Abstract:

In the orthogonal frequency division multiple access (OFDMA) system, one of the efficient and low complex methods to allocate radio resources among multiple users is chunk based resource allocation, which groups a number of adjacent subcarriers into a chunk and allocates resources chunk by chunk. In this paper, performance analysis of chunk-based resource allocation is studied in the multi-cell OFDMA environment. Fractional frequency reuse (FFR) is considered in the cellular OFDMA. Basically, FFR divides each cell into central and edge areas where two different values of the frequency reuse factor are assumed. This paper analytically evaluates how spectral efficiency performance is affected by system parameters, including radius ratio of the central area to the whole cell, transmit signal to noise ratio (SNR), number of users, number of subcarriers per chunk, and coherence bandwidth. The numerical results show that there exists an optimal radius ratio to achieve the highest spectral efficiency in the proposed research. The optimal radius ratio is about 0.7, which is almost irrespective of the SNR, number of users, and number of subcarriers per chunk. In other words, the sizes of the central area and the edge area of the whole cell are almost equal when achieving the optimal performance.

I. INTRODUCTION:

Orthogonal frequency division multiple access (OFDMA) is a multiple access scheme currently exploited in most of modern wireless systems such as long term evolution-advanced (LTE-A) and IEEE 802.11/n to permit wide-band data services. OFDMA is immune against inter-symbol interference (ISI) by dividing wide-band frequency selective fading channel into orthogonal narrow-band flat fading sub-channels [1]. Fractional frequency reuse (FFR) is firstly proposed in [2] to solve the problem of strong co-channel interference encountered mainly by cell-edge users in multi-cell universal frequency reuse systems. FFR divides the macro-cell coverage area into two regions: center region with universal frequency reuse and edge region with reuse factor less than 1 (1/3 in our case). Resource allocation (RA) problem in multi-cell OFDMA system is divided into three sub-problems; sub-channel allocation among users, power loading and bit loading on different sub-channels. These sub-problems should be jointly and efficiently solved with reasonable complexity. Many algorithms in the literature have been proposed to solve RA problem on a sub-channel basis [3], [4] either to maximize system throughput or minimize transmitted power. RA on a sub-channel basis has two main drawbacks: (1) RA algorithm complexity highly grows as number of sub-channels increases and (2) large signaling is required to be fed-back about channel information of each sub-channel. To simplify RA problem and reduce complexity, a number of contiguous subchannels are grouped together into one chunk and RA is done on a chunk basis rather than sub-channel basis. Many algorithms in the literature have been proposed to solve the chunk-based RA problem in the single-cell scenario. Authors in [5] addressed system performance under many aspects such as fixed-size versus free-size chunks, equal power versus variable power per sub-channel and consecutive versus non-consecutive grouping using binary integer programming (BIP) models. Authors in [6] addressed the optimal chunk-based RA problem under bit error rate (BER) constraint considering the effect of dynamic power loading and coherence bandwidth on system throughput. Authors in [7] addressed chunk-based RA problem by dividing it into two separate sub problems (chunk assignment and power loading) to reduce complexity and simplify implementation. Some other works in the literature addressed chunk-based RA problem in MIMO based systems under BER constraint [8], with fairness guarantee [9] or under user rate constraint [10]. The main contribution of this paper is proposing a fairness aware chunk-based RA algorithm for the downlink transmission of multi-user multi-cell OFDMA system with FFR adoption.
We compare our proposed RA algorithm with two different algorithms in the literature in terms of average system spectral efficiency (SE), fairness among users and rates of cell edge users. The first reference algorithm is called capacity maximization (CM) algorithm [11] in which RA is done in a two-step process. The first step is to allocate different chunks among users based on small scale fading channel conditions only such that each user is assigned the chunk with the highest channel condition. Power is then loaded homogeneously among chunks and bit loading is done so as to satisfy BER constraint on a further step. The second algorithm is the Round Robin (RR), the simplest allocation methodology, which allocates chunks among users regardless of their channel conditions. Although additional complexity is added by our proposed algorithm and SINR feedback is required at the transmitter 2 compared to the two reference algorithms, this can be tolerated by the increase in system average SE and fairness among users. The rest of the paper is organized as follows. System model is described in Section II. Our proposed chunk-based RA algorithm is described in Section III. Simulation and results are given in Section IV. The paper is concluded in Section V.

II. SYSTEM MODEL:

Fig. 1 shows the two-layer cellular model with FFR and 19 cells in total. In the two-layer cellular model, each cell is divided into two areas: the central area and edge area, and the same frequency can be reused among the areas with the same pattern of shadow. As shown in the figure, the frequency within the central area of one cell is reused in all other cells, while the frequency within the edge area of one cell is reused every three cells, but not reused among adjacent cells. Therefore, the frequency reuse factor in the central area is one, while the frequency reuse factor in the edge area is 1/3.

For the ith cell, where i = 1, 19, there is one base station (BS), defined as BSi, located in the center of the cell. The first cell, i.e. i = 1, is assumed as the reference cell, and BS1 is the reference BS. The number of active users in the first cell is assumed to be K. The user shown by the black point in the first cell is a reference user or the kth user, where k = 1, K. When the reference user is in the central area of the first cell, all other eighteen BSs outside the first cell may cause co-channel interference to the reference user through downlink transmission due to the frequency reuse factor of one.

However, when the reference user is located in the edge area of the first cell, only six BSs (i.e. BS8, BS10, BS12, BS14, BS16 and BS18) having the same grey shadowing edge areas cause co-channel interference to the reference user.

In order to simplify the analysis, each hexagonal cell is approximated as a circular cell with the equal area as explained in [9][10]. By denoting the radius of the equivalent circular cell as r, the edge length of the hexagonal cell is equal to 1.1r, as shown in Fig. 1. The radius of the central area of the cell is denoted as rc. Further, by denoting the distance, d1, k, between BS1 and the reference user (i.e. the kth user) as xk, i.e. d1,k = xk, and θk as the angle of the user between the line from BS1 to the lower horizontal edge of the first cell and the line from BS1 to the user, the distances, \( \{d_{i,k}\} i=2, \ldots, 19 \), between BSi and the reference user can be expressed as [7].

\[
d_{i,k} = \sqrt{\left[x_k \sin(\phi_{i,k} - \theta_k)\right]^2 + \left[D_{i,1} - x_k \cos(\phi_{i,k} - \theta_k)\right]^2} = \sqrt{x_k^2 - 2x_k D_{i,1} \cos(\phi_{i,k} - \theta_k) + D_{i,1}^2},
\]

where Di,1 is the distance between BSi and BS1, e.g. D4,1 shown in Fig. 1, and \( \phi_{i,k} \) is the angle between the line from BS1 to BSi and the line from BS1 vertically to the lower horizontal edge of the first cell, e.g. \( \phi_{4,1} \) in Fig. 1. Therefore, the location of the kth user is represented by \( (x_k, \theta_k) \) in Fig. 1. It is assumed that there are M subcarriers in the downlink OFDMA system. This downlink system is a single input single output (SISO) system, i.e. each base station has only one transmit antenna and each user has only one receive antenna. Each subchannel between any BS and the reference user is a Rayleigh fading channel which introduces an additive white Gaussian noise (AWGN) with a double-sided power spectral density of N0/2. The frequency response, \( h_{i,k,m} \), m = 1, M, of the mth subchannel between BSi and the kth user contains path loss, fading, and phase, and is given by \( h_{i,k,m} = d^{-\lambda/2} i, k, \lambda, \alpha_i k, m \), where \( \lambda \) is the propagation path loss exponent. \( \alpha_i k, m \),
called the channel fading factor, is the magnitude of the channel fading of the m-th subchannel between BS i and the k-th user, and is identically and independently Rayleigh distributed with unitary mean square, \( E(\alpha^2)_{i,k,m} = 1 \), for all i and k. \( \psi_{i,k,m} \) is the channel phase and is assumed to be uniformly distributed in \([0, 2\pi)\). Therefore, \( h_{i,k,m} \) is identically and independently complex Gaussian distributed with zero mean for all i and k. Its variance is equal to the path loss, i.e. \( E(h_{i,k,m}^2) = \psi_{i,k,m} = d^{-\lambda} i,k \), which is related to the transmission distance and propagation exponent. Although frequency responses of different users have different variances due to different transmission distances, fairness is considered here in terms of the normalized received power. That is, the frequency response of the k-th user from the i-th base station is divided by the square root of the average power \( \psi_{i,k,m} = d^{-\lambda/2} i,k \). And \( h_{i,k,m} = h_{i,k,m}/\psi_{i,k,m} = \tilde{a}_{i,k,m} \) is identically and independently complex Gaussian distributed with zero mean and unitary variance \( E(\tilde{a}^2_{i,k,m}) = 1 \). Similarly, \( h_{i,k,m} \) are identically and independently distributed for all i and k. By defining \( v_{1,2} \) as the correlation coefficient between any two subcarriers (i.e. the m-th and m-2-th subcarriers) of the wireless channel from a BS to a user, \( v_{1,2} \) is given by \( [18] \). 

\[
\rho_{m,m} = \frac{1}{1 + \left( \frac{v_{m-1,m} \Delta f}{f_c} \right)^2} 
\]

where \( \Delta f \) is the frequency separation between two adjacent subcarriers, and \( f_c \) is the channel coherence bandwidth. In the downlink OFDMA system, in order to achieve large throughput, the adaptive modulation scheme is adopted according to channel qualities. By defining L as modulation level, L-ary quadrature amplitude modulation (QAM) is considered as the modulation scheme, and L takes values from the following set: \( L = \{0, 2, 4, 8, 16, 32\} \), where b represents the number of total bits per OFDM symbol, which takes a value from a set of even numbers \( B = \{0, 2, \ldots, B\} \) Subject to the total transmit power constraint, \( PT \), in each cell, in the chunk-based downlink resource allocation, the transmit power allocated to all subcarriers on one chunk is assumed to be the same for one user \([3]\). That is, \( P_{k,n,M} = \ldots = P_{k,(n+1)} \) \( M-1 = P_{k,n} \), where \( P_{k,n} \) is the transmit power to the k-th user on the m-th subcarrier, and \( P_{k,n} \), called chunk-power per subcarrier, is the power allocated on each subcarrier within the nth chunk. The modulation level determined on the nth chunk for the k-th user is denoted as \( L_{k,n} \), and is also assumed to be the same for all subcarriers within one chunk for each user, i.e. \( L_{k,n} = L_{k,n,M+m_\_} \), where \( m_\_ = 0, 1, \ldots, M-1 \) and \( L_{k,m} \) is the level of the QAM modulation for the k-th user on the m-th subcarrier. Therefore, the bits/symbol is the same for all subcarriers within the same chunk, and the bits per symbol per subcarrier (bits/symbol/subcarrier) on the nth chunk for the k-th user is given by \( r_{k,n} = \log_2 L_{k,n} = \log_2 L_{k,n,M+m_\_} \).

### III. PROPOSED CHUNK-BASED RESOURCE ALLOCATION ALGORITHM:

In this section, we propose a chunk-based RA algorithm with fairness provision for the downlink transmission of multiuser multi-cell OFDMA-based systems. During successive chunk allocation, our proposed algorithm employs the FFR concept to cope with co-channel interference and protect cell-edge users in the multi-cell scenario. The algorithm is described in details in Algorithm 1. The proposed algorithm is initialized by zero rates \( R_k = 0 \) for every user \( k \) belongs to the set of all users \( \psi \). The set of chunks \( \Lambda \) is also initialized with null set on every chunk \( c \). The first step of the algorithm is a round robin step such that, successively, each user \( \psi \) is allocated the chunk with the highest channel magnitude among set of remaining chunks \( \Lambda \). If achievable rate by any user \( \psi \) is above zero, the chunk copt is assigned to user \( k \) and removed from the set of available chunks \( \Lambda \). Then, the total rate associated with user \( k \), \( R_k \), is also updated. Otherwise, the loop continues for the next user in the round robin step.

This round robin step prevents starvation for users with poor channel conditions. The next step is a fairness provision step such that the remaining set of chunks available in the set \( \Lambda \) is fairly allocated among users. A set of candidate user’s \( \psi_c \) and is initialized by the set of all users \( \psi \) and the user \( \text{min} = \arg \min \psi \), and \( R_k \) is selected so that fairness among users is implicitly enhanced. The chunk with the highest channel magnitude with respect to user \( \text{min} \) among the set of chunks \( \Lambda \), denoted as copt, is selected for user \( \text{min} \). If the achievable rate by user \( \text{min} \) on the chunk copt is above zero, copt is assigned to user \( \text{min} \), \( R_{\text{min}} \) is updated and copt is removed from the set of chunks \( \Lambda \). Otherwise, user \( \text{min} \) is removed from the set of candidate users \( \psi \) and as it will not achieve rate over any other chunk as long as it doesn’t achieve rate on its optimal chunk.
The algorithm is terminated either if the set of candidate users $\psi_c$ and becomes a null set (i.e. no user can achieve rate over any chunk of the remaining ones) or the set of chunks $\Lambda$ becomes a null set. If the set of available chunks $\Lambda$ is not empty at the algorithm termination, these chunks are considered in outage and unallocated. This fairness step maximizes system spectral efficiency while fairness among users is guaranteed as will be explained in the results section.

**IV. SIMULATION & RESULTS:**

In this section, we evaluate the performance of our proposed RA algorithm in terms of different system metrics compared to the reference algorithm in [11] and the RR algorithm. These metrics include average system spectral efficiency (SE), fairness index (FI) and rates of cell-edge users. Number of users, $K$, is set to 8, average transmit signal to noise power ratio $\text{SNR} = P_{\text{center}} + P_{\text{edge}} \sigma_2 \eta = 20\text{dB}$ and $P_{\text{center}} = P_{\text{edge}}$, number of sub-channels per chunk is set to $M = 12$ and coherence bandwidth is $B_c = 5\Delta f$. Other simulation parameters are summarized in Table I. All results are obtained for 104 channel realizations.

Fig. 2 shows average system SE per sub-channel against FFR radius ratio $R_c/R$ for the different algorithms. Results reveal that up to a specific radius ratio of 0.4, capacity efficiency increases when the radius ratio starts to increase from a very small value and achieves the highest value when the radius ratio is around 0.7. The average spectral efficiency increases with increasing the number of users because of multiuser diversity. It can also been seen from Fig. 3 that the number of users in the system does not have much impact on the value of optimal radius ratio, although the optimal radius ratio increases slightly with increasing the number of the users. Fig. 4 shows the effect of the radius ratio on the average spectrum efficiency when the number of subcarriers per chunk takes various values. It can be seen that again when the radius ratio is about 0.7, the average spectral efficiency achieves the highest value. The number of subcarriers per chunk does not affect the value of the optimal radius ratio. With increasing the number of subcarriers per chunk, the average spectral efficiency decreases. However, the performance degradation is not significant when increasing $M$. This result shows that a proper number of subcarriers can be grouped together to reduce the complexity of resource allocation, while the achieved system performance is very close to the performance of the subcarrier-based resource allocation scheme, in which $M_\_ = 1$.

**VI. CONCLUSION:**

The FFR technology in cellular systems is an efficient method to improve the OFDMA system performance, while the chunk-based resource allocation scheme can effectively reduce the complexity of resource allocation in the OFDMA system. In this paper, the performance of the chunk-based resource allocation is investigated in FFR-based OFDMA system. The effects of the radius ratio of the central area to the whole cell are extensively studied. It is also illustrated by numerical results that irrespective of the signal to noise ratio, the number of users and number of subcarriers per chunk, the optimal radius ratio of the central area to the whole cell is around 0.7, at which the central area and the edge area are the same in areas.
REFERENCES:


