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A Sliding Mode Observer for a Networked DC Motor with Time Delays and Sensor Faults Tolerance

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ABSTRACT

In this paper the effect of time delay upon a Networked Control System(NCS) is studied and the boundary limits of the system with delay is determined theoretically in terms of plant parameters. Then a Sliding Mode Observer is designed to recognize the effects of minor faults and to isolate the effect of them in the presence of time delays and loss of information during transmission. The error during estimation is stabilized based on Lyapunov theory. Optimal integral square error can be achieved by obtaining the controller parameters by solving bilinear matrix inequalities. The overall system is implemented and the results are verified by using simulations of MATLAB/ Simulink.

Index Terms — ***Networked Control System, Sliding Mode Observer, Integral square error, Lyapunov Stability, bilinear matrix inequalities.***

INTRODUCTION

Networked control systems have various applications in a space craft, medical, manufacturing and various industrial applications. They also have reduced cost, good performance and high reliability. In Networked system the information from the sensors, actuators etc., is send through a communication network. The use of communication network will lead to intermittent losses or delays of the communicated information. The transmission of a continuous signal varying in time through a network it requires the signal to be sampled first, then encoded then transmitted and then at the receiving end it must be decoded. The delay occurs during the transfer of information from sensor to controller and between controller and actuator. These delays have an adverse effect upon the system performance and may even destabilize the system. To obtain an optimal estimator and controller the network induced delay should be shorter than the sampling period but in most applications it is always greater than sampling time [1-5]. Before each transmission of information the node monitors the network, if the network before each transmission. When the network is inactive it transmits information immediately else it waits till network becomes free. During multiple signal transmission the signal with the higher priority interrupt is transferred first and the low priority message is terminated and will be retried when the network is idle.

The network induced delay has a greater effect upon these packet dropouts. They can be made bounded and constant by periodic transmission of packets. Single packet transmission delivers information from sensors or actuator at the same time. Multiple packet transmission is preferred than single packet as the line can carry a fixed amount of information. A fault can be detected based upon parity space approaches which employ fault detection filters. Observer based method for fault detection is gaining popularity which performs the same as parity approach. The first step to achieve this is to estimate the states by designing an observer; the information of these states is used to construct the residual signal by weighted output estimation error. The residuals are used to have information about faults. Fault detection based upon unknown input observers has been a greater research area. In this area a lot of papers have been published for controller design, stability NCS [3]-[8]. The delay margin has been calculated theoretically based on system parameters and controller values. The performance of PI controller in presence of delay is studied in detail in [9], in which a disturbance rejection determined. The PI controller gains are computed based on linear matrix inequalities. Suboptimal mixed robust controller design for NCSs and a performance index constraint are minimized. This paper presents a sensor fault tolerant Time delayed DC motor system(TDCMS) with unknown time delays; information loss during transmission, external torque disturbances is studied. At first the delay margin for particular parameters and control values are calculated theoretically and then verified using MATLAB / SIMULINK. Then a sliding mode observer is designed to make the observer tolerant to sensor faults, Lyapunov stability analysis have been used for error minimization. The optimality PI controller designed solving bilinear matrix inequalities [13]. The paper is organized as follows, In section II the modeling of the TDCMS is described in detail. Section III a theoretical method and steps to calculate delay margin is discussed, later in chapter the sliding mode observer equations with state error minimization and optimal PI controller designare mentioned. Then SIMULINK environment is used to explain the results in detail.

MODELING OF TDCMS

The state equations of an TDCMS is given by

$$\frac{di_a(t)}{dt} = -\frac{r_a}{l_a} i_a(t) - \frac{k_b}{l_a}\omega(t) + \frac{1}{l_a} v(i_k h) \quad [1]$$

$$\frac{d\omega(t)}{dt} = -\frac{k}{j} i_a(t) - \frac{b}{j}\omega(t) - \frac{1}{j} T_l \quad [2]$$

Where $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}], k \in N$. The delay at the k^th step for the networked system is denoted by τ_k. Both sensor-to-controller and controller-to-actuator delays are included in inducing the total delay, which is not a known value. In this system, the speed controller sends the packet at instant $i_k h$. It takes τ_k seconds for the local dc-motor chopper to receive the information. The motor states depends upon the transmission data till information at the next instant $i_{k+1} h + \tau_{k+1}$ sec is received. It should be noted that packet losses are rooted in this model if $\tau > h$. Since the overall network delay includes both induced delays, the sensors used to measure the armature current and speed at (t)th second. With $x_a \overset{def}{=} [i_a \quad \omega]^T$ and assuming than the speed sensor output is $\omega(t) + \omega_F(t)$ Where $\omega_F(t) \in R$ represents speed sensor faults the motor state equations are given by

$$\begin{aligned} \dot{x}_a(t) &= A_a x_a(t) + B_a v(i_k h) + E_a T_l \\ y_a(t) &= C_a x_a(t) + D\omega_F(t) \end{aligned} \quad [3]$$

Where

$$A_a = \begin{bmatrix} -\frac{r_a}{l_a} & \frac{k_b}{l_a} \\ -\frac{k}{j} & -\frac{b}{j} \end{bmatrix} \qquad B_a = \frac{1}{l_a} I_2(:,1) \qquad C_a = I_2$$

$$E_a = -\frac{1}{j} D \qquad D = I_2(:,2)$$

If the drive system is speed sensorless or, equivalently, the sensor suffers from complete failure, then $\omega_F(t) = -\omega(t)$, which corresponds to a speed with zero sensor output.

As explained [14] with the assumption that the load torque is an extra state variable with very slow variations compared to motor time constants and $x \overset{def}{=} [x_a^T \quad T_l]^T$, we have

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(i_k h) \\ y(t) &= Cx(t) + D\omega_F(t) \end{aligned} \quad [4]$$

Where

$$A = \begin{bmatrix} -\frac{r_a}{l_a} & -\frac{k_b}{l_a} & 0 \\ -\frac{k}{j} & -\frac{b}{j} & -\frac{1}{j} \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} B_a \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

THEORITICAL DELAY MARGIN CALCULATION

The DC motor transfer function is obtained as follows:

$$G(S)H(S) = \frac{\frac{K}{JL_a}}{s^2 + \left(\frac{r_a}{l_a} + \frac{b}{j}\right)s + \frac{r_a b + k k_b}{l_a j}} \quad [5]$$

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The PI controller is given by the following transfer function:

$$G_C(S) = K_P + \frac{K_I}{S} \quad [6]$$

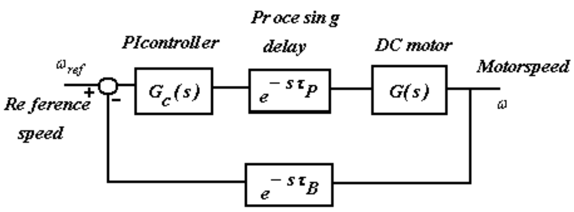

Fig 1. Block Diagram of a time delayed DC motor system

The PI controller is used to for the speed response to reach the desired value As shown in Figure 1, all time delays in the feedback loop are lumped together into a feedback delay between the output and the controller. The characteristic equation of system given by

$$\Delta(s,\tau) = 1 + G(S) + G_C(S)e^{-s\tau} = 0$$

$$\Delta(s,\Gamma) = P(s) + Q(s)e^{-s\Gamma} = 0 \quad [7]$$

where P(s), Q(s) are polynomials in s with real coefficients.These polynomials are given as

$$P(s) = p_3 s^3 + p_2 s^2 + p_1 s + p_0$$

$$Q(s) = q_1 s + q_0 \quad [8]$$

Where

$$p_3 = 1, p_2 = \left(\frac{r_a}{l_a} + \frac{b}{j}\right), p_1 = \frac{r_a b + k k_b}{l_a j}, p_0 = 0$$

$$q_1 = \frac{k k_p}{j l_a}, q_0 = \frac{k k_i}{j l_a}$$

The location of the roots of the characteristic Equation are determined to analyze the stability of DC motor control system. The theoretical method is given in detail below. For stability the roots of characteristic equation should lie in the left half of s-plane. In single delay case, our objective is to compute delay margin τ for different parameters. The characteristic equation $\Delta(s,\tau) = 0$ is an implicit function of s and τ. Because of the complex conjugate symmetry $\Delta(-s,\tau) = 0$ have same roots as $-j\omega_c$. Note to find value of delays such that $\Delta(s,\tau) = 0$ and $\Delta(-s,\tau) = 0$ have a common root at s = $j\omega_c$ This result could be stated as follows:

$$P(s) + Q(s)e^{-s\Gamma}$$

$$P(-s) + Q(-s)e^{s\Gamma} \quad [9]$$

The exponential terms could be easily eliminated and the following new characteristic equation is obtained:

$$P(s)P(-s) - Q(s)Q(-s) = 0 \quad [10]$$

The replacement s by $j\omega_c$ leads to the following polynomial in ω given by

$$W\left(w_c^2\right) = P(jw_c)P(-jw_c) - Q(jw_c)Q(-jw_c) = 0 \quad [11]$$

Finally it becomes

$$W\left(w_c^2\right) = t_6 w_c^6 + t_4 w_c^4 + t_2 w_c^2 + t_0 = 0 \quad [12]$$

Where

$$t_6 = p_3^2, t_4 = p_2^2 - 2p_1 p_3, t_2 = p_3^2,$$

$$t_4 = p_1^2 - q_1^2, t_0 = -q_0^2$$

The computation of real roots is easier then the computation of imaginary roots from this method. The stability of the Networked DC motor speed control system could be easily analyzed by obtaining the roots of W. Depending upon these roots,

i) The new polynomial of W does not possess any positive real roots. In this case, the characteristic equation of will not have any roots on the $j\omega$-axis. Consequently, the DC motor speed control system will be delay-independent stable.

ii) The polynomial W may have at least one positive real root. In this case, the characteristic equation will have at least a pair of complex roots on $j\omega$ As a result; the DC motor speed control system will be delay-dependent stable [11].

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For a positive real root ω_c the corresponding value of delay margin τ can be easily computed using characteristic equation and W:

$$\tau^* = \frac{1}{\omega_c}\left[\frac{\mathrm{Im}\left[\frac{P(j\omega_c)}{Q(j\omega_c)}\right]}{\mathrm{Re}\left[\frac{P(j\omega_c)}{Q(j\omega_c)}\right]}\right] + \frac{2r\pi}{\omega_c}; r = 0,1,2......\infty \quad [13]$$

The characteristic equation has finite real roots

$$[\omega_c] = [\omega_{c1}, \omega_{c2}, \omega_{c3}, ...\omega_{cq}]$$

The delay margins of real roots for each positive root

$[\omega_{cm}], m = 1,2,.........q$ is given as

$$\tau_m^* = [\tau_{m1}^*, \tau_{m2}^*, \tau_{m3}^*, ...\tau_m^*], m=1,2,......q$$

The system delay margin will be the minimum of $\tau_m^* m = 1,2,.....$

$$\tau_m^* = \min(\tau_m^*) \quad [14]$$

The delay margin based on polynomial equations is given as follows:

$$\tau_m^* = \frac{1}{\omega_c} Tan^{-1}\left(\frac{q_0 p_1 \omega_{cm} + (q_1 p_2 - q_0 p_3)\omega_{cm}^3}{(q_0 p_2 - q_1 p_1)\omega_{cm}^2 - q_1 p_3 \omega_{cm}^4}\right) + \frac{2r\pi}{\omega_c}$$

r=0,1,2,…inf

[15]

SLIDING MODE OBSERVER DESIGN

In this section, a sliding-mode observer is designed to estimate the rotor speed and load torque.

The observer structure is given by

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bv(i_k h - e_\tau(t)) + G(y(t) - \hat{y}(t)) + B\Psi(t)\\ \hat{y}(t) &= C\hat{x}(t)\\ \hat{\omega}(t) &= D^T \hat{y}(t)\end{aligned} \quad [17]$$

where the hat-signed variables stand for the estimated ones and $e_\tau(t)$ denotes the estimation error of network-induced delay. Furthermore, G and $\Psi(t)$ are observer gain matrix and auxiliary sliding-mode input, respectively, and will be designed to guarantee asymptotic stability of the observer. Let $e_F = x - \hat{x}$ the state estimation error. Thus, the error dynamic equations are determined to be

$$\dot{e}_F(t) = (A - GC)e_F(t) + B(v(i_k h) - v(i_k h - e_\tau(t))) - GD\omega_F(t) - B\Psi(t) \quad [18]$$

The variations of armature voltage controlled by PWM signal is limited during a time period, or mathematically, the control signal v(t) is Lipchitz, i.e.,

$$|v(i_k h) - v(i_k h - e_\tau(t))| \le \beta |e_\tau(t)| \quad [19]$$

where β is known positive scalar. The error system in (6) is asymptotically stable if

$$\begin{aligned}&(A - GC)^T + (A - GC) + \Lambda < 0\\ &GD = [0 \quad 0 \quad 0]^T\end{aligned} \quad [20]$$

and $\Psi(t)$ selected to be

$$\begin{aligned}&\Psi(t) = \rho \frac{e_i(t)}{|e_i(t)|}\\ &\rho \ge \beta |e_\tau(t)| + \varepsilon\end{aligned} \quad [21]$$

Where $e_i(t) = i_a(t) - \hat{i}_a(t)$ is the armature current estimation error, Λ is a positive-definite matrix, and ε denotes an arbitrary positive scalar. This can be verified by choosing the following Lyapunov function:

$$V_o(t) = e_F^T(t) e_F(t) \quad [22]$$

Therefore, the derivative of the aforementioned function along with the error system trajectory is as follows:

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$$\dot{V}_o(t)=e_F^T(t)\left[(A-GC)^T+(A-GC)\right]e_F(t)+2e_F^T(t)B(v(i_kh)-v(i_kh-e_\tau(t)))$$
$$-2e_F^T(t)GD\omega_F(t)-2e_F^T(t)B\Psi(t)$$

[23]

Since $e_F^T(t)B=(1/l_a)e_i(t)$ and from (21) and (22) and the assumptions in (23)

$$\begin{aligned}\dot{V}_o(t)&\le -e_F^T(t)\Lambda e_F(t)+2(1/l_a)e_i(t)\beta\left|e_\tau(t)\right|-2(1/l_a)e_i(t)\Psi(t)\\&\le -e_F^T(t)\Lambda e_F(t)+2(1/l_a)e_i(t)\left[\beta\left|e_\tau(t)\right|-\Psi(t)\right]\\&\le -e_F^T(t)\Lambda e_F(t)-2(1/l_a)e_i(t)\varepsilon\le -e_F^T(t)\Lambda e_F(t)\\&\le -\lambda_\Lambda\left\|e_F(t)\right\|^2\end{aligned}$$

[24]

Where

$\lambda_\Lambda=\lambda_{\min}(-\Lambda)$.

Consequently, $\lim_{t\to\infty}\left[\hat{\omega}\quad \hat{T}_l\right]=\left[\omega\quad T_l\right]$.

PI CONTROLLER DESIGN

The design of an optimal controller is designed in this section to achieve complete stability of the time delayed DC motor system which can be observed with the DC motor speed $\omega(t)$ tracking reference input r_s.

The steady state armature current is given by

$$i_{ass}=(b/k)r_s+(l/k)T_l \qquad [25]$$

With

$$x_{ass}\overset{def}{=}\left[(b/k)r_s+(l/k)T_l \quad r_s\right] \text{ And}$$

$$e_R=x_a-x_{ass}$$

$$\dot{e}_R(t)=A_a(e_R(t)+x_{ass})+B_av(i_kh)+E_aT_l \qquad [26]$$

To achieve zero steady state error $e_R(t)\to 0$ the control voltage $v(i_kh)\to v_{ss}$ given

$$v_{ss}=\frac{br_a+kk_b}{k}r_s+\frac{r_a}{k}T_l \qquad [27]$$

The DC motor system to track reference speed the controller equation is given by

$$\begin{aligned}v_a(t)&=k_{Po}\hat{e}_\omega(t)+k_{Io}I_{\hat{e}_\omega}(i_kh)\\&=D^Tk_{Po}\hat{e}_R(t)+k_{Io}I_{\hat{e}_\omega}(i_kh)\end{aligned} \qquad [28]$$

Where

$$\tau(t)=t-i_kh,\hat{e}_\omega(t)\overset{def}{=}\hat{\omega}(t)-r_s$$

The integral term is calculate at each and every time instant as

$$I_{\hat{e}_\omega}(i_kh)=\int_0^{i_kh}\hat{e}_\omega(\theta)d\theta \qquad [29]$$

The proportional and integral controls are applied separately to the speed and current error tracking. The armature voltage to dc motor is given by

$$v(i_kh)\overset{def}{=}\frac{br_a+kk_b}{k}r_s+\frac{r_a}{k}\hat{T}_l+v_a(i_kh) \qquad [30]$$

Assuming the reference speed and load torque to be zero. The error in estimation is given by $e_F=\left[e_F^T\quad e_{T_L}\right]^T$, the error vector is given by

$$\hat{e}_R(t)=e_R-e_E$$

Combining equations (14) in (13) we get

$$\begin{aligned}\dot{e}_R(t)&=A_ae_R(t)+k_{Po}B_aD^T\hat{e}_R(i_kh)+k_{Io}I_{\hat{e}_\omega}B_a(i_kh)\\&=A_ae_R(t)+k_{Po}B_aD^T\hat{e}_R(i_kh)-k_{Po}B_aD^Te_E(i_kh)\\&\quad+k_{Io}I_{\hat{e}_\omega}B_aI_{\hat{e}_\omega}(i_kh)\end{aligned} \qquad [31]$$

Where $\quad I_{\hat{e}_\omega}(i_kh)=\hat{e}_\omega(t)=D^T\hat{e}_R(i_kh) \qquad [32]$

The conditions for error minimization and control vector design are taken from [10].

SIMULATION RESULTS

In this section, for a PI controller gains, delay margin is determined based upon given relations in section III. The accuracy of theoretical delay margin results is confirmed by using MATLAB/Simulink environment.

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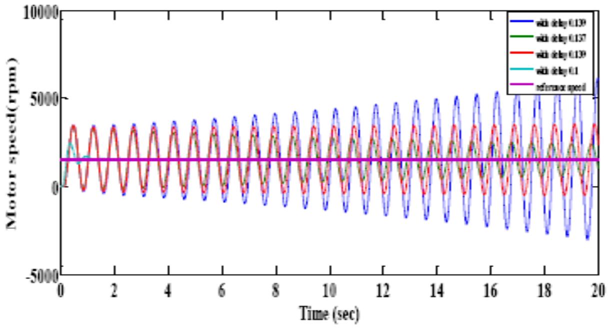

Fig 2. Output response of the system using PI controller with varying time delays.

The DC motor parameters used in this paper are given as follows:

$$r_a = 3.27\Omega, l_a = 1.81*10^{-3} H, k_b = 7.7*10^{-2} V.s/rad$$
$$k = 1.68*10^{-3} N.m/A,\ j = 3.6*10^{-4} kg.m^2\Omega$$
$$b = 80*10^{-4} N.m.s/rad$$

In fig (2) the speed response of the system for different time delays is shown. From this response it shows the time delay effect upon the settling time of the system.

For controller gains $k_p = 2.0, k_i = 2.6$ the delay margin is calculated, the delay margin is obtained as $\tau_m^* = 0.129s$,the theoretical results are then verified using simulink as shown in fig 3.

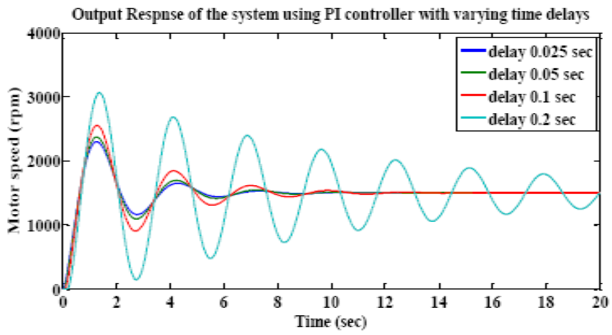

Fig 3. The motor speed for different delays around the theoritically calculated delay margin.

The reference speed and faulty speed is given by

$$r_s = 85u(t) - 10u(t-22)$$

$$f(t) = 10u(t-65))\cos(0.45\pi t)$$

The load torque disturbance is given by $T_l = 0.14u(t-38)$

The sliding mode gain is chosen as $\rho = 3.05$

The observer gain matrix is obtained as [9]

$$G = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}^T$$

The proportional controller gains are obtained as $k_p = 0.1, k_i = 0.3$.

The faulty , actual, estimated and reference speeds are shown in fig 4.

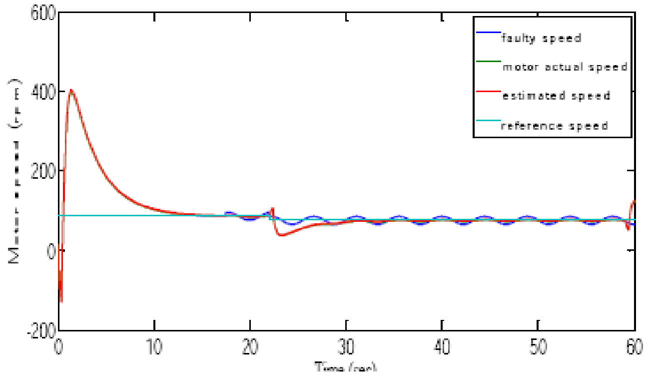

Fig (4) Reference, actual ,estimatedand fault speeds of the networked dc motor in the presence of sensor fault and disturbance with observer-based PI controller.

The effect of packet dropout with sampling time Ts lesser than delay time and greater than delay time are shownin Fig (5).

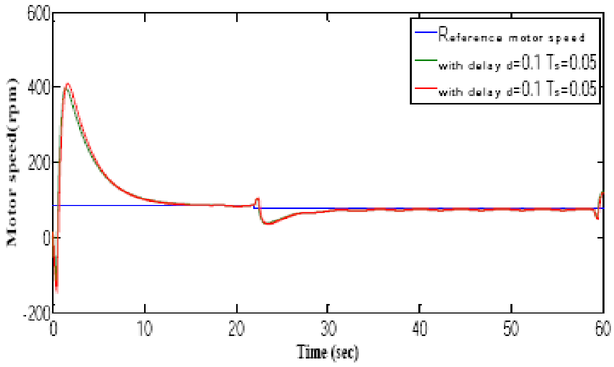

Fig 5. The effect of packet dropout with different sampling time and delay time.

CONCLUSION

This paper has analyzed the closed loop stability of the networked DC motor speed with time delays in its feedback and feed forward paths. A theoretical method is used to compute delay margins which depend on parameters and PI controller values. Then a speed sensorless technique has been used and the error stability between estimated model and original model based upon Lyapunov approach. Then to achieve optimum performance an PI controller based on linear matrix inequalities and sliding mode observer is designed to reduce the effect of varying delays and packet dropouts.

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