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# PI Controller Tuning Using Boundary Locus

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**ABSTRACT**

***Practical systems are highly nonlinear in nature and operating points are changing as the system operating conditions changes. Controlling of such systems needs the design of appropriate controllers. In general, fixed gain controllers are designed at nominal operating conditions. These types of controllers function satisfactorily near the nominal operating point where they are designed and the performance degrades as the operating point changes in the wide range. So, to keep the system performance near its optimum, efficient tracking of the operating point and updating the controller parameters corresponding to the current operating point to obtain better performance is very much essential. In this work, a method to find the PI controller parameters with and without time delay has been developed for the various applications considered in this works. The method is based on the plotting of the stability boundary locus in the ($k_p$, $k_i$) plane and then computing the stabilizing values of the parameters of PI controller. The method does not need the use of Pade approximation and linear programming to solve set of inequalities. To study the effectiveness of the method, studies have been carried out by considering the Air craft pitch control application.***

***Keywords:*** ***Boundary locus; PI controller; pitch control***

## INTRODUCTION

Controllers since these types of controllers have been widely used in industries for several decades. However, many important have been recently reported on There has been a great amount of research work on the tuning of PI (proportional integral), PID (proportional integral derivative) and lag/lead computation of all stabilizing P(Proportional),PI (proportional Integral) & PID controllers[1]. A new and complete analytical solution based on the generalized version of the Hermiter Biehler theorem has been provided for computation of all stabilizing constant gain controllers for a given plant. A linear programming solution for characterizing all stabilizing PI and PID controllers for a given plant has been obtained. This approach, besides being computationally efficient, has revealed important structural properties of PI and PID controllers .For example, it was shown that for a fixed proportional gain, the set of stabilizing integral and derivative gains lie in a convex set.

This method is very important since it can cope with systems that are open loop stable or unstable, minimum or non-minimum phase. However, the computation time for this approach increases in an exponential manner with the order of the system being considered. It also needs sweeping over the proportional gain to find all stabilizing PI and PID controllers, which is a disadvantage of the method. An alternative fast approach to this problem based on the use of the Nyquist plot. A stability boundary locus approach for the design of PI and PID controllers has been given[2]. A parameter space approach for the design of PI and PID controllers. More direct graphical approaches to this problem based on frequency response plots have been given. However, the

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requirement for frequency grinding has become the major problem for this approach. Compensator design in classical control engineering is based on a plant with fixed parameters. In the real world, however, most practical system models are not known exactly, meaning that the system contains uncertainties. Much recent work on systems with uncertain parameters has been based on Kharitonov theorem, there have been many developments in the field of parametric robust control related to the stability and performance analysis of uncertain control systems represented as interval plants.

In this work, a new approach is given for computation of stabilizing PI controllers in the parameter plane, (kp, ki) plane. The result is used to obtain the stability boundary locus over a possible smaller range of frequency. Thus, a very fast way of calculating the stabilizing values of PI controllers for a given SISO (Single input single output) control system is given. The proposed method is also used for computation of PI controllers for relative stabilization and for achieving user specified gain and phase margins. An extension of the method to find all stabilizing values of the parameters of a PID controllers[4], namely kp,ki and kd in the (kp,ki) plane, (kp,kd) plane and (ki,kd) plane, is also given. It is shown that the stability boundary for the convex polygon in the (ki, kd) plane for affixed value of kp can be generated from four straight lines. The equations of these straight lines can be easily derived using the stability boundary of the stabilizing regions obtained in the (kp,ki) and (kp,kd) plane. The proposed method is finally used for computation of PI controllers for the stabilization of interval systems.

## BOUNDARY LOCUS METHOD FOR WITH AND WITHOUT TIME DELAY

The new technique has been proved here for computation of stabilizing PI controllers in the parameter ($k_p$, $k_i$) plane[3]. The proposed method is also used for computation of PI controllers for relative stabilization and for achieving user specified gain and phase margins. An extension of the method to find all stabilizing values of the parameters of a PID controller[7,8], namely $k_p$, $k_i$and, $k_d$ in the ($k_p$, $k_i$) plane, ($k_p$, $k_d$) plane and ($k_i$, $k_d$) plane , is also given. The proposed method is also applicable for interval systems.

### 2.1 Boundary locus method without time delay

#### 2.1.1 Stabilization using PI controller

Consider the single input, single output (SISO) control system of fig 2.1 where[6]

$$G(S) = \frac{N(S)}{D(S)} \qquad (1)$$

Is the plant to be controlled and C(S) is a PI controller of the form

$$C(S) = k_p + \frac{k_i}{S} \qquad (2)$$

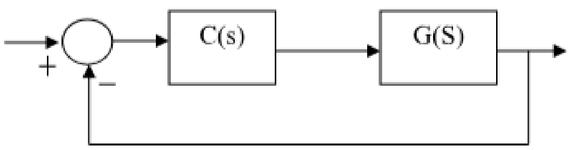


Figure 1 SISO control system with G(s)

The problem is to compute the parameters of the PI controller of Eq.(2.2) that stabilize the system of Fig. 2.1

Decomposing the numerator and the denominator polynomials Eq. (2.1) into their even and odd parts and substituting s=i ω

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega\omega_o(-\omega^2)}{D_e(-\omega^2) + j\omega\omega_o(-\omega^2)}$$

The close loop characteristics polynomial of the system can be written as

$$\begin{aligned}\Delta(S) = &\left[k_i N_e(-\omega^2) - k_p\omega^2 N_o(-\omega^2) - \omega^2 D_o(-\omega^2)\right] \\ &+ j\left[k_p\omega N_e(-\omega^2) + k_i\omega N_o(-\omega^2) + \omega D_e(-\omega^2)\right]\end{aligned} \qquad (3)$$

$$\Rightarrow \Delta(s) = a + jb$$

Equating real and imaginary part to zero

$$k_p\left(-\omega^2 N_o(-\omega^2)\right) + k_i\left(N_e(-\omega^2)\right) = \omega^2 D_0(-\omega^2) \qquad (4)$$

$$k_p\left(-\omega^2 N_e(-\omega^2)\right) + k_i\left(N_o(-\omega^2)\right) = \omega^2 D_e(-\omega^2) \qquad (5)$$

These are rewritten as

$$k_p Q(\omega) + k_i R(\omega) = X(\omega)$$

$$k_p S(\omega) + k_i U(\omega) = Y(\omega)$$

Where

$$Q(\omega) = -\omega^2 N_o\left(-\omega^2\right), R(\omega) = N_e\left(-\omega^2\right)$$

$$S(\omega) = -\omega N_e\left(-\omega^2\right), U(\omega) = \omega N_e\left(-\omega^2\right) \quad (6)$$

$$X(\omega) = \omega^2 D_o\left(-\omega^2\right), Y(\omega) = -\omega D_e\left(-\omega^2\right)$$

From these the equations are given as

$$k_p = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \quad (7)$$

$$k_i = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \quad (8)$$

Solving these two equations simultaneously, the stability boundary locus l ($k_p$,$k_i$, ω ), in the ($k_p$, $k_i$) plane can be obtained . choosing a test point within each stable region that contains the values of stabilizing $k_p$and, $k_i$ parameters can be determined.

#### 2.1.2 Stabilization for specified gain and phase margins

Consider $G_c(s) = Ae^{-j\emptyset}$ in Eq. (2.1), then the characteristic equation is[5]

$$\Delta(S) = \begin{bmatrix} k_p A\left(\omega N_e\left(-\omega^2\right)\sin(\phi) - \omega^2 N_o\left(-\omega^2\right)\cos(\phi)\right) \\ + k_i A\left(N_e\left(-\omega^2\right)\cos(\phi) + \omega N_o\left(-\omega^2\right)\sin(\phi) - \omega^2 D_o\left(-\omega^2\right)\right) \end{bmatrix}$$
$$+ j\begin{bmatrix} k_p A\left(\omega N_e\left(-\omega^2\right)\cos(\phi) + \omega^2 N_o\left(-\omega^2\right)\sin(\phi)\right) \\ + k_i A\left(\omega N_o\left(-\omega^2\right)\cos(\phi) - N_e\left(-\omega^2\right)\sin(\phi)\right) + \omega D_e\left(-\omega^2\right) \end{bmatrix} = 0 \quad (9)$$

$$\Delta(s) = a + jb$$

Equate the real and imaginary part to zero. And by further substitution we have,

$$Q(\omega) = A\left(\omega N_e\left(-\omega^2\right)\sin(\varphi) - \omega^2 N_0\left(-\omega^2\right)\cos(\varphi)\right)$$

$$R(\omega) = A\left(N_e\left(-\omega^2\right)\cos(\varphi) + \omega N_0\left(-\omega^2\right)\sin(\varphi)\right)$$

$$S(\omega) = A\left(\omega N_e\left(-\omega^2\right)\cos(\varphi) + \omega^2 N_0\left(-\omega^2\right)\sin(\varphi)\right) \quad (10)$$

$$U(\omega) = A\left(\omega N_0\left(-\omega^2\right)\cos(\varphi) - N_e\left(-\omega^2\right)\sin(\varphi)\right)$$

$$X(\omega) = \omega^2 D_0\left(-\omega^2\right) Y(\omega) = -\omega D_e\left(-\omega^2\right)$$

To obtain the stability boundary locus for a given value of gain margin A, one needs to set φ =0 in Eq.(2.14). On the other hand, setting A=1 in Eq. (2.14). One can obtain the stability boundary locus for a given phase margin φ .

### 2.2 Boundary locus method with time delay

#### 2.2.1 Stabilization using a PI controller

Consider the single input, single output (SISO) control system of Fig.2.2, where

$$G_p(s) = G(s) = G(s)e^{-TS} = \frac{N(S)}{D(S)}e^{-TS} \quad (11)$$

$$G(j\omega) = \frac{N_e\left(-\omega^2\right) + j\omega\omega_o\left(-\omega^2\right)}{D_e\left(-\omega^2\right) + j\omega\omega_o\left(-\omega^2\right)}$$

Is the plant to be controlled C(s) is a PI controller of the form

$$C(S) = k_p + \frac{k_i}{S} = \frac{k_p S + k_i}{S} \quad (12)$$

Where the closed loop characteristic polynomial is

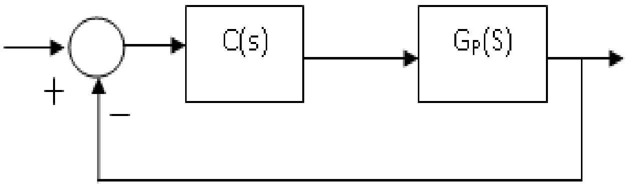


Figure.2: SISO control system with $G_p(s)$

$$\Delta(j\omega) = [(k_i N_e - k_p\omega^2 N_0)\cos(\omega\tau) + \omega(k_i N_0 + k_p N_e)\sin(\omega\tau) - \omega^2 D_0] +$$
$$j[\omega(k_i N_0 + k_p N_e)\cos(\omega\tau) - (k_i N_e - k_p\omega^2 N_0)\sin(\omega\tau) + \omega De] \quad (13)$$

Now evaluate the real and imaginary part to zero

$$k_p\left[-\omega^2 N_o\cos(\omega\tau) + \omega N_e\sin(\omega\tau)\right] + ki\left[N_e\cos(\omega\tau) + \omega N_o\sin(\omega\tau)\right] = \omega^2 D_0$$
$$k_p\left[\omega N_e\cos(\omega\tau) + \omega^2 N_o\sin(\omega\tau)\right] + ki\left[\omega N_o\cos(\omega\tau) - N_e\sin(\omega\tau)\right] = -\omega\, De \quad (14)$$

$$k_p Q(\omega) + k_i R(\omega) = X(\omega)$$

$$k_p S(\omega) + k_i U(\omega) = Y(\omega)$$

From this we compute

$$\mathbf{Q(\omega) = \omega N_e \sin(\omega\tau) - \omega^2 N_o \cos(\omega\tau)}$$

$$\mathbf{R(\omega) = N_e \cos(\omega\tau) + \omega N_o \sin((\omega\tau)}$$

$$\mathbf{S(\omega) = \omega N_e \cos(\omega\tau) + \omega^2 N_o \sin(\omega\tau)}$$

$$\mathbf{U(\omega) = \omega N_o \cos(\omega\tau) - N_e \sin(\omega\tau)}$$

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$$X(\omega) = \omega^2 D_o$$
$$Y(\omega) = -\omega^2 De \qquad (15)$$

From these equations we $k_p$ and $k_i$ as,

$$k_p = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)}$$

$$k_i = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)}$$

Once the stability boundary locus has been obtained then it is necessary to test whether stabilizing controllers exist or not since the stability boundary locus, $l(k_p,k_i,\omega)$, and the line $k_i$ =0 may divide the parameter plane [( $k_p$ $k_i$ ) plane] into stable and unstable regions. Here, the $k_i$ =0 is the boundary line obtained from substituting $\omega = 0$ into Eq.(2.22) and equating it to zero since a real root of A(s) of Eq.(2.22) can cross over the imaginary axis at s = 0. It can be seen that the stability boundary locus is dependent on the frequency w which arrives from 0 to ∞. However , one can consider the frequency below the critical frequency, $\omega_c$ or the ultimate frequency since the controller operates in this frequency range. Thus, the critical frequency can be used to obtain the stability boundary locus over a possible smaller range of frequency such as $\omega \in [O, \omega_c]$ since the phase of $G_p(s)$ at s=j $\omega_c$ is equal to $-180^0$.

$$\tan^{-1}\frac{\omega D_o}{N_e} - \tan^{-1}\frac{\omega D_e}{N_o} - \omega\tau = -\pi$$

$$\tan(\omega\tau) = \frac{\omega(N_e D_e - N_o D_o)}{N_e D_e + \omega^2 N_o D_o} = f(\omega)$$

### 2.2.2 Stabilization for specified gain and phase margins

Phase and gain margins are two important frequency domain performance measures which are widely used in classical control theory for controller design. Consider Fig.2.2 with a gain-phase margin $G(s) = Ae^{-j\emptyset}$, which is connected in the feed forward path.

Then Eq.(2.24) can be written as

$$Q(\omega) = A\left[\omega N_e \sin(\omega\tau) - \omega^2 N_o \cosh\right]$$

$$R(\omega) = A\left[N_e \cosh + \omega N_o \sinh\right]$$

$$S(\omega) = A\left[\omega N_e \cosh + \omega^2 N_o \sinh\right]$$

$$U(\omega) = A\left[\omega N_o \cosh - N_e \sinh\right]$$

$$X(\omega) = \omega^2 D_o$$

$$Y(\omega) = -\omega^2 De$$

Where $h = \omega\tau + \phi$ and $G(s) = Ae^{-j\emptyset}$.

Thus from these set of equation we find $k_p$ and $k_i$

$$k_p = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)}$$

$$k_i = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)}$$

To obtain the stability boundary locus for a give value of gain main A, one needs to set $\phi = 0$ in Eq.(2.26). on the other hand, setting A=1 in Eq.(2.26), one can obtain the stability boundary locus for a given phase margin $\phi$.

## CASE STUDIES

### Aircraft pitch control with PI controller

Aircraft motion governing equation, this paper will not spend time to deduce the pitch control systems transfer function,but gives it directly according to paper[9],[10],[11] $\theta$ and $\delta_e$ represents the aircrafts

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pitch angle and elevator deflection angle. The transfer function is given

$$\frac{\Delta\theta(S)}{\Delta\delta_e(S)} = \frac{1.151S + 0.1774}{S^3 + 0.739S^2 + 0.921S}$$

$$G_{actuator}(s) = \frac{60}{s+60}$$

substitute $S = j\omega$ then equation becomes

$$\frac{\Delta\theta(jw)}{\Delta\delta_e(jw)} = \frac{jw(1.151) + 0.1774}{-0.739w^2 + j[0.921w - w^3]}$$

$$G_{actual}(jw) = \frac{60}{jw+60}$$

$$r = 10.644$$
$$s = 10.644\omega \qquad (16)$$
$$u = 69.06\omega$$
$$x = -60.739\omega^4 + 55.26\omega^2$$
$$y = -\omega^5 + 45.261\omega^2$$

By substituting the p, q, ,r, s, x and y in Eq.( 2.7 and 2.8) equations, then we get $K_p$and $K_i$.

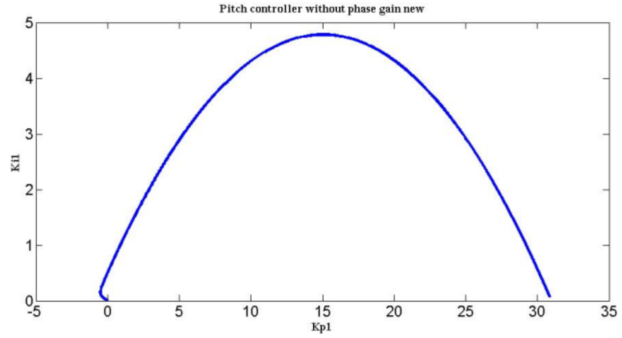


Figure 3 Air craft pitch control response without phase and gain

### 3.2 pitch control for specified phase and gain margin

$$Q(s) = -69.06\omega^2 + a\cos\phi + 10.644\omega$$

$$R(s) = 10.644a\cos\phi + 69.06\omega a\sin\phi$$

$$S(s) = 10.644\omega a\cos\phi + 69.06\omega^2 a\ \sin\phi$$

$$U(s) = 69.06\omega a\cos\phi - 10.644a\sin\phi$$

$$X(s) = -60.739\omega^4 + 55.26\omega^2$$

$$Y(s) = -\omega^5 + 45.261\omega^2$$

- A1=1, Ø1=0
- A2=2, Ø2 = 30
- A3=3, Ø3=45

By substituting A by a1 and Ø by Ø1 in p,q,r,s,x and y.similarly for remaining gain and phases in Eq.(2.7 and 2.8)then we get $K_p$ and $K_i$.

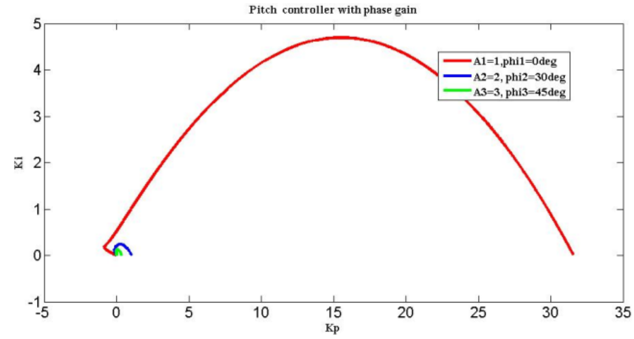


Figure 4 Air craft pitch control response for specified Gain and Phase

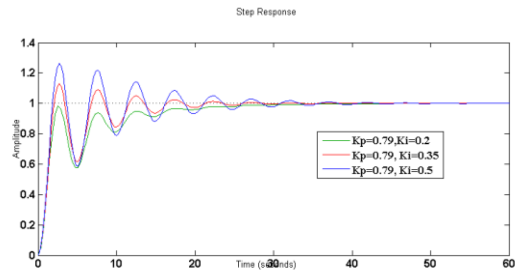


Figure 5:Time response G(s) of pitch control (kp=0.79,ki=0.20 and kp=0.79 ,ki=0.35 and kp=0.79,ki=0.50)

By decreasing the $K_i$ and maintaining $K_p$ at constant value the step response of the system is improved and it is clear from the figure 5. From the figure 6 it is obvious that by increasing $K_p$ and decreasing $K_i$ step response of the system is improved.

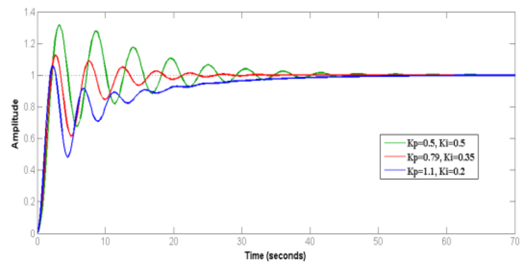


Figure 6:Time response G(s) of pitch control (kp=0.50,ki=0.50and kp=0.79,ki=0.35 and kp=1.10,ki=0.20)

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## CONCLUSION

A method to find the PI controller parameters with and without time delay has been developed for the various applications are developed. To study the effectiveness of the method, studies have been carried out by considering the Vehicle suspension system and Air craft pitch control applications. The method is based on the plotting of the stability boundary locus in the ($k_p$, $k_i$) plane and then computing the stabilizing values of the parameters of PI controller. The method does not need the use of Pade approximation and linear programming to solve set of inequalities. The method has several important advantages over existing results obtained in this direction. Beyond the stabilisation, the method is used to shift all the poles to a shifted half plane that guarantees a settling time of response. Computation of stabilising PI controllers which achieve user specified gain and phase margins are also studied. The method gives the range of gains for which the system is stable which will be useful for the implementation of the method in real-time.

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