Non-Linear Sliding Mode Control With Fuzzy Logic for Speed Control of Permanent Magnet Synchronous Motor (PMSM)

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Abstract—A sliding mode control and disturbance compensation techniques are developed in order to optimize the speed control performance of PMSM in this paper. First a composite control method i.e., a sliding mode control based on one novel sliding mode reaching law (SMRL) is presented, which can dynamically adapt to the variations of controlled system which allows chattering reduction on control input, by maintaining high tracking performance of the controller. To estimate lumped uncertainties, to compensate strong disturbances and achieve high servo precision an extended sliding mode disturbance observer (ESMDO) is proposed. Further fuzzy control is used to achieve better performance. Simulation and experimental results both shows that proposed method achieve a better speed response.

Keywords—permanent magnet synchronous motor (PMSM), sliding-mode control (SMC), sliding mode reaching law (SMRL), chattering reduction, extended sliding mode disturbance observer (ESMDO)

INTRODUCTION

The permanent magnet synchronous motor (PMSM) has been widely used in industrial applications due to its characteristics i.e., high power density, torque-to-inertia ratio and efficiency. The linear control technique i.e., proportional integral (PI) control technique [13] is widely used in PMSM system due to its simple implementation. As the PMSM system is a non-linear system with parameter variations, friction force, unavoidable and unmeasured disturbances. To limit these disturbances it will be very difficult for linear control technique to obtain a sufficiently high performance. Non linear control algorithms thus become a natural solution for controlling the PMSM. With the rapid progress in power electronics and modern control theories, many researchers have aimed to develop non linear control methods for the PMSM and various algorithms have been proposed e.g., adaptive control [8], robust control [4],[5], sliding-mode control (SMC) [6],[7],[10], back stepping control, predictive control [11], intelligent control [13], [14] and so on. These algorithms have improved the control performance of PMSM from different aspects. In industrial applications, PMSM systems are always confronted with different disturbances. These disturbances may come internally (e.g., friction force and unmodelled dynamics) or externally (e.g., load disturbances). Conventional feedback based control methods usually cannot react directly and fast to reject these disturbances, whereas sliding mode control is well known for its invariant properties to certain internal parameters and external disturbances among these methods. It has been successfully applied in many fields [15]. SMC shows a good robustness to disturbances. However, the robustness of sliding mode control can only be guaranteed by the selection of large control gains, while the large gains will lead to the chattering phenomenon, which can excite high frequency dynamics.

To overcome the chattering, some approaches have been proposed such as continuation control, high-order sliding mode control, complementary sliding-mode control, sliding-mode reaching law (SMRL) method [3]. Since chattering is caused by the non ideal reaching at the end of reaching phase; the reaching law approaches deals directly with the reaching process. In [3] authors presented some reaching laws, which can restrain chattering by decreasing gain or making discontinuous
gain a function of sliding mode surface. A system variable was used in reaching law in order to suppress the chattering. However in the aforementioned reaching laws, due to the variations of the functions of the sliding surface, the discontinuous gain rapidly decreases which reduces the robustness of the controller near the sliding surface by increasing reaching time.

In real PMSM applications, it is impossible to measure the disturbances directly; disturbance estimation techniques have to be developed. In order to solve the aforementioned problems, a novel reaching law, which is based on the choice of exponential term that adapts to the variations of sliding-mode surface and system states, is proposed in this paper. This reaching law is able to deal with the chattering or reaching time dilemma. Based on this reaching law, a sliding-mode speed controller of PMSM is developed, to further improve the disturbance rejection performance of SMC method, extended sliding mode disturbance observer (ESMDO). The efficient way of improving system performance is to introduce a feed forward compensation part into the controller besides the conventional feedback part. Thus, a composite control method combining an SMC part and a feed forward compensation part based on ESMDO, called SMC+ESMDO is obtained. Further controlling is done by fuzzy controller to improve the system performance more effectively. The effectiveness of the proposed control approach was verified by simulation and experimental results.

I. PROBLEM DESCRIPTION

In This Section Some Preliminaries Are Introduced, Including Pmsm Models And Basic SMC Design With Sliding-Mode Reaching Law (SMRL) Method, To Facilitate The Proposal Of New Methods.

A. PMSM model:

One PMSM model in the rotor d-q coordinates can be expressed as:

\[ Te = 1.5p\varphi_a i_q \]

\[ Te - T_L = \frac{J}{p} \omega + B \omega \]

\[ u_d = r i_d - \omega L i_q + L i_d \]

\[ u_q = r i_q + \omega L i_d + \omega \varphi_a + L i_q \] (1)

Where \( u_d \) and \( u_q \) represents d and q axes stator voltages; \( i_d \) and \( i_q \) are d and q axes currents; \( L \) is stator inductance; \( r \) is stator resistance; \( T_e \) is electrical magnetic torque; \( T_L \) is load torque; \( p \) is number of pole pairs; \( \varphi_a \) is flux linkage of permanent magnets; \( \omega \) is electrical angular velocity; \( B \) is viscous friction coefficient; \( J \) is rotational inertia.

Sliding mode control (SMC) design with reaching law method:

Sliding mode control (SMC) is more insensitive to internal parameter variations and external disturbance when compared to other nonlinear control methods, once the system trajectory reaches and stays on the sliding surface. To design SMC controller to reduce chattering is crucial, a new reaching law is introduced in this section. A complete study of SMRL theory can be found in [3]. In general SMC design can be divided into two steps, the first step is to choose the sliding-mode surface, and the next step is to design the control input such that system trajectory is forced towards the sliding mode surface, which ensures the system to satisfy the sliding-mode reaching condition that is,

\[ s, \dot{s} < 0 \text{.................(2)} \]

Where \( s \) is the sliding-mode surface.

A second-order non linear model is generally used to describe the SMC system adopting one reaching law method:

\[ x'_1 = x_2 \]

\[ x'_2 = f(x) + g(x) + b(x)u \] (3)

Where \( x = [x_1, x_2]^{T} \) is system state, \( g(x) \) represents system disturbance and \( b(x) \) is not zero.

The sliding mode surface is chosen as:

\[ s_1 = c x_1 + x_2 \] (4)
It can guarantee the asymptotic stability of the sliding mode and the asymptotic rate of convergence is in direct relation with the value of c.

The control input $u$, designed in such a way that the sliding-mode reaching condition is met. Thus reaching law is chosen as:

$$\dot{s}_1 = -k_1 \cdot sgn(s_1) \tag{5}$$

Where $k$ is gain.

Substituting eqn. (4) in eqn. (5) yields

$$c \dot{x}_2 + \dot{x}_2 = -k_1 \cdot sgn(s_1) \tag{6}$$

Substituting eqn. (3) in eqn. (6) gives,

$$c \dot{x}_2 + f(x) + g(x) + b(x) = -k_1 \cdot sgn(s_1) \tag{7}$$

According to eqn. (7) the control input $u$ can be easily expressed as

$$u = -b^{-1}(x)[c \dot{x}_2 + f(x) + g(x) + k_1 \cdot sgn(s_1)] \tag{8}$$

The discontinuous term $-b^{-1}(x)k_1sgn(s_1)$ is contained in the control input, which leads to the occurrence of chattering, and the chattering level is up to the value of $k_1$ directly.

The time required to reach sliding-mode surface can be derived by integrating eqn. (5) with respect to time.

$$t_1 = \frac{|s(0)|}{k_1} \tag{9}$$

Reaching time can also be regulated by the value of $k_1$ directly. If the value of $k_1$ is increased, good robustness and faster reaching time can be obtained. Mean while chattering level on the control input also increases, thus in order to solve this, a novel reaching law is proposed.

II. CONTROL STRATEGY

A. Design of SMC speed controller:

**proposed sliding-mode reaching law:** Sliding-mode reaching law is realized based on the choice of an exponential term that adapts to the variations of the sliding-mode surface and system states. Reaching law is given by,

$$\dot{s} = -eq(x_1,s)sgn(s) + eq(x_1,s) \tag{10}$$

Where $k > 0, 0 < \varepsilon < 1, x_1$ is system state, $\varepsilon$ is convergence value.

$eq(x_1,s)$ converges to $\frac{k}{\varepsilon}$ that is greater than $k$ which indicates faster reaching time can be obtained. If $|s|$ decreases, $eq(x_1,s)$ approaches to $1 + \frac{1}{|x_1|}$ then $eq(x_1,s)$ converges to $\frac{k|x_1|}{1+|x_1|}$ in which system state $|x_1|$ gradually decreases to zero under the control input. When system trajectory approaches to sliding-mode surface, the $eq(x_1,s)$ gradually decreases to zero to suppress the chattering. The controller designed by proposed reaching law can dynamically adapt to the variations of sliding-mode surface and system states $|x_1|$ by making $eq(x_1,s)$ vary between $\frac{k}{\varepsilon}$ and zero.

According to eqn. (10) reaching time $t$, which is required for system state to reach $s$ is

$$s[\varepsilon + (1 + \frac{1}{|x_1|})e^{-\delta s}] = -k\cdot sgn(s) \tag{11}$$

With $s(t)=0$, integrating eqn.(11) from 0 to $t$

$$t = \frac{1}{k}[\varepsilon s(0)] + \frac{(1+\frac{1}{|x_1|})\varepsilon}{\delta}(1 - e^{-\delta s(0)}) \tag{12}$$

since $1 - e^{-\delta s(0)} < 1$

$$t < \frac{1}{k}[\varepsilon s(0)] + \frac{(1+\frac{1}{|x_1|})\varepsilon}{\delta} \tag{13}$$

If parameter $\delta$ is chosen, such that
and can always satisfy between 0 & t.

Eqn. (13) can be simplified as

\[ t < \frac{\varepsilon |s(0)|}{k} \] (14)

According to eqn. (9) and (14), the time difference between t & t₁, with condition that gain \( k = k₁ \)

\[ t - t₁ < \frac{\varepsilon |s(0)|}{k} - \frac{|s(0)|}{k₁} \]

\[ = \frac{|s(0)|}{k}(\varepsilon - 1) \] (15)

\[ \frac{|s(0)|}{k} \] is strictly +ve and \( \varepsilon - 1 \) is always –ve then from eqn. (15) implies that \( t - t₁ < 0 \)

This shows novel reaching law improves the reaching speed of sliding-mode surface with the same again (i.e., \( k = k₁ \)).

If reaching time \( t \) satisfying \( t₁ = t \) is chosen, then

\[ k < \varepsilon k₁ \] (16)

It shows gain (k) less than gain (k₁), which shows that proposed reaching law will reduce the chattering of SMC with same reaching speed.

The reaching law is computed at discrete instants and applied to the system during sampling interval, the discrete form of proposed reaching law is introduced when sliding mode surface is near to 0. According to proposed reaching law (10), if sliding-mode surface \( s \) is near to zero, the denominator term of eq(\( x₁, s \)) approaches \( 1 + \frac{1}{|x₁|} \) then the eq(\( x₁, s \)) converges to

\[ \frac{k|x₁|}{1+|x₁|} \]

Therefore proposed reaching law (10) can be simplified as

\[ \dot{s} \approx \left( -\frac{k|x₁|}{1+|x₁|} \right) sgn(s) \]

Its discrete expression is given by

\[ s(n+1) - s(n) \approx -\frac{k|x₁|T}{1+|x₁|} \cdot sgn(s(n)) \] (17)

Where \( T<0 \) is sampling period. Assume that the system trajectory reaches the sliding-mode surface in a finite time, which implies that \( s(n)=0^+ \) or \( s(n)=0^- \).

The following equation can be obtained in next period with \( s(n)=0^+ \):

\[ s(n+1) \approx -\frac{k|x₁|T}{1+|x₁|} \] (18)

It can also be written with \( s(n)=0^- \):

\[ s(n+1) \approx \frac{k|x₁|T}{1+|x₁|} \] (19)

Therefore, the width of the discrete sliding-mode band for (17) is

\[ \Delta \approx \frac{k|x₁|T}{1+|x₁|} \] (20)

In the same manner, equal reaching law stated in (5) can also be written in discrete form as:

\[ s₁(n+1) - s₁(n) = -k₁ sgn(s₁(n)) \] (21)

Width of the discrete sliding-mode band is

\[ \Delta₁ = k₁T \] (22)

The bandwidth of the above equation is constant, which means the system states under the control of equal reaching law cannot reach the equilibrium point (0,0) which would generate the chattering phenomenon between \( k₁T \) and \(-k₁T \).

However from (20) it can also be found that the bandwidth decreases with decreasing system state \( |x₁| \), which indicates that the system states under the control of proposed reaching law can reach the equilibrium point (0,0). Therefore, the chattering of the proposed reaching law is limited effectively, which is a substantial advantage over the conventional SMC.
B. Speed controller design based on proposed reaching law:

Speed-control algorithms should keep the actual speed track of speed reference $\omega_{ref}$ accurately under the occurrence of disturbances. In order to achieve this control objective, tracking error is defined as $e = \omega_{ref} - \omega$. According to aforementioned sliding-mode design method, the following sliding-mode surface is chosen:

$$s = e = \omega_{ref} - \omega$$  \hspace{1cm} (23)

which is called linear sliding-mode surface.

The time derivative of sliding-mode surface yields

$$\dot{s} = \omega'_{ref} - \dot{\omega}$$  \hspace{1cm} (24)

According to (1), the dynamic equation of motor can be expressed as follows, parameter variations are also taken into account.

$$\dot{\omega} = a_n i_q - b_n T_L - c_\omega$$

$$= a_n i_q - b_n T_L - c_n \omega + \Delta a_i q - \Delta b T_L - \Delta c \omega$$

$$= a_n i_q - c_n \omega + r(t)$$  \hspace{1cm} (25)

Where $a = a_n + \Delta a = \frac{3p^2 q_a}{2J}$

$$b = b_n + \Delta b = \frac{P}{J}$$

$$c = c_n + \Delta c = \frac{B}{J}$$

$a_n$, $b_n$, $c_n$ are nominal parameters, $\Delta a$, $\Delta b$, $\Delta c$ are parameter variations.

$$r(t) = \Delta a i_q - \Delta c \omega - b T_L$$

represents the lumped disturbances including internal parameter variation, friction force and external load disturbances which is assumed to be bounded.

$$|r(t)| \leq l$$  \hspace{1cm} (26)

$l$ is upper bound of lumped disturbances. Substituting (25) and novel reaching law (10) into (24) gives,

$$\dot{s} = \omega'_{ref} + c_n \omega - r(t) - a_n i_q$$

$$= -eq(x_1, s).sgn(s)$$  \hspace{1cm} (27)

Therefore control input $i_q^*$ is designed as:

$$i_q^* = a_n^{-1}(\omega'_{ref} + c_n \omega - r(t) + eq(x_1, s).sgn(s))$$  \hspace{1cm} (28)

In this control input, the lumped disturbances $r(t)$ is included, which is unknown. Thus control input (28) is not yet complete. In order to deal with this problem, the lumped disturbances $r(t)$ is replaced by upper bound $l$, the following control input is designed:

$$i_q^* = a_n^{-1}(\omega'_{ref} + c_n \omega + [l + eq(x_1, s)].sgn(s))$$  \hspace{1cm} (29)

From above equation, it can be found that upper bound $l$ has an important effect on control performance. It is difficult to select upper bound in practical application, because lumped disturbances are difficult to know the exact value and measure. Some methods like trail and error control can be used to select upper bound, these approaches are time consuming and cannot provide enough robustness. To overcome this drawback an sliding-mode control with disturbance compensation method is presented.

<table>
<thead>
<tr>
<th>TABLE I.</th>
<th>PARAMETERS OF PMSM</th>
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<tbody>
<tr>
<td>d- and q-axes inductances</td>
<td>$L_d = L_q = 11.5 mH$</td>
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<tr>
<td>Stator phase resistance</td>
<td>$r = 3.5 \Omega$</td>
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<tr>
<td>Viscous friction coefficient</td>
<td>$B = 0.00001 \text{N.m.s/rad}$</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>$P = 3$</td>
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<tr>
<td>Rotational inertia</td>
<td>$J = 0.00044 \text{kg.m}^2$</td>
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<tr>
<td>Flux linkage of permanent magnets</td>
<td>$\varphi_a = 0.107 \text{Wb}$</td>
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</table>
C. Extended sliding-mode disturbance observer (ESMDO):

The control performance will be degraded due to the existence of lumped disturbances, if the corresponding compensation method is not able to suppress it. In order to this, ESMDO is proposed to estimate lumped disturbances $r(t)$ on-line. The estimated disturbances are considered as feedforward part to compensate the disturbances of aforementioned SMC method in (28).

According to (25), also disturbances $r(t)$ are regarded as the extended system states, an extended dynamic equation can be obtained as

$$\dot{\omega} = a_n i_q - c_n \omega + r(t)$$

$$\dot{r}(t) = d(t)$$

(30)

Where $d(t)$ is the variation rate of system disturbances $r(t)$.

Then ESMDO can be constructed for (30) as

$$\dot{\omega} = a_n i_q - c_n \dot{\omega} + \dot{r}(t) + u_{smo}$$

$$\dot{r}(t) = g u_{smo}$$

(31)

Where $\dot{\omega}$ is an estimate of speed $\omega$, $\dot{r}(t)$ is an estimate of lumped disturbances $r(t)$, $g$ is sliding-mode parameter and $u_{smo}$ represents the switching signal, designed as

$$u_{smo} = \eta \cdot \text{sgn}(s)$$

(32)

Where $\eta$ is negative and sliding-mode surface $S$ is same as (23).

The error equation can be obtained by subtracting (30) from (31) gives,

$$\dot{e}_1 = -c_n e_1 + e_2 + u_{smo}$$

$$\dot{e}_2 = g u_{smo} - d(t)$$

(33)

Where $e_1 = \dot{\omega} - \omega$ is speed estimation error and $e_2 = \dot{r}(t) - r(t)$ is disturbance estimation error. Parameter $\eta$ should be selected reasonably to ensure sliding mode occurring to satisfy reaching condition.

D. Fuzzy logic controller with (SMC+ESMDO):

Fuzzy logic is a very powerful tool for dealing quickly and efficiently with imprecision and nonlinearity as it is a multi-valued logic. Fuzzy logic incorporates a rule-base structure in attempting to make decisions. In fuzzy logic control, the term "linguistic variable" refers to whatever state variables the system designer is interested in. Linguistic variables that are often used in control applications include Speed, Speed Error, Position, and
Derivative of Position Error. The fuzzy variable is perhaps better described as a fuzzy linguistic qualifier. The total fuzzy inference system is a mechanism that relates the inputs to a specific output or set of outputs. First, the inputs are categorized linguistically (Fuzzification), then the linguistic inputs are related to outputs (fuzzy inference) and, finally, all the different outputs are combined to produce a single output (Defuzzification).

![Fuzzy inference system](image)

fig. 2. Fuzzy inference system

A fuzzy controller converts a linguistic control strategy into an automatic control strategy, and fuzzy rules are constructed by expert experience or knowledge database. Firstly, input voltage $V_{dc}$ and the input reference voltage $V_{dc-ref}$ have been placed of the angular velocity to be the input variables of the fuzzy logic controller. Then the output variable of the fuzzy logic controller is presented by the control Current $I_{max}$.

To convert these numerical variables into linguistic variables, the following seven fuzzy levels or sets are chosen as: NB (negative big), NM (negative medium), NS (negative small), ZE (zero), PS (positive small), PM (positive medium), and PB (positive big). The fuzzy controller is characterized as: Seven fuzzy sets for each input and output; Fuzzification using continuous universe of discourse; Implication using Mamdani's ‘min’ operator; De-fuzzification using the ‘centroid’ method.

Table II Fuzzy Rules

<table>
<thead>
<tr>
<th>Δe</th>
<th>e</th>
<th>NL</th>
<th>NM</th>
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Where, $e$ represents error and $Δe$ represents change in error.

E. Simulation results:

To demonstrate the effectiveness of proposed SMC, ESMDO, FUZZY approach simulation results of PI method and SMC+ESMDO method and SMC+ESMDO+FUZZY method in one PMSM system were made. Simulations are established in MATLAB/Simulink.

![Simulation model of SMC+ESMDO+FUZZY](image)

Fig.3. simulation model of SMC+ESMDO+FUZZY

![Simulation results under PI controller: torque and speed](image)

Fig.3.(a). Simulation results under PI controller: torque and speed

![Simulation results under SMC+ESMDO: torque and speed](image)

Fig.3.(b). Simulation results under SMC+ESMDO: torque and speed
IV. CONCLUSION

A non-linear SMC algorithm is proposed and has been experimentally applied to a PMSM system, to avoid chattering occurring and to suppress disturbances. The major work includes: a novel SMRL method, to control chattering; to estimate system disturbances, extended sliding-mode disturbance observer is presented; a fuzzy controller is introduced; a composite control method that combines SMC, ESMDO and FUZZY is developed to further improve the disturbance rejection ability of SMC system.

REFERENCES


