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A Modified Mixed Stable Method Using Model Order Reduction and Design

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ABSTRACT:
The present paper deals with a approximating method for large time – delays of multi-input multi-output (MIMO) dynamical systems. Time delay terms of the state space equations are described by delay matrix in the complex domain. A mixed model reduction method of matrix Pade-type-Routh model for the multivariable linear systems was presented. Matrix Pade-type Routh model approximation can largely reduce the instability and the overshoot, so the fast response property is improved. Simulation results of the proposed method are presented to illustrate the correctness and effectively.

Keywords: ***multi-input multi-output Systems; Time-Delays; matrix Pade-type model reduction, Routh table.***

INTRODUCTION

A time delay in input-output relations is a common property of many industrial processes control [1], [2], such as thermo technical processes, chemical processes etc. The effects of time delay are essential. Take a freeze dryer for example, the temperature control system is a first order large inertia system produce dynamic temperature fluctuations, which lead the freeze dried products cannot fulfil the high quality demand. The time-delay property should not be neglected, that when unknown greatly complicates the control problem. In the analysis of a high degree multivariable system, it is often necessary to compute a lower degree model so that it may be used for a analogue or digital simulation of the system. The denominator polynomial of the reduced model is obtained from the Routh table and its numerator matrix polynomial is obtained by the matrix Pade-type Routh Model [6],[7].However, majorities of these ways engage in the analysis of single time-delay variable. Pade-type Routh model is popular method to approximate a scalar pure delay exponential function $e^{-\tau s}$. In this paper, the multi-input multi-output multivariable matrix Pade-type approximation, the basic concept is defined and applied to the state-space approximation problem of multivariable linear systems.

This paper has five sections, section II states matrix Pade-type-Routh model reduction method. Section III explains the state equation of MIMO delay system. Section IV presents two simulation examples with different large time delay based on the proposed method, the step responses are plotted. Section V gives the conclusion.

MATRIX PADE-TYPE-ROUTH MODEL REDUCTION METHOD

Let the transfer function of a higher order system be represented by [6], [7]

$$G(s)=\frac{D_0+D_1 s+\ldots+D_{k-1}s^{k-1}}{e_0+e_1 s+\ldots+e_k s^k}=\frac{D(s)}{E(s)}, \qquad (1)$$

Where D_i, i=0,1,…, k-1 are constant l x r matrices, and e_i, i=0,1,…, k are scalar constants. G (s) can be expanded into a power series of the from

$$G(s)=C_0+C_1 s+C_2 s^2+\ldots \qquad (2)$$

Where the C_i, i=0,1,…., are l x r constant matrices which satisfy the relation

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$$C_0 = \frac{1}{e_0} D_0,$$
$$C_i = \frac{1}{e_0}[D_i - \sum_{j=0}^{i-1} e_{i-j} C_j], i = 1,2..... \quad (3)$$

Thus using Eq. (3) the matrix transfer function may be expanded into a power series.

Assume that the reduced model R (s) of order n is required, and let it be of the form

$$R(s) = \frac{D_n(s)}{E_n(s)} = \frac{A_0 + A_1 s + ... + A_{n-1} s^{n-1}}{b_0 + b_1 s + ... + b_{n-1} s^{n-1} + b_n s^n}, \quad (4)$$

here the A_i, i=0,1,…, n -1 are constant l x r matrices, and b_i, i=0,1,…., n are scalar constants.

Algorithm 1

Step 1 The denominator E_n (s) of reduced model transfer function can be constructed from the Routh Stability array of the denominator of the system transfer function as follows.

The Routh stability array is formed by the following

$$b_{i,j} = b_{i-2,j+1} - \frac{b_{i-2,1} b_{i-1,j+1}}{b_{i-1,1}}, \quad (5)$$

$$where\, i \geq 3 and 1 \leq j \leq \left[\frac{(k-i+3)}{2}\right]$$

The Routh table for the denominator of the system transfer function is given as

$$\begin{array}{lllll} b_{11} = e_k & b_{12} = e_{k-2} & b_{13} = e_{k-4} & b_{14} = e_{k-6} & \dots \\ b_{21} = e_{k-1} & b_{22} = e_{k-3} & b_{23} = e_{k-5} & b_{24} = e_{k-7} & \dots \\ b_{31} & b_{32} & b_{33} & ..... & \\ ..... & & & & \\ b_{k-1,1} & b_{k-1,2} & & & \\ b_{k,1} & & & & \\ b_{k+1,1} & & & & \end{array} \quad (6)$$

E_n (s) may be easily constructed from the (k+1-n)-th and (k+2-n)-th and (k+2-n)-th rows of the above to give

$$E_n(s) = \sum_{j=0}^{n} b_j s^j$$
$$= b_{k+1-n.1} s^n + b_{k+2-n.1} s^{n-1} + b_{k+1-n.2} s^{n-2} + ... \quad (7)$$

Step 2 The numerator D_n (s) of reduced model transfer function by (5) and (6) can be obtained from

$$D_n(s) = s^{n-1} \phi\left(\frac{\tilde{E}_n(x) - \tilde{E}_n(s^{-1})}{x - s^{-1}}\right), \quad (8)$$
$$where\, \tilde{E}_n(s) = s^n E_n(s^{-1}).$$

Thus the reduced model transfer function is given by

$$R(s) = \frac{D_n(s)}{E_n(s)} \text{ s}$$

STATE EQUATION OF MIMO DELAY SYSTEM

Consider a MIMO continuous-time system with delays

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix} \begin{bmatrix} u_1(t-\tau_1) \\ u_2(t-\tau_2) \\ \vdots \\ u_m(t-\tau_m) \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \quad (10)$$

$$\dot{x} = Ax + Bu \quad (11)$$

$$y = Cx \quad (12)$$

Where

$x \in R^n, u \in R^m, y \in R^1, and\, A \in R^{nxn}, B \in R^{mxm}, C \in R^{mxn} are$ the situation of input and output vectors, respectively. Laplace transform of Eq. (11) and Eq. (12) respectively, then the transform function matrix of the MIMO system with delays can be obtained

$$\begin{aligned} Y(s) &= C(sI - A)^{-1} B \tau(s) U(s) \\ &= G_1(s) G_2(s) U(s) \\ &= G(s) U(s) \end{aligned} \quad (13)$$

Where G_1(s), G_2(s), are without and with time delay parts of MIMO system G(s), τ (s) is pure delays diagonal matrix which is given by

$$G_2(s) = \tau(s) = \begin{bmatrix} e^{-\tau_1 s} & 0 & \dots & 0 \\ 0 & e^{-\tau_2 s} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & e^{-\tau_m s} \end{bmatrix} \quad (14)$$

Thus the time delay is represented in transfer function form as :

$$e^{-\tau s} = \frac{2 - \tau s}{2 + \tau} \quad (15)$$

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SIMULATION EXAMPLE

Consider MIMO continuous-time system with delays

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}2&1&-8\\3&4&-4\\0&1&-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}4&5&6\\1&0&1\\3&1&1\end{bmatrix}\begin{bmatrix}u(t-32)\\u(t-100)\\u(t-800)\end{bmatrix}$$

$$\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

Input time delays are $\tau_1 = 32\,s$, $\tau_2 = 100\,s$, $\tau_3 = 800\,s$, respectively.

$$Y(s) = C\,(sI-A)^{-1}\,B\,\tau(s)\,U(s)$$

$$\frac{Y(s)}{U(s)} = G_1(s)G_2(s)$$

Where G_1 (s) is linear MIMO System

G_2 (s) is Purely time delay

$$G_2(s)=\tau(s)$$

Dut to pure delays component $G_2(s)$ is a diagonal matrix similarity transformation approach is used to obtain the decoupled state space equation, such that each output is corresponding to one input. Fig.1, Fig.2 and Fig.3 gives step response .

$$\tau(s)=\begin{bmatrix}e^{-32s}&0&0\\0&e^{-100s}&0\\0&0&e^{-800s}\end{bmatrix}$$

$$G_1(s)=\frac{\begin{bmatrix}s^2-9s+6 & 2s^2+6s+28 & -8(1+s)\\ s^2-20s-9 & 6-12s & s^2-7s-8\\ 2s^2+s+1 & s^2+2 & s^2+s\end{bmatrix}}{s^3+7s^2+14s+8}$$

By applying pade – type Routh model the order reduced system transfer function is obtained as follows :

Reduced order denominator (by applying Routh table):

s^3	1	14
s^2	7	8
s^1	12.85	0
s^0	8	

$$E_2(s) = 7s^2+12.85s+8$$

Reduced order numerator (by applying pade –type method):

$$C_0=\frac{1}{e_0}D_0=\begin{bmatrix}0.75&3.5&-1\\-1.125&0.75&-1\\0.125&0.25&0\end{bmatrix}$$

$$C_1=\frac{1}{e_0}[D_1-e_1C_0]=\begin{bmatrix}-2.437&-5.375&0.75\\0.531&-2.8125&0.875\\0.093&-0.437&0.125\end{bmatrix}$$

$$D_0=e_0\,c_0=\begin{bmatrix}6&28&-8\\-9&6&-8\\1&2&0\end{bmatrix}$$

$$D_1=e_0c_1+e_1c_0=\begin{bmatrix}-9.851&1.975&-6.85\\-10.208&-12.862&-5.85\\2.35&-0.283&1\end{bmatrix}$$

Thus the reduced order transfer function is

$$R_2(s)=\frac{D(s)}{E_2(s)}$$

$$\therefore R_2(s)=\frac{\begin{bmatrix}6-9.851s & 28+1.98s & -8-6.85s\\ -9-10.2s & 6-12.86s & -8-5.85s\\ 1+2.35s & 2-0.283s & s\end{bmatrix}}{7s^2+12.85s+8}$$

By the addition of time delay to the original linear transfer function is

$$T(s)=\frac{Y(s)}{U(s)}=G_1(s)\,G_2(s)$$

$$T(s)=\frac{\begin{bmatrix}-s^3+8.9s^2-16s+0.03 & -s^3-5.9s^2+28s-0.05 & 8s^2+7.9s-0.01\\ -s^3+20s^2+8.9s-0.01 & 12s^2-6s+0.01 & -s^3+7s^2+7.9s-0.01\\ -2s^3-7s^2-s+0.002 & -s^3+0.002s^2-2s+0.004 & -s^3-s^2+0.002s\end{bmatrix}}{s^4+7s^3+14s^2+8s+0.016}$$

Reduced order denominator (by applying Routh Table):

$$E_2'(s)=12.67s^2+8.019s+0.016$$

Reduced order numerator (by applying Pade-type method) :

$$D'(s)=\begin{bmatrix}0.03-16s & -0.05+28.04s & -0.01+8s\\ -0.01+7.9s & 0.012+6s & -0.016+8s\\ 0.002-s & 0.004+2.4s & 0.002s\end{bmatrix}$$

Thus the reduced order transfer function with time delay is

$$R_2'(s)=\frac{\begin{bmatrix}0.03-16s & -0.05+28.04s & -0.01+8s\\ -0.01+7.9s & 0.012+6s & -0.016+8s\\ 0.002-s & 0.004+2.4s & 0.002s\end{bmatrix}}{12.67s^2+8.019s+0.016}$$

The simulation results for the original and reduced order systems can be seen from Fig.1, Fig.2 and Fig.3. These are the step responses with time delay for the original transfer function.

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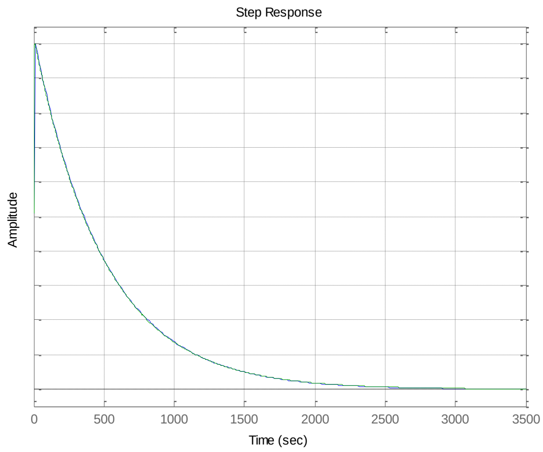

Fig.1: Step response of first output with time delay.

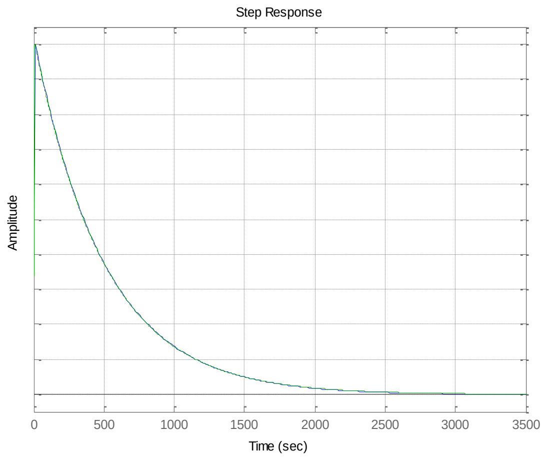

Fig.2:.Step response of second output with time delay.

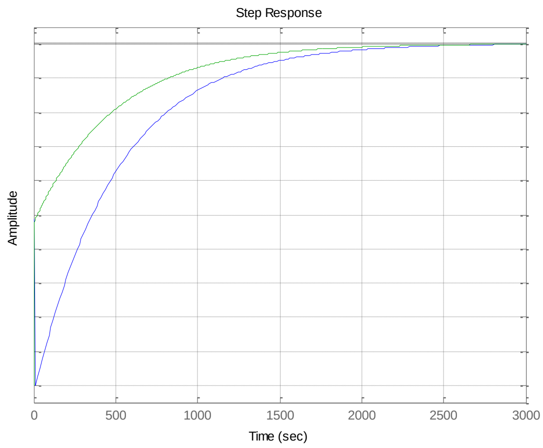

Fig3. Step response of third output with time delay.

And we can observe the step responses for original and reduced order system without time delay in Fig.4, Fig.5 and Fig.6.

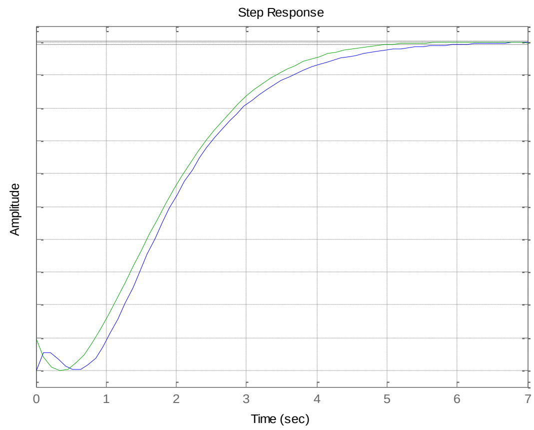

Fig4.Step response of first output without time delay

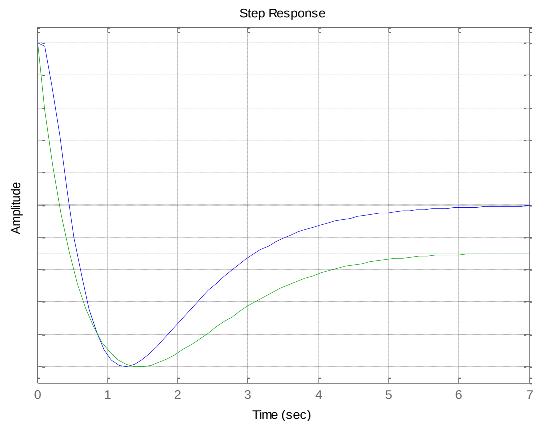

Fig5. Step response of second output without time delay

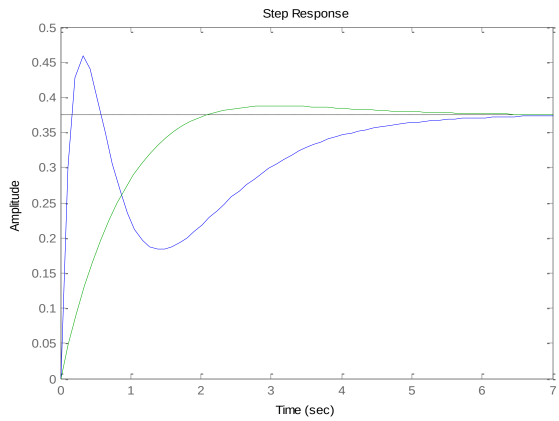

Fig6.Step response of third output without time delay

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IV. CONCLUSION

In this paper, we observed step response of both original and reduced orders systems with and without time delay. A multivariable matrix Pade – type Routh Model for approximating the mulitinput- multioutput (MIMO) large time delays control system is presented. The proposed method is based on the right – coprime matrix Pade –Routh model, decoupling the MIMO state space equation, and estimating the minimum variance error to ensure the stability respectively. The method is simple and can be applied to practical control engineering.

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