\boldsymbol{G}_k$ is obtained from

$$G_k = \int_0^{T_S} \varnothing(\tau) G d\tau$$

If we forget about the gain times the residual portion of the filtering equation, we can see that the projected state is simply

$$\bar{x}_k = \varnothing_k \hat{x}_k + G_k u_{k-1}$$

For this problem

$$G = Gu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g \end{bmatrix}$$

Therefore, the $\boldsymbol{G}_k$ becomes

$$G_k = \int_0^{T_S} \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g \end{bmatrix} \mathrm{d}\tau = \begin{bmatrix} 0 \\ 0 \\ \frac{gT_s^2}{2} \\ -gT_s \end{bmatrix}$$

and our projected state is determined from

$$\bar{x}_k = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix} \hat{x}_{k-1} + \begin{bmatrix} 0 \\ 0 \\ \frac{gT_s^2}{2} \\ -gT_s \end{bmatrix}$$

converting the preceding matrix equation for the projected states to four scalar equations

$\bar{x}_{T\,k}=\hat{x}_{T\,k-1} + T_s\hat{\dot{x}}_{T\,k-1}$

$\bar{\dot{x}}_{T\,k}=\hat{\dot{x}}_{T\,k-1}$

$\bar{y}_{T\,k}=\hat{y}_{T\,k-1} + T_s\hat{\dot{y}}_{T\,k-1}$-0.5g$T_s^2$

$\bar{\dot{y}}_{T\,k}=\hat{\dot{y}}_{T\,k-1} - gT_s$

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The next portion of the Kalman filter uses gains times residuals. Because the measurements are non linear, the residuals are simply the measurements minus the projected values of the measurements (i.e., we do not want to use the linearized measurement matrix) [3].Therefore, the projected value of the angle and range from the radar to the projectile must be based upon the projected state estimates or

$$\bar{\varnothing}_k = tan^{-1}\left(\frac{(\bar{y}_{T_{k-1}} - y_R)}{\bar{x}_{T_{k-1}} - x_R}\right)$$

$$\bar{r}_k = \sqrt{(\bar{x}_{T_{k-1}} - x_R)^2 + (\bar{y}_{T_{k-1}} - y_R)^2}$$

Now the extended Kalman-filtering equations can be written simply as

$$\hat{x}_{T_k} = \bar{x}_{T_k} + K_{11_k}(\theta_k^* - \bar{\theta}_k) + K_{12_k}(r_k^* - \bar{r}_k)$$
$$\hat{\dot{x}}_{T_k} = \bar{\dot{x}}_{T_k} + K_{21_k}(\theta_k^* - \bar{\theta}_k) + K_{22_k}(r_k^* - \bar{r}_k)$$
$$\hat{y}_{T_k} = \bar{y}_{T_k} + K_{31_k}(\theta_k^* - \bar{\theta}_k) + K_{32_k}(r_k^* - \bar{r}_k)$$
$$\hat{\dot{y}}_{T_k} = \bar{\dot{y}}_{T_k} + K_{41_k}(\theta_k^* - \bar{\theta}_k) + K_{42_k}(r_k^* - \bar{r}_k)$$

Where $\boldsymbol{\varnothing}_k^*$ and r_k^* are the noisy measurements of radar angle and range. Again, notice that we are using the actual nonlinear measurement equations in the extended Kalman filter.

5. SIMULATION RESULTS & ANALYSIS:

The preceding equations for the Kalman filter and Riccati equations were programmed and are shown, along with a simulation of the real world. We can see that the process-noise matrix is set to zero in this example (i.e., $\varnothing_s$=0). In addition, we have initialized the states of the filter close to the true values. The filter's position states are in error by 1000ft/s. The initial covariance matrix reflects those errors. Also, because we have two independent measurements, the Riccati equation requires the inverse of a 2*2 matrix. This inverse is done exactly using the matrix inverse formula for a 2*2 matrix. The extended Kalman filter was run for the nominal conditions described. The Estimated trajectory is shown in Figure 1. The radar measurement errors are shown in Figure 2.

The estimated positions and velocities are shown in Figure 3. The errors in estimated positions and velocities are shown in Figure 4 and are within acceptable limits, there by proving Kalman filter being one of the powerful mathematical tool in estimation of Projectile motion through Radar Measurements.

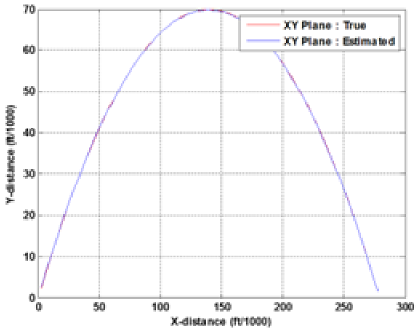

Fig 1. Path of Projectile in XY-Plane

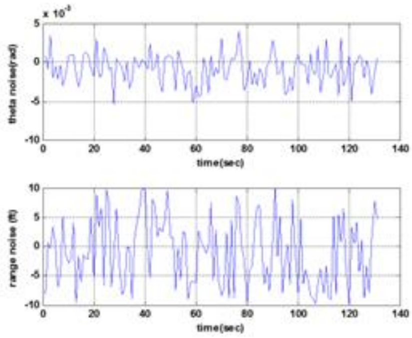

Fig 2. Radar Measurement errors

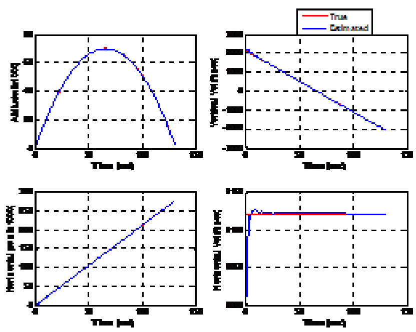

Fig 3. True and Estimated Position & Velocities

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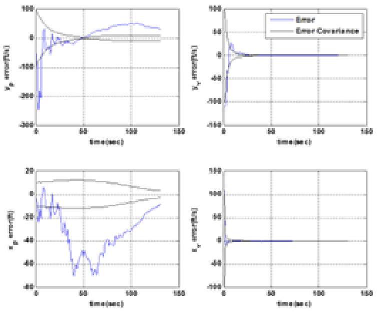

Fig 4. Errors in Position & Velocities

6. APPLICATIONS OF KALMAN FILTER:

Kalman filtering technique is used in

a). Signal processing: In signal processing, the **weiner filter** is a filter used to produce an estimate of a desired or target random process by linear time-invariant filtering of an observed noisy process, assuming known stationary signal and noise spectra, and additive noise.

b). Image processing: The median filter is a nonlinear digital filtering technique, often used to remove noise from an image or signal. Such noise reduction is a typical pre-processing step to improve the results of later processing.

c). Sensor fusion: Sensor fusion is a process by which data from several different sensors are "fused" to compute something more than could be determined by any one sensor alone.

The other applications of kalman filter are

1) In the design and development of GPS receive
2) Tracking objects (e.g., missiles)
3) Navigation
4) Fusing data from radar, laser scanner and stereo-cameras for depth and velocity measurement
5) Radar Tracking
6) Inertial Navigation
7) Vehicle navigation and control
8) Guidance of commercial airplanes

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8. CONCLUSION:

In this paper, we have presented a real time and accurate method for tracking the projectile motion by using extended kalman filter. We have determined the position, velocity and acceleration of the radar by using the kalman filter algorithm. This algorithm is very fast and uncomplicated, so it is possible to detect a moving object better and it has broad applicability. It can control some problems of object tracking such as appearance and disappearance of objects, and missing of an object.

9. FUTURE SCOPE:

The path of the projectile may not be as smooth as parabola. It can be of any random shape modeled with the non-linear dynamic state equation. Hence the present work can be extended further by using more tracking algorithms and comparing their performance accordingly to achieve more accuracy. The work can be further extended to track the object in three dimensions.

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