

Mathematical Modelling of Electromagnetic Actuator for Propulsion Force

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Abstract:

By modeling the system with the mechanical, electrical, and magnetic field, the system equation is derived for the actuator modeling. As it is not easy to conclude the output signal from the equations, Simulink has been used to simulate and check the performance aspect of the system. The force generated depends mainly on the acceleration and the weight of the mover. This force thus generated by using electromagnetic principle can be used as a driving mechanism in real time applications.

Keywords—electromagnetic actuator, linear actuator, system modeling, equivalent magnetic circuit.

Introduction

Propulsion force is required for driving any mechanism and also, this actuator should meet the condition that it could exert the required force condition, with operating periodically [1].

Among actuators that meet the condition, we selected the electromagnetic actuator which can exert proper force within a short time. Also, by giving a periodic voltage source, we can make the actuator operate periodically.

However, the actuator does not operate properly at some state (e.g., high-frequency input, low current input, or else). And for better usage of the electromagnetic actuators, it is needed to analyze the operating system of the actuator [2].

MODELING ANALYSIS

The magnetic actuator operates when a magnetic force is applied to the mover. This magnetic force is calculated from the current, and the current is derived from the voltage input [3].

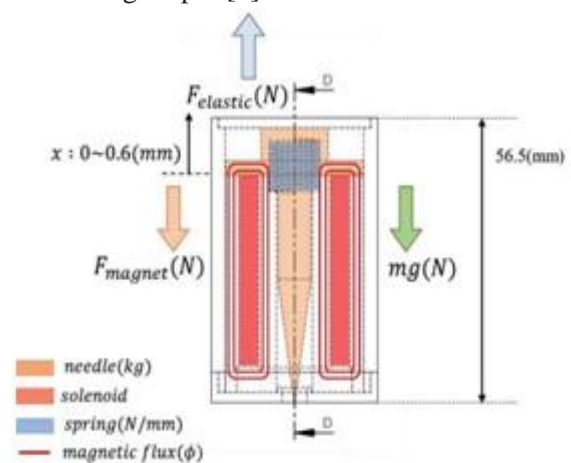


Fig: 1 Components, forces acting on actuator

A. Mechanical analysis

In total, three forces are acting on mover: elastic force, gravity force, and magnetic force (F_{magnet}). This relationship is plotted in Fig.1. In the modeling, we ignored friction and damper because the friction is so small that it cannot be measurable. Therefore, the model equation for the mechanical system could be described as below equation.

$$m\ddot{x} = k(x' - x) - mg - F_{magnet} \quad (1)$$

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Among this equation, a sum of elastic force and gravity force could be substituted as a new elastic equation with new equilibrium point (x_o).

$$k(x' - x) - mg = k(x_o - x) \quad (2)$$

So system equation (1) could be corrected as below equation.

$$m\ddot{x} = k(x_o - x) - F_{magnet} \quad (3)$$

In the equation (3), the magnetic force could be calculated by differentiating magnetic energy (U_{magnet}) with the parameter of the position of the mover (x).

$$F_{magnet} = \frac{dU_{magnet}}{dx} \quad (4)$$

Magnetic energy is calculated from magnetic density (B), which can be derived from magnetic flux per area (Φ/A).

$$U_{magnet} = \int \frac{B^2}{2\mu} \cdot A dx$$

$$= \int \frac{1}{2\mu} \left(\frac{\Phi}{A}\right)^2 \cdot dx \quad (5)$$

The magnetic flux (Φ) in equation (5) could be calculated more precisely using equations introduced in the next chapter. Therefore, in this section by applying equation (5) to (4), the magnetic force to (3), we can conclude the equation as

$$m\ddot{x} = k(x_o - x) - \frac{1}{2\mu} \frac{\Phi^2}{A} \quad (6)$$

B. Electrical analysis

A basic model of the electric system is LR-circuit, due to the solenoid in the magnetic actuator. This could be described as equation (7).

$$V = Ri + L \frac{di}{dt} \quad (7)$$

However, due to the movement of the mover, a magnetic path that magnetic flux passes through changes, inducing a change in inductance value.

So we need to consider the change of inductance value ($L(x)$) concerning the position of the mover (x) [6]. Relationship of current and magnetic resistance (R_m) in the solenoid (8) and the relation of inductance and magnetic flux are considered to calculate the inductance.

$$N_c i = R_m(x) \Phi \quad (8)$$

$$Li = N_c \Phi \quad (9)$$

By modifying (8) about equation of current

$$i = \frac{R_m(x) \Phi}{N_c} \quad (10)$$

And by substituting i in (9), we could get equation that calculating inductance as below

$$L(x) = \frac{N_c^2}{R_m(x)} \quad (11)$$

Also, because magnetic resistance and magnetic flux changes with the position of mover (x) changes, this induces voltage through the solenoid. This is related with the Faraday's law of induction, and the induced voltage is calculated as below

$$V_{ind}(t) = \frac{d\Phi(t)}{dt} \quad (12)$$

From (11), (12) we can conclude electrical system as below

$$V(t) - V_{ind}(t) = Ri(t) + L(x) \frac{di(t)}{dt} \quad (13)$$

C. Magnetic Analysis

Calculating the magnetic system is important related to the magnetic force (4), (5) and inductance (11) and induced voltage (12).

To calculate the magnetic flux and magnetic resistance, we

considered an equivalent magnetic circuit. From the equivalent circuit, we can calculate magnetic resistance that magnetic flux passes through. Also, from the equation of magneto motive force of the solenoid, magnetic flux could be calculated.

Magneto motive force is same with a multiplication of coil turn number and current and also same with multiplication of magnetic flux and magnetic resistance.

$$F_m = N_c i = \Phi R_m \quad (14)$$

In equation (14) we can know the value of N_c , and calculate current (i) from (13). If magnetic resistance can be calculated, we can derive magnetic flux, and calculate magnetic force, inductance, and induced voltage.

Magnetic resistance can be calculated by dividing region, which magnetic flux passes through. A divided region is shown in Fig.2. From the equation of

calculating magnetic resistance:

$$R_{magnet} = \int \frac{dl}{\mu A} \quad (15)$$

We can calculate the total magnetic resistance of the circuit as below

$$R_m = R_{a1} + R_{a2} + R_{s1} + R_{s2} + R_u + R_f$$

$$= \frac{x}{\mu_o \pi (r_2^2 - r_1^2)} + \frac{x}{\mu_o \pi (r_4^2 - r_3^2)} + \frac{l_s}{\mu_s \pi (r_2^2 - r_1^2)} + \frac{l_s}{\mu_s \pi (r_4^2 - r_3^2)} + \frac{\ln(\frac{r_4}{r_1})}{2\pi \mu_{slu}} + \frac{\ln(\frac{r_4}{r_1})}{2\pi \mu_{slf}} \quad (16)$$

By classifying the equation with variable parts, including the mover position (x), and constant part, the total magnetic equation (16) is substituted as equation (17).

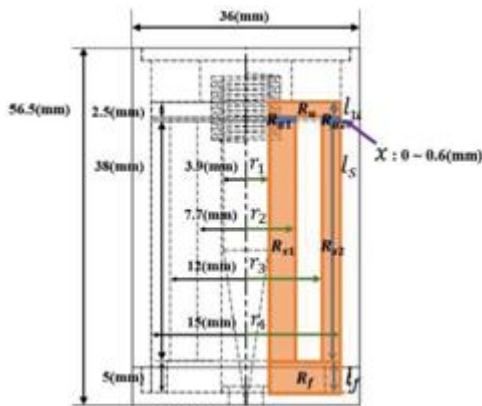


Fig:2 segmentation for calculating magnetic circuit

$$R_m = C_1 x + C_2 \quad (17)$$

$$C_1 = \frac{1}{\mu_o \pi (r_2^2 - r_1^2)} + \frac{1}{\mu_o \pi (r_4^2 - r_3^2)}$$

$$C_2 = R_{s1} + R_{s2} + R_u + R_f$$

By substituting magnetic resistance in (14) with (17), we could derive magnetic flux as below equation.

$$\phi = \frac{Ni}{C_1 x + C_2} \quad (18)$$

D. Concluded Equation

From equation (4), (5), (11), (12), (17) and (18) we can conclude overall system model equations as below. All the parameters for calculating concluded equations are listed in

$$F_{magnet} = \frac{N_c^2}{2\mu_o A_o} \frac{i(t)^2}{(C_1 x(t) + C_2)^2} \quad (19)$$

$$L(x) = \frac{N_c^2}{(C_1 x(t) + C_2)^2} \quad (20)$$

$$m\ddot{x} = k(x_o - x) = \frac{N_c^2}{2\mu_o A_o} \frac{i(t)^2}{(C_1 x(t) + C_2)^2} \quad (21)$$

$$V(t) - \frac{d}{dt} \left\{ \frac{Ni(t)}{C_1 x(t) + C_2} \right\} = Ri(t) \frac{N_c^2}{(C_1 x(t) + C_2)^2} \frac{di(t)}{dt} \quad (22)$$

$$(A_o = \pi(r_4^2 - r_3^2) + \pi(r_2^2 - r_1^2))$$

CALCULATION OF OPERATIONAL CONDITION

Calculation of the magnetic system is essential as it is related to the magnetic force (4), (5) and inductance (11) and induced voltage (12).

To operate the actuator, we need to find the condition for the operation (e.g., voltage, current, or else). From the mechanical model (21), acceleration, movement of the mover, is related to current value.

So we considered two cases. In the first case, we turn on the actuator by sending current, moving the mover from maximum position to minimum position ($x: x_{max} \rightarrow x_{min}$). The other case is turning off the actuator by switching off current, which means that the mover position moves from minimum to maximum ($x: x_{min} \rightarrow x_{max}$).

When the actuator turns on, acceleration of mover should be minus (-) value. In that situation, we consider that the position of the mover (x) has not changed. From the equation (21), to make acceleration of mover become (-) value, without the position of the mover (x) changing, we need to increase current value above a certain point to turn on the actuator (i_{on}). The following process could calculate this point.

$$\ddot{m}x < 0$$

$$k(x_o - x) - \frac{N_c^2}{2\mu_o A_o} \frac{i(t)^2}{(C_1 x(t) + C_2)^2} < 0$$

$$k(x_o - x) < \frac{N_c^2}{2\mu_o A_o} \frac{i(t)^2}{(C_1 x(t) + C_2)^2}$$

$$i > \frac{(C_1 x(t) + C_2) \sqrt{2\mu_o A_o k(x_o - x)}}{N_c}$$

$$i_{on} = \frac{(C_1 x_{max} + C_2) \sqrt{2\mu_o A_o k(x_o - x_{max})}}{N_c} \quad (23)$$

When we turn off the actuator, we need to move mover from minimum position to maximum position. This means we need to make plus (+) acceleration value. Condition when the actuator is about to turn off, position of mover does not change. By the equation (19), this means magnetic force should be decreased by decreasing current below a certain point to turn off the actuator (i_{off}). This condition could be calculated from following process.

$$\begin{aligned}
 \dot{m}x > 0 \\
 k(x_o - x) - \frac{N_c^2}{2\mu_0 A_0} \frac{i(t)^2}{(C_1 x(t) + C_2)^2} > 0 \\
 k(x_o - x) < \frac{N_c^2}{2\mu_0 A_0} \frac{i(t)^2}{(C_1 x(t) + C_2)^2} \\
 i < \frac{(C_1 x(t) + C_2) \sqrt{2\mu_0 A_0 k(x_o - x)}}{N_c} \\
 i_{off} = \frac{(C_1 x_{min} + C_2) \sqrt{2\mu_0 A_0 k(x_o - x_{min})}}{N_c} \quad (24)
 \end{aligned}$$

SIMULATION RESULTS

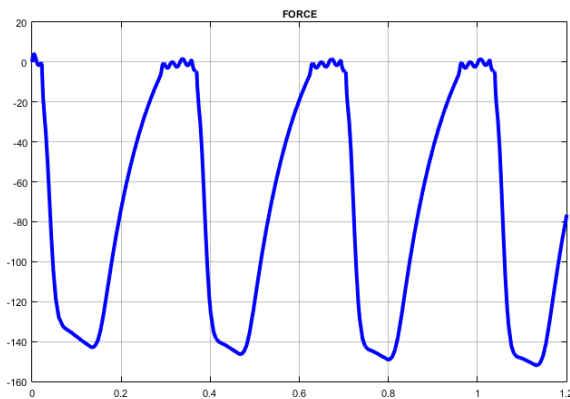


Fig3: Force waveform for 3500 turns

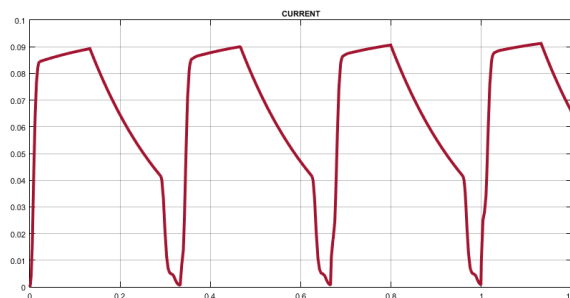


Fig4: Input current waveform for 3500 turns

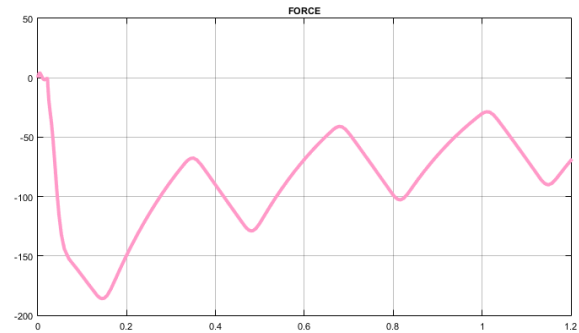


Fig5: Force waveform for 5500 turns

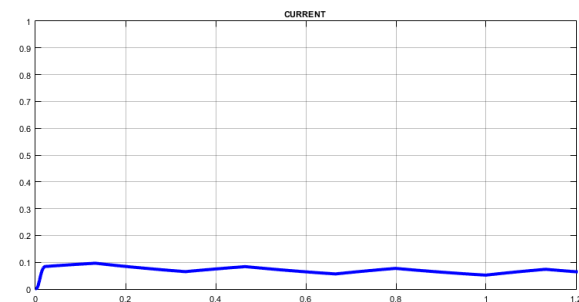


Fig6: Input current waveform for 5500 turns

According to the equation of magnetic force (19) and mechanical system (21), many factors have relation with exerted force (e.g., coil turn number (N_c), a permeability of steel μ_s), spring constant (k), equilibrium position (x_0), or else). By changing parameter values in simulation, reducing the error is possible. Compare to the results of Fig.3, it was successful to get more powerful exerted force, as shown in the result of Fig.5. In the result, we increased the magnetic force by increasing coil turn number (N_c) from 3500 to 5500, keeping other parameters same. However, even though we increase the force, performance frequency of actuator was reduced to 1Hz in Fig.5, compare to the results of Fig.3 which has 3Hz performance frequency. In actual operation, the actuator works worse at 4Hz. Instead, it works normally at 3Hz, with same voltage input. We found out that this is because an increase of coil turn number (N_c) leads to an increase of inductance ($L(x)$), which can be deduced from (20). Due to inductance increase, current slowly decreases below the certain value for turning off (i_{off}), delaying time for the actuator being turned off so that the performance frequency gets lowered. Not only the

coil turn number(N_c), we also find out that other parameter values also affect the performance of the actuator like exerted force and performance frequency. The effect of the parameters could be inferred from the equation about inductance (20) and exerted force (21).

CONCLUSION

From the modeling analysis, we derived the mechanical and electrical system equation. Also, using Simulink, we can check the performance aspect of the system, and calculate the exerted force and operating condition. We try to increase simulation force by adjusting the parameter we used. However, parameters adjustment changes the performance frequency of the actuator a lot, so it was not easy to adjust used parameters to get an optimal result. In the future study, we will consider the response of the system, and study on the performance frequency of the system for better analysis of the electromagnetic actuator.

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