

Denoising of an Image with SVD-Based Method

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Abstract

Nonlocal self-similarity of images has attracted considerable interest in the field of image processing and led to several state-of-the-art image denoising algorithms, such as BM3D, LPG-PCA, PLOW and SAIST. In this paper, we propose a computationally simple denoising algorithm by using the nonlocal self-similarity and the low-rank approximation. The proposed method consists of three basic steps. Firstly, our method classifies similar image patches by the block matching technique to form the similar patch groups, which results in the similar patch groups to be low-rank. Next, each group of similar patches is factorized by singular value decomposition (SVD) and estimated by taking only a few largest singular values and corresponding singular vectors. Lastly, an initial denoised image is generated by aggregating all processed patches. For low-rank matrices, SVD can provide the optimal energy compaction in the least square sense.

The proposed method exploits the optimal energy compaction property of SVD to lead a low-rank approximation of similar patch groups. Unlike other SVD-based methods, the low-rank approximation in SVD domain avoids learning the local basis for representing image patches which usually is computationally expensive. Experimental results demonstrate that the proposed method can effectively reduce noise and be competitive with the current state-of-the-art denoising algorithms in terms of both quantitative metrics and subjective visual quality.

INTRODUCTION

During acquisition and transmission, images are inevitably contaminated by noise. As an essential and important step to improve the accuracy of the possible subsequent processing, image denoising is highly desirable for numerous applications, such as visual enhancement, feature extraction and object recognition.

The purpose of denoising is to reconstruct the original image from its noisy observation as accurately as possible, while preserving important detail features such as edges and textures in the denoised image. To achieve this goal, over the past several decades, image denoising has been extensively studied in the signal processing community, and numerous denoising techniques have been proposed in the literature. Generally, denoising algorithms can be roughly classified into three categories: spatial domain methods, transform domain methods and hybrid methods. The first class utilizes the spatial correlation of pixels to smooth the noisy image, the second one exploits the sparsity of representation coefficients of the signal to distinguish the signal and noise, and the third one takes advantage of spatial correlation and sparse representation to suppress noise.

PROPOSED METHOD:

Based on the analysis of SVD in Section II, we propose an efficient method to estimate the noise-free image by combining patch grouping with the low-rank approximation of SVD (abbreviated as LRA-SVD), which leads to an improvement of denoising performance. The main motivation to use SVD in our method is that it provides the optimal energy compaction in the least square sense, which implies that the signal

and noise can be better distinguished in SVD domain. Fig. 1 illustrates block diagram of the proposed approach. Concretely, the patch grouping step identifies similar image patches by the Euclidean distance based similarity metric. Once the similar patches are identified, they can be estimated by the low rank approximation in the SVD-based denoising step. In the aggregation step, all processed patches are aggregated to form the denoised image. The back projection step uses the residual image to further improve the denoised result.

For ease of presentation, let \mathbf{Y} denote a noisy image defined by

$$\mathbf{Y} = \mathbf{X} + \mathbf{E} \quad (10)$$

where \mathbf{X} is the noise-free image, and \mathbf{E} represents additive white Gaussian noise (AWGN) with standard deviation τ which, in practice, can usually be estimated by various methods such as median absolute deviation (MAD), SVD based estimation algorithm and block-based ones. In this paper, we use a vectorized version of the model, i.e.

$$\mathbf{y} = \mathbf{x} + \mathbf{e}. \quad (11)$$

Given a noisy observation \mathbf{y} , our aim is to estimate \mathbf{x} as accurately as possible. As similarly done in BM3D and LPG-PCA, the proposed method also has two stages: the first stage produces an initial estimation of the image \mathbf{x} , and the second stage further improves the result of the first stage. Different from them, our method adopts the low-rank approximation to estimate image patches and uses the back projection to avoid loss of detail information of the image. Each stage contains three steps: patch grouping, SVD-based denoising and aggregation. In the first stage, the noisy image \mathbf{y} is firstly divided into M overlapping patches denoted by $\{\mathbf{y}_i\}_{i=1}^M$, where \mathbf{y}_i is a vectorized format of the i -th image patch. For each patch \mathbf{y}_j , its similar patch group is formed by searching similar patches from $\{\mathbf{y}_i\}_{i=1}^M$. Next, each similar patch group is denoised by the low-rank

approximation in SVD domain. Thirdly, the denoised image \mathbf{b}_x is achieved by aggregating all denoised patches. In the second stage, the final denoised image is obtained by applying the processing steps described above on the image \mathbf{e}_y produced by the back projection process. In the rest of this section, the procedures of our proposed method will be described in detail.

Patch Grouping:

Grouping similar patches, as a classification problem, is an important and fundamental issue in image and video processing with a wide range of applications. While there exist many classification algorithms available in the literature, e.g., block matching, K -means clustering, nearest neighbor clustering and others [40], we exploit the block matching method for image patch grouping due to its simplicity. For each given reference patch \mathbf{y}_j with size $\sqrt{m} \times \sqrt{m}$ the block matching method finds its similar patches from $\{\mathbf{y}_i\}_{i=1}^M$ by a similarity metric. In [22], the Euclidean distance from the transform coefficients is used to identify the similar square patches. A shape-adaptive version of this similarity metric is presented in [23], whereas it leads to a high computational cost. The simplest measure of similarity between two patches

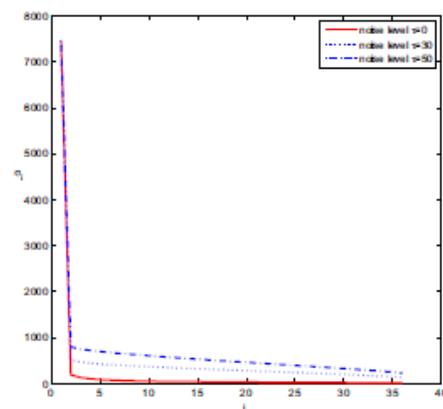


Fig. 2. Singular values of group matrices of Lena image with different noise levels.

is the Euclidean distance directly in the spatial domain. Thus we employ the spatial Euclidean distance as our similarity metric, which is defined by

$$S(\mathbf{y}_j, \mathbf{y}_c) = \|\mathbf{y}_j - \mathbf{y}_c\|_2^2 \quad (12)$$

where $\|\cdot\|_2$ denotes the Euclidean distance and \mathbf{y}_c is a candidate patch. The smaller $S(\mathbf{y}_j, \mathbf{y}_c)$ is, the more similar \mathbf{y}_j and \mathbf{y}_c are. The reference patch \mathbf{y}_j and its L -most similar patches denoted by $\{\mathbf{y}_{c,i}\}_{i=1}^L$, are chosen to construct a group matrix by using each similar patch as a column of the group matrix, and its corresponding group matrix \mathbf{P}_j is formed by

$$\mathbf{P}_j = [\mathbf{y}_j, \mathbf{y}_{c,1}, \dots, \mathbf{y}_{c,L}]. \quad (13)$$

Due to \mathbf{P}_j being made up of the noisy patches, it can be represented as

$$\mathbf{P}_j = \mathbf{Q}_j + \mathbf{N}_j \quad (14)$$

where \mathbf{Q}_j and \mathbf{N}_j denote the noise-free group matrix and the noise matrix, respectively

In general, the number L of similar patches in the group matrix cannot be too small. Too small L leads to too few patches within each group matrix, which makes the SVD-based denoising less robust. On the contrary, too large one leads to dissimilar patches being grouped together, which results in an incorrect estimation of \mathbf{P}_j . Similarly, the patch size $\sqrt{m} \times \sqrt{m}$ also has an impact on the performance of our method.

SVD-Based Denoising:

For simplicity of description, we will use \mathbf{Q} and \mathbf{P} instead of \mathbf{Q}_j and \mathbf{P}_j by a slight abuse of notation. Now our task is to estimate the noise-free group matrix \mathbf{Q} from its noisy version \mathbf{P} as accurately as possible. Ideally, the estimate $\hat{\mathbf{Q}}$ should satisfy

$$\|\mathbf{P} - \hat{\mathbf{Q}}\|_F^2 = \tau^2 \quad (15)$$

where $\|\cdot\|_F$ is the Frobenius norm and τ is the standard deviation of noise.

The similarity between patches within the noise-free image \mathbf{x} leads to a high correlation between them, which means that \mathbf{Q} is a low-rank matrix. Fig. 2 illustrates the low-rank property of \mathbf{Q} by displaying the singular values of group matrices of *Lena* image with different noise levels, where each point is the average i -th singular value over all group matrices. The estimate of \mathbf{Q} can be obtained by the low-rank approximation in the least square sense. Therefore, we can estimate \mathbf{Q} from \mathbf{P} by solving the following optimization problem

$$\hat{\mathbf{Q}} = \arg \min_{\mathbf{Z}} \|\mathbf{P} - \mathbf{Z}\|_F^2 \quad \text{s.t.} \quad \text{rank}(\mathbf{Z}) = k \quad (16)$$

where $\text{rank}(\cdot)$ denotes the rank of matrix \mathbf{Z} . In SVD domain, \mathbf{P} can be represented as

$$\mathbf{P} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T. \quad (17)$$

Let

$$\mathbf{P}_k = \mathbf{U}\mathbf{\Sigma}_k\mathbf{V}^T \quad (18)$$

where $\mathbf{\Sigma}_k$ is obtained from the matrix $\mathbf{\Sigma}$ by setting the diagonal elements to zeros but the first k singular values, i.e.

$$\mathbf{\Sigma}_k = \text{diag}(\sigma_1, \dots, \sigma_k, 0, \dots, 0). \quad (19)$$

\mathbf{P}_k is the solution of (16), which is a classical result given by the Eckart-Young-Mirsky Theorem. *Theorem 1 (Eckart-Young-Mirsky)*: For any real matrix \mathbf{P} , if the matrix \mathbf{Q} is of rank k , then

$$\|\mathbf{P} - \mathbf{Q}\|_F^2 \geq \sum_{i=k+1}^n \sigma_i^2 \quad (20)$$

where $\sigma_i (i=1, \dots, n)$ are the singular values of \mathbf{P} , and equality is attained when $\mathbf{Q} = \mathbf{P}_k$ defined by (18). This theorem shows that \mathbf{P}_k is the optimal solution for (16) in the Frobenius norms sense. Thus, we have

$$\hat{\mathbf{Q}} = \mathbf{P}_k. \quad (21)$$

The key issue for this method is to determine the value of k . By comparing (15) with (20), we can find that \mathbf{P}_k is

the ideal estimate of \mathbf{P} when $\sum_{i=k+1}^n \sigma_i^2$ is equal to τ^2 . Therefore, k can be determined by the following criterion

$$\sum_{i=k}^n \sigma_i^2 > \tau^2 \geq \sum_{i=k+1}^n \sigma_i^2. \quad (22)$$

Aggregation:

Until now, we have estimated each group matrix by applying the low-rank approximation defined by (21). Then the denoised patches can be obtained by rearranging column vectors of each denoised group matrix. As a result of taking the L nearest neighbors of each patch to construct a group matrix, a single patch might belong to several groups, and multiple estimates of this patch can be obtained. Thus we aggregate different estimates of this patch to obtain its denoised version by the following averaging process.

$$\hat{\mathbf{x}}_i = \frac{1}{n} \sum_{j=1}^n \hat{\mathbf{x}}_{i,j} \quad (23)$$

where $\hat{\mathbf{x}}_i$ is the denoised version of a patch \mathbf{y}_i , and $\hat{\mathbf{x}}_{i,j}$ ($j=1, \dots, n$) denote n different estimates of \mathbf{y}_i . The next step is to synthesize the denoised image from the denoised patches. Since the patches are sampled with overlapping regions for avoiding block artifacts at the boundaries of patches, multiple estimates are obtained for each pixel. Thus these estimates of each pixel in the image need to be aggregated to reconstruct the final denoised image. The common method of combining such multiple estimates is to perform a weighted averaging of them. Meanwhile, the weighted averaging procedure can suppress noise further. The simplest form of aggregation is the uniformly weighted averaging which assigns the same weight to all estimates. However, the uniform weights will lead to an over-smoothed result. In general, the adaptive weights derived from various biased and unbiased estimators, such as variance-based weights, SURE-based weights and exponential weights [10], can lead to better results. Different from these

adaptive weights, in this paper, we exploit the weights depending on the rank k of each group matrix due to its simplicity. For the j -th group matrix \mathbf{bQ}_j , our weight is defined by

$$w_j = \begin{cases} 1 - \frac{k}{L+1}, & k < L + 1, \\ \frac{1}{L+1}, & k = L + 1. \end{cases} \quad (24)$$

If $k < L + 1$, it means that patches in the group matrix are linearly correlated. The higher the degree of correlation of patches is, the smaller the rank k of the group matrix is. The estimate of patches yielded from the low-rank approximation is better. Thus, this estimate needs to be assigned a high weight. If $k = L + 1$, there exists no correlation among patches. The simplest uniform weight is used. Based on the weights defined in (24), the denoised estimate for the i -th pixel of the image can be expressed as

$$\hat{x}_i = \frac{1}{W} \sum_{j \in \Gamma(x_i)} w_j \hat{x}_{i,j} \quad (25)$$

where W is a normalizing factor defined by

$$W = \sum_j w_j, \quad (26)$$

$\Gamma(x_i)$ denotes the index set of all similar group matrices containing the pixel x_i , which is described as

$$\Gamma(x_i) = \{j | x_i \in \mathbf{Q}_j, j = 1, \dots, C\}, \quad (27)$$

and $\hat{x}_{i,j}$ denotes the denoised estimate of the i -th pixel in the j -th similar group matrix \mathbf{Q}_j . Once all pixels are estimated by (25), the final denoised image can be obtained by reshaping the estimates of all pixels.

Back Projection

Although most of noise can be removed by using the denoising procedures described before, there still exists a small amount of noise residual in the denoised image. The noise residual stems from the fact that noise in the original noisy image affects the accuracy of the patch grouping, which leads to an inaccurate group. The

grouping errors in turn affect the SVD-based denoising. In addition, there exists another reason for noise residual. Ideally, based on the discussion in Section III-B, the optimal estimate \mathbf{Q} satisfies.

$$\begin{aligned} \|\mathbf{P} - \hat{\mathbf{Q}}\|_F^2 = \|\mathbf{P} - \mathbf{Q}\|_F^2 &\implies \|\mathbf{P} - \hat{\mathbf{Q}}\|_F^2 = \|\mathbf{N}\|_F^2 \\ &\implies \sum_{i=k+1}^n \sigma_i^2 = \tau^2. \end{aligned} \quad (28)$$

Unfortunately, the left side of (28) is not usually equal to the right side. In most cases, it is that $\tau^2 > \sum_{i=k+1}^n \alpha_i^2$. Therefore, we need to further improve the denoising performance of our method.

The commonly used way to further improve the performance of a denoising method, as used by K-LLD [43] and SAIST, is to develop an iterative version for the basic denoising method. While the iterative strategy for image denoising has been widely used in the literature, it has a very high computational cost which limits the scope of applications. An alternative way exploited by BM3D and LPG-PCA is the two-stage approach, in which the basic estimate of the noisy image yielded by the denoising method is used as a reference image to perform improved grouping and parameter estimation.

In this paper, unlike the iteration-based or the reference-based strategies, we make use of the two-stage strategy with a back projection step to further suppress the noise residual. Back projection is an efficient method that uses the residual image to improve the denoised result [44], [45]. In fact, the use of the residuals in improving estimates can date at least back to John Tukey's classic book [46], in which this idea is termed *twicing*. This concept is also known by several names, such as Bregman iterations, *l2*-boosting, and biased diffusion. A recent paper [47] provides a good overview of these methods. The basic idea of back projection is to generate a new noisy image $\tilde{\mathbf{y}}$ adding filtered noise back to the denoised image, i.e.

$$\tilde{\mathbf{y}} = \hat{\mathbf{x}}_0 + \delta(\mathbf{y} - \hat{\mathbf{x}}_0) \quad (29)$$

where $\delta \in (0, 1)$ is a constant projection parameter and $\hat{\mathbf{x}}_0$ is the denoised result produced by the first stage. Note that when $\delta \rightarrow 0$, $\tilde{\mathbf{y}} \rightarrow \hat{\mathbf{x}}_0$. On the contrary, if $\delta \rightarrow 1$, $\tilde{\mathbf{y}} \rightarrow \mathbf{y}$. For simplicity, in our experiments, we set $\delta = 0.5$, which is a trade-off between 1 and 0. Now we can achieve an improved result of $\hat{\mathbf{x}}_0$ by denoising $\tilde{\mathbf{y}}$ with the proposed three processing steps in previous subsections, i.e., *patch grouping*, *SVD-based denoising*, and *aggregation*. It is necessary to point out that the noise variance of $\tilde{\mathbf{y}}$, denoted as $\tilde{\tau}^2$, needs to be updated in the SVD-based denoising step. We employ the estimator presented in [47] to determine $\tilde{\tau}^2$, which is written as

$$\tilde{\tau} = \gamma \sqrt{\tau^2 - \|\mathbf{y} - \hat{\mathbf{x}}_0\|_F^2} \quad (30)$$

where γ is a scaling factor

PROPOSED ALGORITHM:

To summarize, the complete procedure of our proposed method is algorithmically described in the following Algorithm 1.

Algorithm 1 The proposed denoising algorithm

Input: Noisy image \mathbf{y}

Output: Denoised image $\hat{\mathbf{x}}$

- 1: $\tau \leftarrow$ Estimate noise standard deviation by computing the median absolute deviation (MAD) of the finest wavelet coefficients as described in [36];
 - 2: $\{\mathbf{P}_j\}_{j=1}^C \leftarrow$ Group image patches with the similarity metric defined by Eq. (12) and form group matrices;
 - 3: **for each** $j \in [1, C]$ **do**
 - 4: $\hat{\mathbf{Q}}_j \leftarrow$ Calculate the low-rank approximation of \mathbf{P}_j via Eq. (21);
 - 5: $w_j \leftarrow$ Compute the weight for $\hat{\mathbf{Q}}_j$ via Eq. (24);
 - 6: **end for**
 - 7: $\hat{\mathbf{x}}_i \leftarrow$ For each patch existing multiple different estimates, aggregate its estimates via Eq. (23).
 - 8: $\hat{\mathbf{x}}_0 \leftarrow$ Aggregate all $\hat{\mathbf{Q}}_j$ via Eq. (25);
 - 9: $\tilde{\tau} \leftarrow$ Update the noise standard deviation τ via Eq. (30);
 - 10: $\tilde{\mathbf{y}} \leftarrow$ Generate a new noisy image via the back projection described by Eq. (29);
 - 11: $\hat{\mathbf{x}} \leftarrow$ Obtain the final denoised image by performing *Step 2 to Step 8* for $\tilde{\mathbf{y}}$.
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BLOCK DIAGRAM:

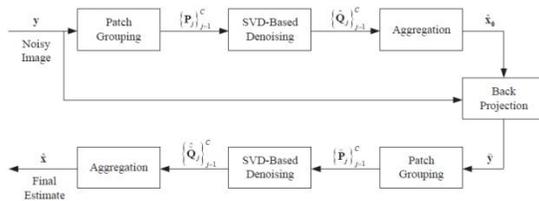


Fig. 1. Block diagram of the proposed denoising algorithm.

Let \mathbf{X} be a grayscale image. The basic principle of linear image representation is that the signal of interest can be decomposed into a weighted sum of a given representation basis. Thus, \mathbf{X} can be represented as

$$\mathbf{X} = \sum_{i=1}^N a_i \phi_i \quad (1)$$

where $a_i (i = 1, \dots, N)$ are the representation coefficients of the image \mathbf{X} in term of the basis functions $\phi_i (i = 1, \dots, N)$. ϕ_i can either be chosen as a prespecified basis, such as, curvelet, contourlet, shearlet and other directional basis, or designed by adapting its content to fit a given set of images. In general, an adaptive basis has better performance than the prespecified one. In [20], Aharon *et al.* proposed a learning method to achieve a set of adaptive basis (also called dictionary). This method extracts all the $\sqrt{m} \times \sqrt{m}$ patches from the image \mathbf{X} to form a data matrix $\mathbf{S} = (s_1, s_2, \dots, s_n) \in R^{m \times n}$, where m is the number of pixels in each patch, $s_i (i = 1, \dots, n)$ are image patches ordered as columns of \mathbf{S} and n is the number of patches. Then the dictionary is learned by solving the following optimization problem

$$\min_{\Phi, \mathbf{A}} \sum_{i=1}^n \|\mathbf{s}_i - \Phi \mathbf{a}_i\|_2^2 \quad s.t. \quad \|\mathbf{a}_i\|_0 \leq \beta \quad (2)$$

where $\Phi \in R^{M \times P}$ is the dictionary of p column atoms, $\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) \in R^{P \times N}$ is a matrix of coefficients, β indicates the desired sparsity level of the solution, and the notation $\|\mathbf{a}_i\|_0$ stands for the count of the nonzero entries in \mathbf{a}_i . Based on the learned dictionary Φ , \mathbf{S} can be represented as

$$\mathbf{S} = \Phi \mathbf{A} \quad (3)$$

Another method for image representation with adaptive basis selection is principle component analysis (PCA) [32], which determines the basis from the covariance statistics of the data matrix \mathbf{S} . The principal components transform of \mathbf{S} is calculated as

$$\mathbf{A} = \Phi^T (\mathbf{S} - E(\mathbf{S})) \quad (4)$$

with Φ defined by

$$\Omega_S = \Phi \Lambda \Phi^T \quad (5)$$

where $E(\mathbf{S})$ is the matrix of mean vectors, Ω_S is the covariance matrix of \mathbf{S} , Φ is the eigenvector matrix, and $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_m)$ is the diagonal eigenvalue matrix with

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m \quad (6)$$

It can easily be derived that the covariance matrix Ω_A of the matrix \mathbf{A} equals to

$$\Omega_A = \Phi^T \Omega_S \Phi = \Lambda \quad (7)$$

which implies that the entries of \mathbf{A} are uncorrelated. This property of PCA can be used to distinguish between the signal and noise. It is because the energy of noise is generally spread over the whole transform coefficients, while the energy of a signal is concentrated on a small amount of coefficients. One major shortcoming of the adaptive dictionary and PCA is that they impose a very high computational burden. An alternative method for adaptive basis selection is the singular value decomposition (SVD). The SVD of the data matrix \mathbf{S} is a decomposition of the form

$$\mathbf{S} = \mathbf{U} \Sigma \mathbf{V}^T = \sum_{i=1}^n \sigma_i \mathbf{u}_i \mathbf{v}_i^T \quad (8)$$

where $\mathbf{U} = (\mathbf{u}_1, \dots, \mathbf{u}_n) \in R^{m \times n}$ and $\mathbf{V} = (\mathbf{v}_1, \dots, \mathbf{v}_n) \in R^{n \times n}$ are matrices with orthonormal columns, $\mathbf{U}^T \mathbf{U} = \mathbf{V}^T \mathbf{V} = \mathbf{I}$, and where the diagonal matrix $\Sigma = \text{diag}(\sigma_1, \dots, \sigma_n)$ has nonnegative diagonal elements appearing in nonincreasing order such that

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0 \quad (9)$$

The diagonal entries σ_i of Σ are called the singular values of S , while the vectors u_i and v_i are the left and right singular vectors of S , respectively. The product $u_i v_i^T \sigma_i$ in (8) can be considered as an adaptive basis, and σ_i as representation coefficient. In fact, SVD and PCA are intimately related. PCA can be performed by calculating the SVD of the data matrix $p1Ns$ (Refer to [35] for more details). In addition, if a matrix is low rank, we can easily estimate it from its noisy version by the low-rank approximation in SVD domain. Thus, we propose a new denoising method by using SVD instead of PCA in the following section, which has a low computational complexity.

EXPERIMENTAL RESULTS:

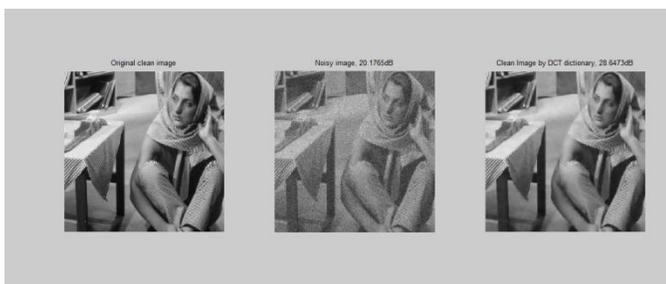


Fig: original, noisy, filtered images

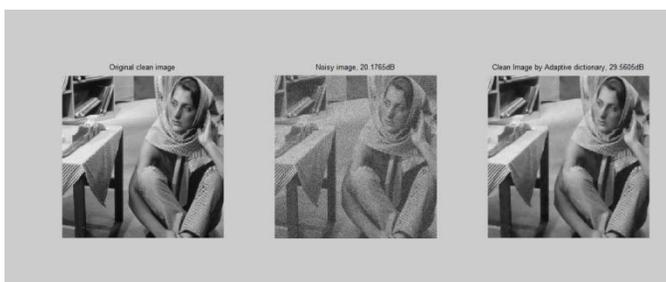


Fig: original, noisy, filtered images

To demonstrate the efficacy of the proposed denoising algorithm, in this section, we give our experimental results concerning simulations that have been conducted on natural grayscale images with size 512×512 . These images have been commonly used to validate many state-of-the-art denoising methods. The

noisy images are generated by adding zero mean white Gaussian noise with different levels to the test images. The noise level τ is from 10 to 50, and the intensity value for each pixel of the images ranges from 0 to 255.

Evaluation Criteria:

Two objective criteria, namely peak signal-to-noise ratio (PSNR) and feature-similarity (FSIM) index [48], are adopted to provide quantitative quality evaluations of the denoising results. PSNR is the mostly widely used quality measure in the literature, even though it is often inconsistent with human eye perception. FSIM measures the similarity between two images by combining the phase congruency feature and the gradient magnitude feature, which is based on the fact that human visual system (HVS) understands an image mainly according to its low-level features. The aforementioned criteria can comprehensively reflect the performance of the denoising methods.

CONCLUSION:

In this paper, we have presented a simple and efficient method for image denoising, which takes advantage of the nonlocal redundancy and the low-rank approximation to attenuate noise. The nonlocal redundancy is implicitly used by the block matching technique to construct low-rank group matrices. After factorizing by SVD, each group matrix is efficiently approximated by preserving only a few largest singular values and corresponding singular vectors. This is due to the optimal energy compaction property of SVD. In fact, the small singular values have little effect on the approximation of the group matrix when it has a low-rank structure. Experimental results demonstrate the advantages of the proposed method in comparison with current state-of-the-art denoising methods. The computational complexity of the proposed algorithm is lower than most of existing state-of-the-art denoising algorithms but higher than BM3D. The fixed transform used by BM3D is less complex than SVD, whereas it is



lessadapted to edges and textures. The main computational cost of our algorithm is the calculation of SVD for each patch group matrix. As each group matrix could potentially be processed independently in parallel, our method is suitable for parallel processing. Therefore, in practice, we can use a parallel implementation to speed it up, which will make it feasible for real-time or near real-time image denoising. In addition, while developed for grayscale images, our method can be extended for shape-adaptive color image and video denoising by taking into account the shape-adaptive patches and the temporal redundancy across color components and frames. This further work will be studied in the future.

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