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Methodology for Computation of Stability Regions using PI and PID Controllers

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ABSTRACT

In this paper the methodology for stabilizing PI controllers for third order systems are obtained using Boundary locus and Kronecker summation method. For stabilizing PID controllers for higher order systems are obtained using σ-Hurwitz stability criterion method. Stabilizing PI controllers in the (k_p, k_i) plane and PID in the (k_p, k_i, k_d) controllers in the guarantees the stability of a feedback system The Kronecker summation method needs the explicit equation in terms of controller parameters that defining the stability boundary in parameter space. The explicit expression needs the characteristic equation. Using this explicit expression, auxiliary characteristic equation is formulated using Kronecker summation operation. This auxiliary characteristic equation defines the stability boundary. The stabilizing region obtained using this method is compared with boundary locus method. The σ-Hurwitz stability criterion method presented to its require sweeping over the parameters are required to find stabilizing set of PID controller. The third and higher order systems are considered to show the effectiveness of these methods and are simulated using Matlab.

Keywords— Boundary locus method, Kronecker Summation method, σ-Hurwitz stability criterion PI controllers, PID Controllers, Stability regions.

1. INTRODUCTION

There have been great amount of research work on the tuning of PI controllers. In this paper, three approaches are given for the computation of stabilizing PI controllers in the parameter (k_p, k_i) plane and PID Controllers in the (k_p, k_i, k_d) plane. The result is used to obtain the stability boundary locus over a possible smaller range of frequency. Thus a very fast way of calculating the stabilizing values of PI controllers for a SISO (single input single output) control system is given. The calculation of robustly stabilizing controllers can be done using the stability boundary locus [1] or alternatively with Kronecker summation method [3]. These are array of techniques for the computation of stabilizing PI controllers. In Kronecker summation method the stability region is found by using Kronecker summation of two matrices. The novel approach makes use of the extraordinary feature of the Kronecker summation operation and explicit equation is obtained as the function of PI controllers which lie on the boundary of stability region must satisfy.

The set of controllers of a given structure that stabilizes the closed loop is of fundamental importance since every design must belong to the set and any performance specifications that are imposed must be achieved over this set. So, this setis known as Stability Set denoted by $\mathbf{S^0}$:(δ(S, K_p K_i K_d)).The three dimensional set S^o is simply described but not necessarily simple to calculate.

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This is new method for the calculation of all stabilizing PI controllers is given. The Basically we use Routh-Hurwitz criterion which will be very difficult due to formation of inequalities. Therefore to simplify the process we use a revolutionary method known as Signature Method. proposed method is based on plotting the stability boundary locus in the (kp, Ki)-plane and then computing the stabilizing values of the parameters of a PI, PID controller. The technique presented to require sweeping over the parameters and also it's need linear programming to solve a set of inequalities. Thus it offers several important advantages over existing results obtained in this direction. Beyond stabilization, the method is used to shift all poles to a shifted half plane that guarantees a specified settling time of response. It is shown via an example that the stabilizing region in the (kp, ki)-plane is always a convex set. The limiting values of a PID controller which stabilize a given system are obtained in the convex set of(kp, ki)-plane, and (ki, kd)plane and 3-D view of stabilizing sets of (kp, ki, kd) observed in the simulation results and.

Once the stabilizing region is obtained, the stability of the third order systems are verified in simulation using an arbitrary point. This paper is organized as follows: - The proposed methods are described in section 2,3 and 4. Examples shown are described in section 5. Conclusion is given in section 6.

2. BOUNDARY LOCUS METHOD.

Computation of Stability regions for PI Controllers:

Assume the classical and very well known feedback control system shown in fig.1, where $C(s)$ and $G(s)$ represent controller and plant respectively. The block diagram of this system is shown in fig.1 and $G(s)$ is defined using equation 1 and $C(s)$ is a PI controller defined by using equation 2.

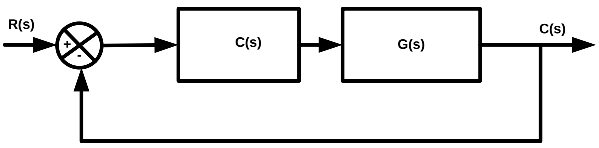

Fig.1 A SISO control system.

Consider a single input, single output (SISO) control system of Fig.1 where

$$G(s) = \frac{N(s)}{D(s)} \qquad (1)$$

is the plant to be controlled and C(s) is a PI controller defined using equation (2).

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \qquad (2)$$

The problem is to compute the parameters of the PI controller of Eq.(2) that stabilize the system of Fig.1.Decomposing the numerator and denominator of polynomials of Eq.(1) into their even and odd parts and substituting $s = j\omega$,gives

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega N_0(-\omega^2)}{D_e(-\omega^2) + j\omega D_0(-\omega^2)} \qquad (3)$$

The closed loop characteristic equation of the system can be defined using (4).

1+G(s)C(s) =0 (4)

$$1 + \left(\frac{N_e + j\omega N_0}{D_e + j\omega D_0}\right)\left(\frac{k_p j\omega + k_i}{j\omega}\right) = 0$$

On simplifying the above equation leads (5).

$$D_e j\omega + j^2\omega^2 D_0 + k_p N_e j\omega + k_p j^2\omega^2 N_0 + k_i N_e + k_i j\omega N_0 = 0 \qquad (5)$$

Substituting $j^2 = -1$ in (5), (6) is obtained.

$$\begin{aligned} &D_e j\omega - \omega^2 D_0 + k_p N_e j\omega - k_p\omega^2 N_0 + k_i N_e + k_i j\omega N_0 = 0 \\ &-k_p\omega^2 N_0 + k_i N_e - \omega^2 D_0 + j(k_p N_e\omega + k_i N_0\omega + D_e\omega) = 0 \end{aligned} \qquad (6)$$

The closed loop characteristic equation be denoted as $\Delta(s)$

$$\Delta(s) = R_\Delta + jI_\Delta = 0$$

Then, equating the real and imaginary parts of $\Delta(s)$ to zero, the equations (7) is obtained.

$$\begin{aligned} &-k_p\omega^2 N_0 + k_i N_e = \omega^2 D_0 \\ &k_p N_e\omega + k_i N_0\omega = -D_e\omega \end{aligned} \qquad (7)$$

Let

$$\begin{aligned} &Q(\omega) = -\omega^2 N_0,\ R(s) = N_e \\ &S(\omega) = \omega N_e\ ,U(\omega) = \omega N_0 \\ &X(\omega) = \omega^2 D_0\ ,Y(\omega) = -\omega D_e \end{aligned} \qquad (8)$$

Then equation (7) can be written as

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$$\begin{aligned} k_p Q(\omega) + k_i R(\omega) &= X(\omega) \\ k_p S(\omega) + k_i U(\omega) &= Y(\omega) \end{aligned} \qquad (9)$$

Using the equations (7) to (9) k_p and k_i are obtained as

$$k_p = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \qquad (10)$$

And

$$k_I = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \qquad (11)$$

The equations (10) and (11) further simplified as

$$kp = \frac{\omega^2 D_0 N_0 + N_e D_e}{-\omega^2 N^2{}_0 - N^2{}_e} \qquad (12)$$

$$ki = \frac{\omega^2 N_0 D_e - \omega^2 N_e D_0}{-\omega^2 N^2{}_0 - N^2{}_e} \qquad (13)$$

Solving the equations (12) and (13) simultaneously, the stability boundary locus, $l(k_p, k_i, \omega)$, in the (k_p, k_i) plane can be obtained. Choosing a test point from that region (k_p, k_i) the stability of the third–order systems are verified.

3.KRONECKER SUMMATION METHOD

The stabilizing regions are also obtained using a second method Kronecker Summation method. The procedural steps for computation of the stabilizing regions using Kroneckecker summation method for feedback system shown in Fig.(1)., are enumerated as follows.

(i). Let the controlled plant is defined using equation (1).

(ii). The controller is defined using (2).

(iii). The characteristic equation of the closed loop system is defined in equation (14)

$$\begin{aligned} CE(s) &= sD(s) + (k_p s + k_i)N(s) \\ &= f_n(k_p, k_i)s^n + \ldots + f_1(k_p, k_i)s + f_0(k_p, k_i) = 0 \end{aligned} \qquad (14)$$

(iv). Transform equation (14) into phase variable companion form (i.e. differential equation matrix form)

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\vdots$$
$$\dot{x}_n = -\frac{f_0(k_p, k_i)}{f_n(k_p, k_i)} x_1 - \frac{f_1(k_p, k_i)}{f_n(k_p, k_i)} x_2 - \ldots - \frac{f_{n-1}(k_p, k_i)}{f_n(k_p, k_i)} x_n$$

which can be written as

$$\dot{X} = MX \qquad (15)$$

where $X = [x_1 \quad x_2 \quad \cdots\cdots \quad x_n]^T$

(v). Obtain the system matrix "M" in phase variable companion form.

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\frac{f_0(k_p, k_i)}{f_n(k_p, k_i)} & -\frac{f_1(k_p, k_i)}{f_n(k_p, k_i)} & -\frac{f_2(k_p, k_i)}{f_n(k_p, k_i)} & \cdots & \cdots & -\frac{f_{n-1}(k_p, k_i)}{f_n(k_p, k_i)} \end{bmatrix}$$

(vi). The relation between (14) and (15) is given as

$$CE(s) = f_n(k_p, k_i)\det(sI - M) = 0 \qquad (16)$$

where s is the root of eq.(14) as well as eigenvalue of matrix M. M is a constant matrix, the complex conjugates of s also satisfies eq.(16) i.e.,

$$\det(sI^* - M) = 0 \qquad (17)$$

(vii). The stability boundary in (k_p, k_i) plane is defined using the determinant of Kronecker summation [6]. Define the auxiliary characteristic equation as in (18)

$$ACE = \det[M \oplus M] = 0 \qquad (18)$$

(viii). After obtaining the matrix M, derive the auxiliary characteristic equation as mentioned above, for some values of 'ω', k_p and k_i are identified such that selected pair of (k_p, k_i) leads to $\det[M \oplus M] = 0$ as both $s = j\omega$ and $s^* = -j\omega$ are the roots of characteristic equation and sum of the eigen values of $s = j\omega$ and $s^* = -j\omega$ is zero. Finally equation (18) defines the boundary in (k_p, k_i) plane that divides this plane into stable and unstable regions.

4. σ -HURWITZ STABILITY METHOD

Consider the plant, with rational transfer function

$$P(s) = \frac{N(S)}{D(S)};$$

With the PID feedback controller

$C(s) = k_p s + k_i + k_d s^2/s(1+sT)$, T>0 ……….(19)

The closed loop characteristics polynomial is $\delta(s) = S^*D(s)(1+sT) + (k_p s + k_i + k_d s^2)^*N(s)$ ………..(20)
we form the new polynomial $v(s) = \delta(s)^*N(-s)$……..(21)
note that the even odd decomposition of v(s) is of the form $v(s) = v_{even}(s^2(k_i, k_d)) + S^*v_{odd}(s^2(k_p))$ the polynomial v(s) exhibits the parameter separation property, namely,

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that k_p appears only in the odd part and (k_i, k_d) only in the even part. By sweeping over the values of (kp, ki) from stable boundary locus (kp vs ki) plane space region. after fixing range of $K_p = K_p^*$,There exists sets of linear inequalities in terms of (K_i, K_d) to satisfying the signature condition.

Signature(V) = n-m+1+2Z$^+$;
This will facilitate the computation of the stabilizing set using signature concepts.
N(s),D(s) are the numerator, denominator of polynomial degrees 'm', 'n' of Plant P(s) respectively.
The closed loop system is stable if and only if, $\sigma(v)$= n-m+2+2z ……….. …………….(22)
closed loop stability is equivalent to the requirement that then n+2 zeros of $\delta(s)$ lie in the open LHP .
this is equivalent to $\sigma(\delta)$=n+2
and to $\sigma(v)$=n+2+z$^+$-z$^-$
n+2+z$^+$-(m-z$^+$) =(n-m)+2+2z$^+$
z^-, z^+ are denote the no. of roots on the S-plane LHP, RHP of numerator N(s).

Sgn[q(w(0$^+$), K_p)] = j;

j = Sgn[$V_{odd}(0^+, K_p)$; ……(23)

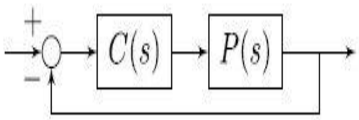
n-m+1+2Z$^+$ = j(i_0 + $2\sum_{t=1}^{l-1}$ $(-1)^t i_t$) ,if n + m is odd…(24)

= **j**(i_0 + $2\sum_{t=1}^{l-1}$ $(-1)^t i_t + (-1)^l i_l$), if n + m is even ---.(25)

Based on this, we can develop the following procedure to calculate S^0: the stabilizing set S^0:($d(s, k_p, k_i, k_d)$) getting from 3-D graph results.

4a. PROCEDURE TO DO SIGNATURE METHOD:

consider unity feedback loop with PID controller c(s) and plant P(s)



$$P(s)= \frac{N(S)}{D(S)}; \quad C(s)= K_p + K_d S + \frac{k_i}{S};$$

N(s), D(s) are the numerator, denominator of polynomial degrees 'm', 'n' of Plant P(s) respectively.
closed loop characteristic equation of polynomial is

$$d(s, k_p, k_i, k_d) = (1+[P(s)*C(s)])$$

$$d(s, k_p, k_i, k_d) = 1+[K_p + K_d S + \frac{k_i}{S}][\frac{N(S)}{D(S)}]$$

$$d(s, k_p, k_i, k_d) = S\ D(s) + [K_i + K_d\, s^2]N(s) +[K_p S][N(s)]. \ \ldots\ldots (9)$$

we Assume that N(s) and D(s) are co-prime, that is, they have no common roots and N(0) $\neq$ 0.
closed loop characteristic equation of polynomial is

$$\delta(s, k_p, k_i, k_d) = S\ D(s) +[K_p S + K_i + K_d\, s^2]N(s).$$

Assume N(s) has no roots on the imaginary axis.

$$v(s) := \delta(S, k_p, k_i, k_d)N(-s).$$

The main motto of the V(s) is the achieve a separation of the gains into the Real and Imaginary parts and also the divide into the Even and odd parts of S.
normally S= j w into the V(S) and then divides into k_p, k_i, k_d into the real and imaginary parts and also the Even and odd parts to be divided. If $d(s, k_p, k_i, k_d)$ is multiplied with the N(-s) ,then only divided properly even part section consists k_i, k_d and odd part section includes K_p, otherwise both are not separated. so that

Even part of V(w) is(k_i, k_d) and odd part is K_p
V(w) = P(w) + j q(w);
V(w) = Even part + odd part (or) real part +imaginary part;
For PI controller (using stability boundary locus method) Even part and odd parts are equal to zero, then

$$V(w) = [P_1(w)+K_iP_2(w)] + j[\, q_1(w)+ K_p q_2(w)].$$

$$K_i = -\frac{P1(w)}{P2(w)}; \quad K_p = -\frac{q1(w)}{q2(w)}; \ \ldots.(9)$$

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For PID controller:

$V(w)= [P_1(w)+ \{k_i - k_d w^2\}P_2(w)]+ j\{q_1(w)+ k_p q_2(w)\}$; ….(10)

In v(s), k_p only appears in the odd degree terms of **S**, while k_i and k_d only appears in the even degree terms of **S.** Now equate the odd degree of or imaginary part of S is equal to zero and odd part of k_p terms equal to zero and then by using the (i) R-H criteria, Basically we use Routh-Hurwitz criterion which will be very difficult due to formation of inequalities. Therefore to simplify the process we use a revolutionary method known as Signature Method.

(ii) stability boundary locus method or above PI controller technique ,To find out the range of k_p.

for fixed range of $k_p = K_p^*$

There exists sets of linear inequalities in terms of (k_i, k_d) to satisfying the signature condition.

Signature(V) = n-m+1+2Z$^+$;

z^-, z^+ are denote the no. of roots on the S-plane LHP, RHP of numerator N(s).

The range of K_p such that q(w) is the odd part of V(w) and roots of q(w) consider only real and positive roots(w_0,w_1,w_2,w_3..),distinct, finite zeros with odd multiplicity was determined by K_p range.

sgn[$q(w(0^+), K_p)$] = j;

$j = \mathrm{sgn}[V_{odd}(0^+, K_p)]$;

$I_1 = \{i_0, i_1, i_2, i_3....\}$; $I_2= \{i_0, i_1, i_2, i_3....\}$; $I_3 = \{i_0, i_1, i_2, i_3....\}$.

I_1, I_2, I_3 ......are the admissible string sets and must satisfies the signature of V.

σ(v)-signature $(v)= n+1-m+2z^+$.

$$n-m+1+2Z^+ = j(i_0 + 2\sum_{t=1}^{l-1}(-1)^t i_t)\text{ ,if } n + m \text{ is odd,}$$

$$= j(i_0 + 2\sum_{t=1}^{l-1}(-1)^t i_t + (-1)^l i_l)\text{, if } n + m \text{ is even}$$

.by sweeping over K_p values into the fixed range and then string sets follows that the stabilizing (k_i, k_d) must satisfies the string of inequalities:

$p1(w_0) +(k_i - k_d W_0^2)P_2(w_0) < 0$

$p1(w_1) +(k_i - k_d W_1^2)P_2(w_1) > 0$

$p1(w_2) +(k_i - k_d W_2^2)P_2(w_2) > 0$

Substituting for w_0,w_1,w_2,w_3 in the above expressions, we obtain set of values of (k_i, k_d) form of equations solved by linear programming and denoted by sets $S1,S2,S3,S4,S5….S_X$**.** by sweeping over different K_p values within the interval and repeating above procedure at each stage, we can generate the set of stabilizing (k_p, k_i, k_d) values.

To show the effectiveness of these stabilizing PI controller design methods third-order systems are considered for simulation in MATLAB.

Example 1:

Case (a) : Boundary locus method:

Consider the third-order system [2] described using a transfer function $G_1(s)$. The stabilizing PI controller parameters are obtained using Boundary Locus method the detailed step-by step computation procedure is given as below.

$$G_1(s) = \frac{2.925}{175.5s^3 + 137.5s^3 + 22 + 1}$$

Substituting $s = j\omega$

$$G_1(j\omega) = \frac{2.925}{(-137.5\omega^2 + 1) + j\omega(-175.5\omega^2 + 22)}$$

Dividing the numerator and denominator into even and odd parts, and represented using $N_{01}, N_{e1}, D_{01}, D_{e1}$.

$N_{01} = 0$, $N_{e1} = 2.925$

$D_{01} = 22 - 175.5\omega^2$, $D_{e1} = -137.5\omega^2 + 1$

K_{p1} and K_{i1} are obtained using equations (12) and (13)

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$k_{p1} = 47\omega^2 - 0.341$ $k_{i1} = 7.521\omega^2 - 60\omega^4$

K_{p1} and K_{i1} both are functions of ω ,and by choosing a suitable range of ω the k_p and k_i values are plotted and the stabilizing PI controller region is shown in Fig.(2).

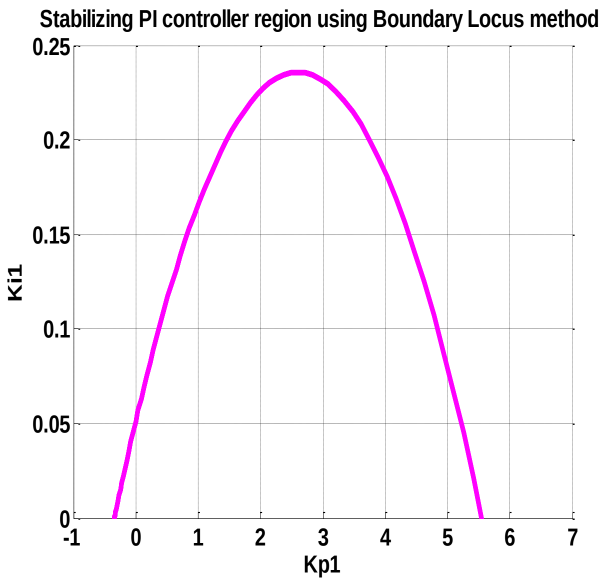

Fig.(2). Stabilizing PI controller region for $G_1(s)$ using Boundary Locus method.

Case (b) : Kronecker Summation method:

The detailed step-by step computation procedure for stabilizing PI controller parameters using Kronecker summation method is given as below. The characteristic equation of the system whose block diagram is shown in Fig.(1) is $1+G_1(s)C(s)=0$. Substituting the $G_1(s)$ and $C(s)$ in the equation results,

$$1+\left(\frac{2.925}{175.5s^3+137.5s^3+22+1}\right)\left(\frac{k_p s+k_i}{s}\right)=0$$

On simplification

$$175.5s^4+137.5s^3+22s^2+(2.925k_p+1)s+2.925k_i=0$$

Comparing the above equation with standarded equation the coefficients are obtained and listed below.

$f_0 = 2.925k_i$, $f_1 = 2.925k_p + 1$

$f_2 = 22$, $f_3 = 137.5$, $f_4 = 175.5$

find $[M \oplus M]$ here ,

By substituting the above values the value of M is obtained.

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-f_o}{f_4} & \frac{-f_1}{f_4} & \frac{-f_2}{f_4} & \frac{-f_3}{f_4} \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.0166k_i & -0.0166k_p-0.005 & -0.125 & -0.783 \end{bmatrix}$$

Apply the Kronecker Summation operation and plot the locus .The stabilizing PI region for the $G_1(s)$ is shown in Fig.(3).

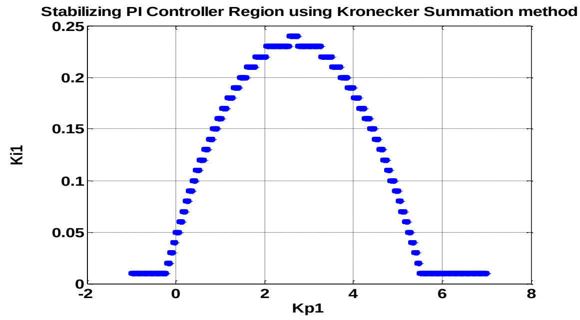

Fig.(3). Stabilizing PI controller region for $G_1(s)$ using Kronecker Summation method.

The third-order system should be stable for any arbitrary (k_p, k_i) point inside the boundary region and should be unstable outside the region. For verifying this fact an arbitrary test point is considered inside and also outside the region and simulation is performed. The time-response of the third-order system with a test point (k_p, k_i) =(1,0.05) which is in the region is simulated with step disturbance of magnitude 0.2 performed at100 seconds and shown in Fig.(4).

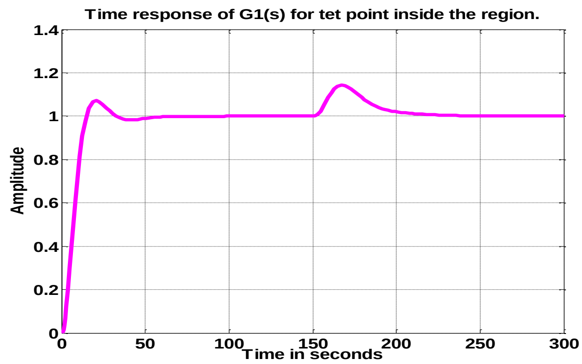

Fig.(4). Time response of $G_1(s)$ with test point inside the region.

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Let us consider a test point (k_p, k_i)=(7,0.2) which is outside the region, the time response is shown in Fig.(5).

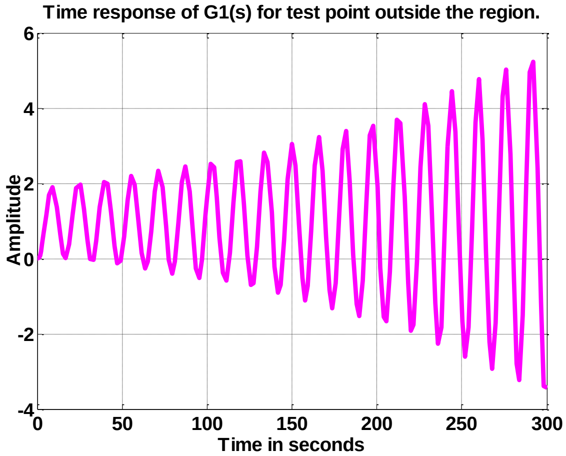

Fig.(5). Time response of $G_1(s)$ with test point outside the region.

Example 2:

Case (a): Boundary locus method:

Consider the third-order system [5] described using a transfer function $G_2(s)$. The stabilizing PI controller parameters are obtained using Boundary Locus method the detailed step-by step computation procedure is given as below.

$$G_2(s) = \frac{5}{s^3 + 2s^2 + 3s + 4}$$

substituting $s = j\omega$

$$G_2(j\omega) = \frac{5}{(4 - 2\omega^2) + j\omega(3 - \omega^2)}$$

Dividing the numerator and denominator into even and odd parts, and represented using $N_{01}, N_{e1}, D_{01}, D_{e1}$.

$$N_{02} = 0, N_{e2} = 5$$

$$D_{02} = 3 - \omega^2, D_{e2} = 4 - 2\omega^2$$

K_{p2} and K_{i2} are obtained using equations (12) and (13)

$$k_{p2} = 0.4\omega^2 - 0.8 \quad k_{i2} = 0.6\omega^2 - 0.2\omega^4$$

K_{p2} and K_{i2} both are functions of ω ,and by choosing a suitable range of ω the kp and ki values are plotted and the stabilizing PI controller region is shown in Fig.(6).

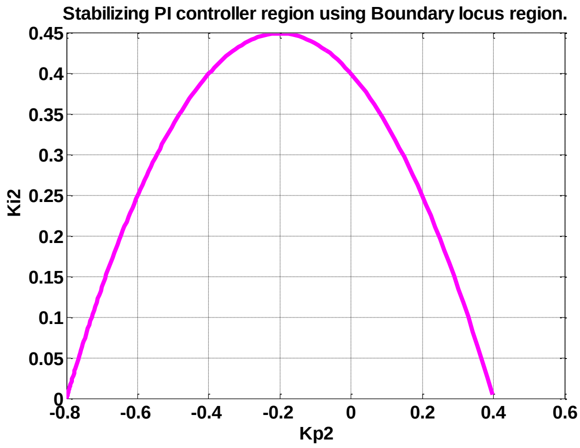

Fig.(6). Stabilizing PI controller region for $G_2(s)$ using Boundary Locus method.

Case (b) : Kronecker Summation method:

The detailed step-by step computation procedure for stabilizing PI controller parameters using Kronecker summation method is given as below. The characteristic equation of the system whose block diagram is shown in Fig.(1) is $1 + G_2(s)C(s) = 0$. Substituting the $G_2(s)$ and $C(s)$ in the equation results,

$$1 + \left(\frac{5}{s^3 + 2s^2 + 3s + 4}\right)\left(\frac{k_p s + k_i}{s}\right) = 0$$

On simplification

$$s^4 + 2s^3 + 3s^2 + (5k_p + 4)s + 5k_i = 0$$

Comparing the above equation with standarded equation the coefficients are obtained and listed below.

$$f_0 = 5k_i,\ f_1 = 5k_p + 4,\ f_2 = 3$$

$$f_3 = 2,\ f_4 = 1$$

find $[M \oplus M]$ here ,

By substituting the above values the value of M is obtained.

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-f_o}{f_4} & \frac{-f_1}{f_4} & \frac{-f_2}{f_4} & \frac{-f_3}{f_4} \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5k_i & -5k_p - 4 & -3 & -2 \end{bmatrix}$$

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Apply the Kronecker Summation operation and plot the locus .The stabilizing PI region for the $G_2(s)$ is shown in Fig.(7).

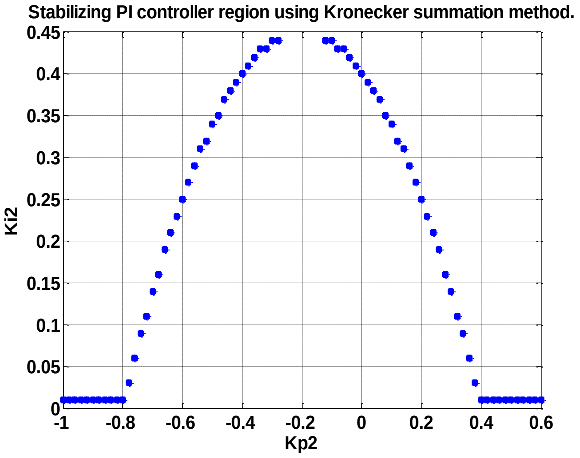

Fig.(7). Stabilizing PI controller region for $G_2(s)$ using Kronecker Summation method.

The third-order system should be stable for any arbitrary $(k_{p,}k_i)$ point inside the boundary region and should be unstable outside the region. For verifying this fact an arbitrary test point is considered inside and also outside the region and simulation is performed. The time-response of the third-order system with a test point $(k_{p,}k_i)$=(0,0.1) which is in the region is simulated with step disturbance of magnitude 0.2 performed at100 seconds and shown in Fig.(8).

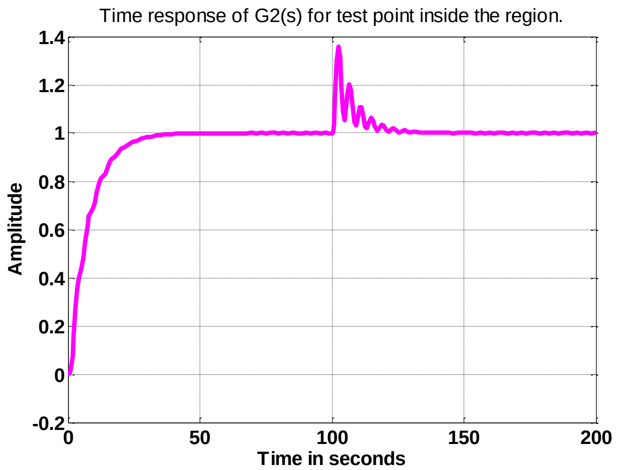

Fig.(8). Time response of $G_2(s)$ with test point inside the region.

Let us consider a test point $(k_{p,}k_i)$=(0,0.6) which is outside the region, the time response is shown in Fig.(9).

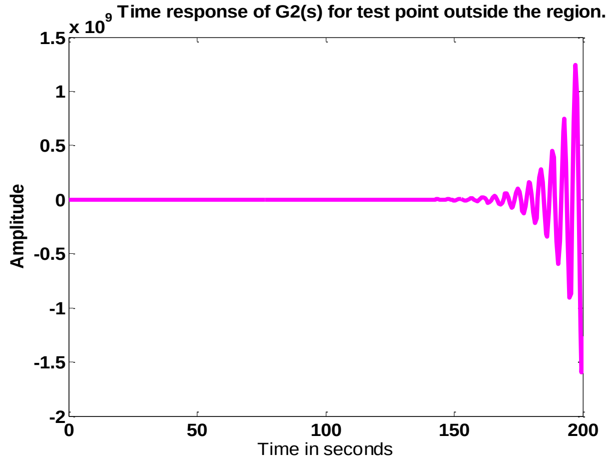

Fig.(9). Time response of $G_2(s)$ with test point outside the region.

(1).Example for Signature Method

Design the problem of determining stabilizing set of PID gains for the plant $P(s)= \frac{N(S)}{D(S)}$;

where $N(s) = s^3-2s^2-s-1$;$D(s) = s^6+2s^5+32s^4+26s^3+65s^2-8s+1$

We use the PID controller with T=0. The closed loop characteristic polynomial is

δ (s, kp, ki, kd) = $s*D(s)+(k_i+k_ds^2)\,N(s)+K_P*S*N(s)$

Here n=6 and m=3

$N_{even}(S^2) = -2S^2 -1$, $N_{odd}(S^2) = S^2 -1$,

$D_{even}(S^2) = S^6 + 32S^4+65S^2 +1$, $D_{Odd}(S^2) = 2S^4+ 26S^2 - 8$

$N(-s) = (-2s^2-1)-s(s^2-1)$

Therefore, we obtain

V(S) = $\delta\,(s, k_p, k_I, k_d)\,N(-s)$

$= \{S^2(-S^8 - 35S^6-87S^4+54S^2+9) +(k_i+k_ds^2)\ (-S^6+ 6S^4+3S^2+1)\}$

$+S*[(-4S^8- 89S^6- 128S^4 - 75S^2 -1) +k_p*(-S^6 + 6S^4 +3S^2 +1)]$

So that

$V\ (j\omega,\ kp,\ ki,\ kd) = [p_1(w)+(k_i-k_d\omega^2)\ \ p_2(\omega)] +j[q_1(\omega)+k_pq_2(\omega)]$;

To get the results, we need to separate the even and odd parts equal to zero

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For PI CONTROLLER $K_i = -\frac{P1(w)}{P2(w)}$; $K_p = -\frac{q1(w)}{q2(w)}$;

For PID CONTROLLER:

$V(w) = [P_1(w) + \{K_i - K_d w^2\} P_2(w)] + j\{q_1(w) + K_p q_2(w)\}$;

Where

$P_1(w) = w^{10} - 35w^8 + 87w^6 + 54w^4 - 9w^2$

$P_2(w) = w^6 + 6w^4 - 3w^2 + 1$

$q_1(w) = -4w^9 + 89w^7 - 128w^5 + 75w^3 - w$

$q_2(w) = w^7 + 6w^5 - 3w^3 + w$

We find that $z^+ = 1$ so that the signature requirement on $v(s)$ for stability is, $\sigma(v) = n-m+1+2z^+ = 6$ Since the degree of $v(s)$ is even, we see from the signature formulas that $q(\omega)$ must have at least two positive real roots of odd multiplicity The range of k_p such that q (ω, kp) has at least 2 real, positive, distinct, finite zeros with odd multiplicities was determined to be (-24.7513, 1) which is the allowable range of k_p. For a fixed $k_p \epsilon$ (-24.7513,1), for instance k_p= -18, we have

$q(\omega, -18) = q_1(\omega) - 18q_2(\omega)$

$= -4\omega^9 + 71\omega^7 - 236\omega^5 + 129\omega^3 - 19\omega$

Then the real, nonnegative, distinct finite zeros of q (ω, -18) with odd multiplicities are

ω_0=o, ω_1=0.5195, ω_2=0.6055, ω_3=1.8804, ω_4=3.6648

Also define $\omega_5 = \infty$.since

Sgn [q (0, -18)] = -1

It follows admissible string I= $\{i_0, i_1, i_2, i_3, i_4, i_5\}$ Must satisfy

$\{i_0 - 2i_1 + 2i_2 - 2i_3 + 2i_4 - i_5\}.(-1) = 6$

Hence the admissible strings are

I_1= {-1, -1, -1, 1, -1, 1};

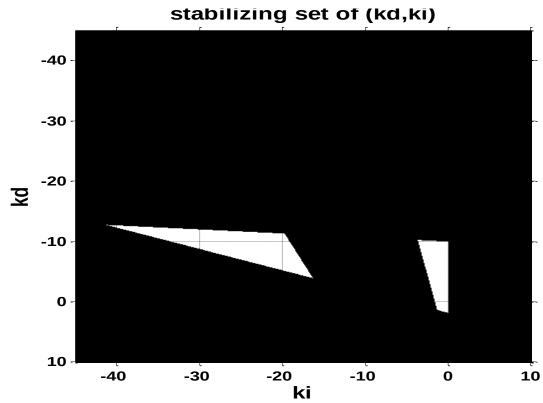
I_2= {-1, 1, 1, 1, -1, 1};

I_3= {-1, 1, -1, -1, -1, 1};

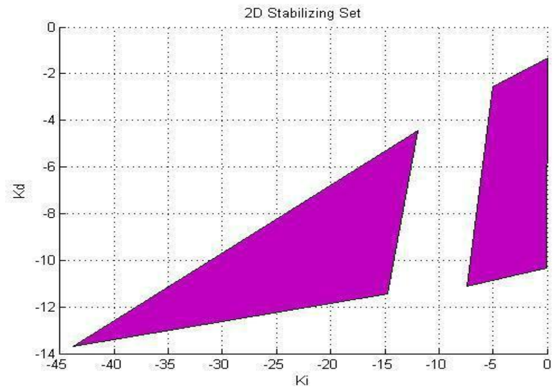
I_4= {-1, 1, -1, 1, 1, 1}

I_5= {1, 1, -1, 1, -1, -1};

For I_1 it follows that stabilizing set () values corresponding to k_p=-18 in fig(10,11)



fig(10) 2-D view of stabilizing set of Ki vs Kd



fig(11) fixed Kp value for varying set of (Ki,Kd)

Must satisfy string k_i, kd of inequalities

$P_1(\omega_0) + (k_i - k_d\omega_0^2)\, p_2(\omega_0) < 0$

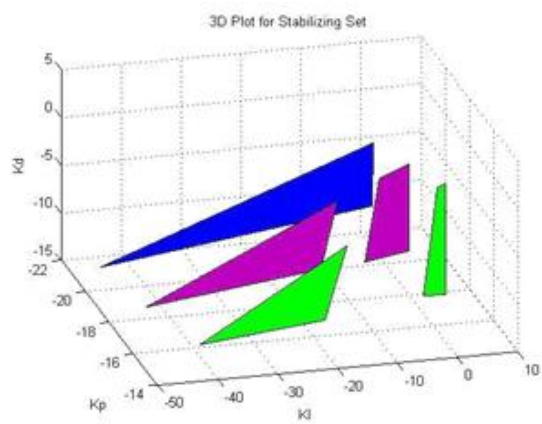
$P_1(\omega_1) + (k_i - k_d\omega_1^2)\, p_2(\omega_0) < 0$

$P_1(\omega_2) + (k_i - k_d\omega_2^2)\, p_2(\omega_0) < 0$

$P_1(\omega_3) + (k_i - k_d\omega_3^2)\, p_2(\omega_0) > 0$

$P_1(\omega_4) + (k_i - k_d\omega_4^2)\, p_2(\omega_0) < 0$

$P_1(\omega_5) + (k_i - k_d\omega_5^2)\, p_2(\omega_0) > 0$



fig(12) 3-D View of stabilizing set

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Substituting for ω_0, ω_1, ω_2, ω_3, ω_4 and ω_5 in the above expressions, we obtain

$k_i<0$
$k_i-0.2699k_d<-4.6836$
$k_i-0.3666k_d<-10.0797$
$k_i-3.5358k_d>3.912$
$k_i-13.5777k_d<140.2055$

the set values of (k_i, kd) for which the above equations hold can be solved by linear programming observed in fig(12) and is denoted by S_1.for I_2, we have

$k_i<0$
$k_i-0.2699k_d<-4.6836$
$k_i-0.3666k_d<-10.0797$
$k_i-3.5358k_d>3.912$
$k_i-13.5777k_d<140.2055$

The set values of (k_i, kd) for which the above equations hold can be solved by linear programming and is denoted by S_2. similarly, we obtain

$S_3 = \varphi$ for I_3
$S_4 = \varphi$ for I_4
$S_5 = \varphi$ for I_5

Then the stabilizing set of (k_i, kd) values when k_p=-18 is given by $S_{(-18)} = \cup_{x=1,2\ldots3} \ldots s_x$
$= s_1 \cup s_2$

From the intersections of plots in space that's 3-dimensional plot fig(12) in which we can select vertex of stabilizing set (K_p, Ki, Kd) values which satisfies the stabilizing criterion.

5.CONCLUSION

This paper deals with the three approaches for computation of stability regions for PI controllers. Stabilizing PI controller parameters are obtained using Boundary locus method, Kronecker Summation method, signature method.. It is observed that both methods are giving identical stabilizing regions.Third order systems are considered for simulation in matlab. Examples are given clearly to find out stability region .

This paper also dealt with an approach has been presented for computation of stabilization of PI,PID controllers using boundary locus and stabilization set of PID gains of (K_p ,K_i, K_d), which can be easily obtained by equating the real and imaginary parts of characteristic equation to zero. The proposed method has further has been used find stabilizing region of PI parameters plant with uncertain parameters and this signature method involves require sweeping over parameters. Also, it needs linear programming to solve set of inequalities used in the signature method for the further solving of region of stabilizing set of PID controller gains in effective method of approach for higher order systems.

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